E C O N O M I C S   B U L L E T I N

An Efficiency Wage Model With Persistent Cycles

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Abstract

This note develops an efficiency wage model which displays persistent cycles under perfect foresight. Limit cycles arise from the dependence of current labor supply on both recent labor market conditions and the expected rate of job creation.

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1 Introduction

Kimball (1994) extended the Shapiro and Stiglitz (1984) efficiency wage model to address out of steady state dynamics. In that model, firms set wages in order to elicit work effort from workers and then set employment to maximize profits given that wage. The wage is decreasing in the expected cost of job loss to workers, which itself is decreasing in the expected ease of finding a new job. The ease of finding a new job is increasing in the level of employment in the economy (due to the normal turnover of workers in existing jobs) and in the rate of growth of employment (this is Kimball's extension to out-of-steady-state dynamics). Thus, the wage is increasing in both the level and expected rate of change of employment. This dynamic specification of labor supply causes employment to respond sluggishly to shocks to labor demand.

In this paper, I add a dynamic model of the disutility of unemployment to a simple representation of Kimball's model. The resulting model displays perfect foresight equilibria with persistent cycles. These cycles are driven by endogenous responses of the efficiency wage to labor market conditions. First, as noted above, the efficiency wage depends positively on labor market tightness and thus on both the level of aggregate employment and on it's rate of change. Second, a recent history of labor market tightness tends to additionally raise the efficiency wage over time, as the subjective valuation of time spent in unemployment rises.

2 Analysis

Without substantial loss of generality, I consider the following simple representation of Kimball's model. L is employment and w is the real wage rate. The equilibrium conditions are:

$$w_t = \bar{w}_t + \frac{\dot{L}_t^{e(t)} + b}{1 - L_t} \tag{1}$$

$$w_t = c - dL_t \tag{2}$$

where $\dot{L}_t^{e(t)}$ is the time t expectation of the derivative of employment with respect to time, and a, b, c, d are positive constants.

The first equation gives an expression for the efficiency wage. For its workers not to 'shirk,' the wage of any firm must be greater than or equal to this level, which reflects the perceived degree of aggregate labor market tightness. As the levels of aggregate employment and expected aggregate employment growth increase, the expected duration of unemployment spells decreases. Workers thus expect a lower cost of job loss and require a greater wage to not shirk. The parameter b corresponds to the exogenous rate of job separation in Kimball's model, and $\bar{w}_t$ reflects the momentary utility of a worker if she is unemployed at time t.

The second equation is the standard profit maximization condition stating that firms hire on their marginal product schedules. Here we assume quadratic production functions, so that the marginal product $f'(L)$ is linear, and that $c > \bar{w}_t \quad \forall t$.

Under perfect foresight, equations (1–2) imply

$$\dot{L}_t = (1 - L_t)(c - dL_t - \bar{w}_t) - b \tag{3}$$

In order to close the model, we must specify an equation of motion for $\bar{w}_t$. We could for example, following Kimball, assume that $\bar{w}_t$ is an exogenous positive constant:

$$\bar{w}_t = \bar{w}. \quad \forall t \tag{4}$$

The steady state equilibrium L^* of (3-4) is less than one (there is unemployment in steady state equilibrium) and locally asymptotically stable.[1] Thus in this case, given an arbitrary initial condition L_0 for employment, employment would grow (shrink) monotonically toward L^* just fast enough for the efficiency wage (1) to equal the marginal product of labor at all times.[2] Faster (slower) growth than this would induce firms to raise (lower) their wages and thus make the current level of employment L_t unprofitable.

Here we take a different approach. We assume that $\bar{w}_t$ depends on the recent history of labor market conditions. It is intuitively plausible that workers' subjective valuations of a unit of time spent in unemployment might rise (relative to time spent working) following prolonged periods of low unemployment and fall following prolonged periods of high unemployment. For example, recent labor market tightness may make leisure more desirable, unemployment less stigmatized, and/or search time less demoralizing.

