E C O N O M I C S B U L L E T I N

Endogenous longevity and the value−maximizing firm

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Abstract

We develop a simple analytical framework where the longevity of profit−maximizing firms requires costly resources. We show that a firm's longevity and value are positively related to the firm's pricing power, cash reserves, honesty, and ratio of equity to debt financing.

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1 Introduction

Traditional economic analysis of the firm assumes that firms will make choices to maximize lifetime profits. The focus of theoretical research has mainly been on the production (Tirole, 1988) and financing (Hart, 1995) decisions of firms. Survival actions that increase lifetime profits by altering the firm's lifespan have received less attention. Usually the horizon of the firm is determined by factors extrinsic to the firm such as market organization and legal constraints that affect the probabilities of defaulting or exiting the market. But absent external forces that threaten a firms survival or cause involuntary hardship, there does not exist an internal rationale for the longevity of firms and economists may be missing an essential element of the firm's life cycle. An active empirical literature on the turnover and mobility of firms (Caves, 1998) has uncovered regularities on firm survival and exits that emphasize external forces, but significant heterogeneity across firms suggests other factors may be relevant as well.

We develop a simple model where firms choose production levels and their lifespan to maximize the lifetime present discounted value of profits. We assume that firms increase their life expectancy at a cost, by setting aside longevity-enhancing resources. We find that the longevity and the value of the firm depend positively on the profitability of the firm, cash reserves, and honesty when moral hazard is an issue. The rationale is that more resources can be devoted to longevity. Our model, also, suggests that equity-financing encourages longevity by freeing up cash for survival, while debt-financing discourages longevity by tieing up resources. Thus, longevity considerations break the equivalence of debt and equity for firm valuations as originally posited by Miller and Modigliani (1961).

2 A Simple Model of Firm Longevity

Firms are assumed to maximize the expected present discounted value of profits or the value of the firm. We analyze a very simple stationary environment where firms must set aside resources to increase their longevity. Longevity-enhancing resources can be thought of as all costly activities – such as quality improvements, advertising, lobbying, client service and so on – that help the firm live longer but do not necessarily improve the production process.

We consider a representative firm with an endogenous maximum lifespan of length T. Firms have a finite life expectancy $E(T) \equiv \int_0^T \delta t e^{-\delta t} dt$ that is determined by the instantaneous death probability $e^{-\delta t}$. The death rate δ covers exogenous forces that lead to default and market exits. Because $E(T)$ and T have a one-to-one mapping, we refer to them interchangeably as longevity. Clearly, without the risk of untimely death, $\delta = 0$ and $T = E(T)$.

The total resources spent by the firm to increase their longevity are aggregated into a lump-sum payment M. Since longevity-enhancing resources are a cost of doing business, they must be subtracted from conventional lifetime profits. Assuming these resources affect longevity according to a concave production function $T(M)$, where $T' \geq 0 > T''$, firms can choose M directly and thus determine T indirectly. Because rational firms understand the connection between the intermediate target M and the ultimate target T, they can invert the longevity production function $T(M)$ and choose longevity directly. Inverting $T(M)$ yields $M(T)$, which is convex with $M' \geq 0$ and $M'' > 0$.

The value of the firm for a maximum lifespan of T is

$$V = \int_0^T e^{-(r+\delta)t} \left[p(y)y - c(y)\right] dt - M(T)$$

where $p\left(y\right)$ is the price that can be charged and $c(y)$ is the cost at each production level y. We assume that $p'(y) < 0$ and $c'(y) > 0$. The effective discount rate of $r + \delta$ rises with the real interest rate r and the force of mortality and makes a firm act more impatiently.

We can write the value of the firm more compactly for stationary outcomes. If we define the annuity value of a security paying one dollar every year until T as $A(T) \equiv \int_0^T e^{-(r+\delta)t} dt = \frac{1}{\rho+\delta}\left(1 - e^{-(r+\delta)T}\right)$, then discounted lifetime profits are

$$V(y, T) = A(T)\left[p(y)y - c(y)\right] - M(T) \tag{1}$$

We also assume that $M(T) = A(T)^{\gamma}$ with $\gamma > 1$. The parameter γ represents the difficulty of survival. A higher γ means that a given percentage change in longevity requires a larger percentage change in longevity-enhancing resources. Having $M(T)$ be a function of $A(T)$ rather than some other function is purely for analytical convenience.

Firms choose annual output levels and a lifespan to maximize the value of the firm.

Optimal output y^* equates the firm's marginal revenue to the marginal cost, or

$$p(y)(1 + \mu) = c'(y) \tag{2}$$

where $\mu \equiv \frac{p'y}{p}$. Thus, optimal output y^* is a function of the demand and the cost parameters and independent of longevity. The separation of production and longevity decisions is convenient here, but it is unlikely to be a feature of more general environments.

Because of the separation result we can proceed in two stages. First, we find the optimal stationary output and re-insert it into the value function or

$$V(y^*, T) = A(T)\pi(y^*) - M(T) \tag{3}$$

where for simplicity we define the indirect profit function as $\pi(y^*) = p(y^*)y^* - c(y^*)$. Next, we let firms choose the lifespan that maximizes the value of the firm given y^*.

3 Optimal Longevity and the Value of the Firm

From now on we only consider the undiscounted case. This assumption is made for clarity and does not in any way limit our insights. That is, we analyze the case where $r + \delta$ goes to zero, $A(T)$ is replaced by T, and the value of the firm in (3) simplifies to

$$V(y^*, T) = T\pi(y^*) - M(T) \tag{4}$$

To choose optimal longevity, firms equate the marginal revenues from living longer py to the marginal cost from living longer $c + M_T$. Rearranging, we find that

$$\pi(y^*) = M_T \tag{5}$$

Thus, optimal longevity equates profits in the last year of life to the marginal cost of longevity-enhancing resources. Because longevity production is Cobb-Douglas and $\gamma > 1$, the marginal cost is a multiple of the average cost or $M_T = \gamma\frac{M}{T}$. Thus, a rise in γ tends to reduce M/T and longevity. Intuitively, a higher γ raises the marginal cost of living longer, which encourages firms to reduce longevity in order to lower the marginal cost.

Solving the first order condition for the optimal lifespan yields

$$T^* = \left[\gamma^{-1}\pi(y^*)\right]^{\frac{1}{\gamma-1}} \tag{6}$$

Thus, the difficulty of survival lowers the firm's life expectancy. Also, conventional profits encourage longevity because part of profits are earmarked for longevity enhancement, or $T^*\pi(y^*) = \gamma M(T^*)$ from the first order condition. Conceivably, one could estimate γ from a cross-section of living and deceased firms and then calculate T^*. The estimated γ may vary by sector, industry, and location and with this information one could compute $V(y^*, T)$.

The optimal value of the firm $V(y^*, T^*)$ can be easily calculated as a function of cost, demand and production parameters by inserting y^* and T^* into (4). Instead, we choose an indirect approach that shows how conventional valuations might be biased. If we rearrange (5) and substitute for $M(T^*)$ in (4) we get

$$V(y^*, T^*) = T^*\pi(y^*)\left(1 - \gamma^{-1}\right) \tag{7}$$

The share of profits after accounting for longevity payments is $1 - \gamma^{-1}$ and adjusts conventional discounted profits given that analysts have computed T^*. The adjustment factor is useful since γ may be estimated even if M is not observable.

Most real-world valuation exercises choose an arbitrary horizon T (Cornell, 1993). Usually this horizon is long under the premise that a firm's maximum lifespan is effectively infinite. Compared to the true value $V(y^*, T^*)$, the value with arbitrary horizon is

$$V(y^*, T) = T\pi(y^*) \tag{8}$$

Thus, the practitioners' formula (8) has two biases compared to (7). The bias of not recognizing longevity maintenance costs is captured by $1-\gamma^{-1}$ and rises as survival becomes easier. Even if one argues that survival expenditures are already captured in conventional costs, there remains the bias of failing to use the optimally chosen longevity. This bias is large when firm longevity is small relative to the arbitrary horizon. Optimal firm longevity is small when survival is difficult, as may have been the case recently for many dot-coms.

4 Optimal Longevity with Working Cash

Many start ups tend to have some cash reserves, little revenue, but high costs. With cash reserves the value of the firm may still be positive even when conventional profits are negative. In the previous section without cash reserves we required $\pi(y^*) \geq 0$, but now we relax this assumption and only require that the value of the firm is not negative.

Let U be the amount of cash reserves of the firm. This cash comes without strings attached and may be a result of previous savings. Assume the firm distributes its reserves over its lifetime to pay for business expenses. If average cash reserves U/T are treated as exogenous by the firm who puts this cash to work, then the value of the firm after setting $U/T \equiv \widetilde{U}$, where $\widetilde{U}$ is treated as exogenous for the maximization, is

$$V(y^*, T) = T\left[\widetilde{U} + \pi(y^*)\right] - M(T) \tag{9}$$

The optimality condition for longevity with working cash is then

$$\widetilde{U} + \pi(y^*) = M_T$$

After substituting for $\widetilde{U}$ we find that

$$\frac{U}{T} + \pi(y^*) = M_T \tag{10}$$

Because the left side of (10) is decreasing in T and the right side is increasing, equation (10) yields a unique solution for longevity denoted $\widetilde{T}$. It is not too hard to see that, like profits, more cash in hand raises the optimal lifespan. The reasoning behind this finding is simply that more money can be used for longevity maintenance.

The optimal value of the firm with optimal longevity $\widetilde{T}$ is $V(y^*, \widetilde{T})$ or

$$\widetilde{V} = \widetilde{T}\left[\frac{U}{\widetilde{T}} + \pi(y^*)\right]\left(1 - \frac{1}{\gamma}\right) \tag{11}$$

while the value with arbitrary horizon T is

$$V = T\left[\frac{U}{T} + \pi(y^*)\right] \tag{12}$$

Again the practitioner's valuation V is biased upward if longevity maintenance costs are ignored. There is also an upward bias if optimal longevity is less than the arbitrary horizon and $\pi(y^*) > 0$ or if optimal longevity is greater than the arbitrary horizon and $\pi(y^*) < 0$. In either case, the upward bias increases the more cash is on hand. In other words, the horizon bias when $U > 0$ exceeds our earlier bias when $U = 0$ and $\pi(y^*) > 0$.

The two cases indicate that the method of financing a firm's business operations matters for longevity and the firm's value. We may interpret the value in the previous section as an interest-free loan with repayment and no moral hazard. That is, after receiving a loan of U, the firm lives to produce a lifetime profit of $T^*\pi^*(1-\gamma^{-1})$ and then repays the funds U. Our working cash example may be interpreted as funding without strings attached or no repayment requirement. This has some of the characteristics of funding from an equity offering. Thus, because $V(y^*,\widetilde{T}) > V(y^*,T^*)$ and $\widetilde{T} > T^*$ for positive profits, our examples suggest that equity-financing may create more value and encourage longer lifespans than the same amount of debt-financing. The reason is that equity-financing may be used for longevity enhancement, while debt has no such usefulness.[1]

5 Longevity, Debt and Moral Hazard

We now consider a simple example of moral hazard where there is the possibility that the firm absconds with loaned funds. Although the "take the money and run" scenario is extreme, it indicates how debt covenants may affect longevity and the firm's value.

If firms are well-behaved, the value of the firm with a loan in the undiscounted case is:

$$V(y^*,T) = T\pi(y^*) - M(T) \tag{13}$$

It is also possible, however, that the firm absconds with the money after it receives the loan. Lenders would like to create incentives that prevent this outcome. Thus, they only lend funds if the value of the firm after repayment exceeds the value of absconding U. Abstracting from the cost to the firm of absconding, which would be subtracted from U, the lender's requirement translates into the condition

$$V(y^*,T) \geq U \tag{14}$$

We note that because of the separation of production and longevity decisions, optimal output will not be affected in either case.

[1]Because equity financing allows firms to exist with negative profits, the firm's risk of default may increase. From this perspective, debt financing is safer because it constrains profits to be positive.

To maximize the value function subject to the incentive compatibility constraint, we form the Lagrangian:

$$V(y^*, T) + \lambda \{U - V(y^*, T)\}$$

Thus, optimal longevity must satisfy the Kuhn-Tucker condition

$$(1 - \lambda)(\pi(y^*) - M_T) = 0 \tag{15}$$

As the Lagrangian multiplier satisfies $\lambda = \frac{dV}{dU} > 0$, we compare its magnitude with unity to evaluate the Kuhn-Tucker condition. When $1 \neq \lambda$, the incentive compatibility constraint is not binding and we revert to the unconstrained solution of the second section. When $1 = \lambda$, the incentive compatibility constraint is binding and determines longevity $\overline{T}$, or

$$V(y^*, \overline{T}) = \overline{T}\pi(y^*) - M\left(\overline{T}\right) = U \tag{16}$$

We conclude that longevity and the value of the firm are lower for morally-conflicted firms that receive debt financing than for honest firms. Because $V(y^*, T^*) > V(y^*, \overline{T})$, honest firms that do not require an incentive compatibility constraint have a higher value than potentially dishonest firms. Also, optimal longevity in the honest case is larger than in the dishonest case, or $T^* > \overline{T}$, because the value function is increasing in T for values less than the unconstrained optimum. The reason is the incentive constrained firm operates only as long as it takes to fulfill its debt obligations.

Moral hazard exaggerates the difference in value between firms that only use equity financing and those that only use debt financing. This is an implication of our earlier result that $V(y^*, \widetilde{T}) > V(y^*, T^*)$ and that $\widetilde{T} > T^*$ as long as profits are positive. Thus, moral hazard strengthens our conclusion that an all-equity firm lives longer and has a higher value than an all-debt firm for positive profits.

6 Conclusion

Firms do not exist forever but their actions can postpone their demise. This idea is developed in a dynamic model where firms can boost their lifespan by setting aside longevity-enhancing resources. We find that a firm's optimal longevity responds intuitively to a variety of factors and argue that longevity is a crucial element of a firm's life-cyle and valuation decisions.

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