Specifically, assume that

$$\dot{\bar{w}}_t = \delta \int_{-\infty}^{t} e^{-\delta(t-s)} g(L_s, \bar{L}) ds. \tag{5}$$

for some function g satisfying $g(\bar{L}, \bar{L}) = 0$, $g_1 > 0$, $g_2 < 0$. The rate of change of $\bar{w}_t$ depends on a weighted average of past labor market conditions relative to an exogenous reference point $\bar{L} < 1$. The more (weighted) time that the labor market has spent above $\bar{L}$, the greater will be the rate of change of $\bar{w}_t$, and the sign of this rate of change depends on the sign of the weighted average. Thus, if the labor market has recently stayed above $\bar{L}$ for a prolonged period of time, the efficiency wage will be rising for any given L_t and $\dot{L}_t$.[3] The higher is the discount rate δ, the greater the relative weight placed on recent experience in the weighted average.

Equation (5) implies that $\bar{w}_t$ evolves according to

$$\ddot{\bar{w}}_t = -\delta(\dot{\bar{w}}_t - g(L_t, \bar{L})) \tag{6}$$

Equations (3) and (6) now determine the dynamics of employment L_t and the labor supply (efficiency wage) shifter $\bar{w}_t$ given arbitrary initial conditions for L, $\bar{w}$, and $\dot{\bar{w}}$.

The steady state equilibrium for this system is $L = \bar{L}$ and $\bar{w} = \bar{w}^*$, where $\bar{w}^*$ solves (3) for $\dot{L} = 0$ and $L = \bar{L}$. This equilibrium may or may not be locally asymptotically stable.

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[1] The eigenvalue for the equation of motion linearized around L^* is $-b/(1-L^*) - d \cdot (1-L^*) < 0$.

[2] Here we are treating employment as though it is a predetermined variable. A rationale for this is given in Georges (1995).

[3] This process could be thought of as a process of habit formation (e.g., Campbell and Cochrane (1999), Carroll, Overland and Weil (2000)).

To evaluate the local and global dynamics of the model, we can rewrite the dynamical system (3)(6) as a set of three first order differential equations

$$\begin{aligned} \dot{L}_t &= (1 - L_t)(c - dL_t - \bar{w}_t) - b \\ \dot{\bar{w}}_t &= q_t \\ \dot{q}_t &= -\delta(q_t - g(L_t, \bar{L})) \end{aligned} \tag{7}$$

The Jacobian of system (7) linearized at the steady state $(\bar{L}, \bar{w}^*, 0)$ can be written

$$J = \begin{pmatrix} -(1-\bar{L})d - (w^* - \bar{w}^*) & -(1-\bar{L}) & 0 \\ 0 & 0 & 1 \\ \delta g_1(\bar{L}, \bar{L}) & 0 & -\delta \end{pmatrix} \tag{8}$$

where $w^* = c - d\bar{L}$ is the steady state equilibrium value of the wage w.

Whereas the steady state equilibrium is independent of the function g, its local stability properties and the global dynamics of the system are not. We thus proceed by numerically simulating the model under three specific functional forms for g: (a) linear; (b) logarithmic; (c) quadratic;

$$g(L_t, \bar{L}) = \theta \cdot (L_t - \bar{L}) \tag{9a}$$
$$g(L_t, \bar{L}) = \theta \cdot \log(L_t/\bar{L}) \tag{9b}$$
$$g(L_t, \bar{L}) = \theta \cdot (L_t^2 - \bar{L}^2) \tag{9c}$$

where θ is a positive constant.

At $\delta = 0$, the eigenvalues of J are $(j_{11}, 0, 0)$, where $j_{11} < 0$ is the first element of J. Numerical simulations suggest that, for each of the three specifications of the function g, as δ increases from zero, the first root of J remains negative, decreasing monotonically. Thus, for $\delta > 0$, the long run qualitative dynamics of the system are determined by the characteristics of the latter pair of roots of J.[4] Just above $\delta = 0$ this pair of roots are complex and may have negative or positive real parts. For absolute values of j_{11} that are sufficiently large relative to θ, the real parts immediately become negative as δ rises from zero, whereas for moderate absolute values of j_{11} relative to θ these two roots have real parts that first rise and then fall in δ, eventually passing from positive to negative. Thus, if j_{11} and θ lie within a certain range, there is a Hopf bifurcation in δ, implying that there are closed orbits away from the steady state for a range of values of δ. Denote the value of δ at which this bifurcation occurs δ_B. An example is given in Figure I.

[4] See e.g., Guckenhiemer and Holmes (1983) Ch. 3.2.

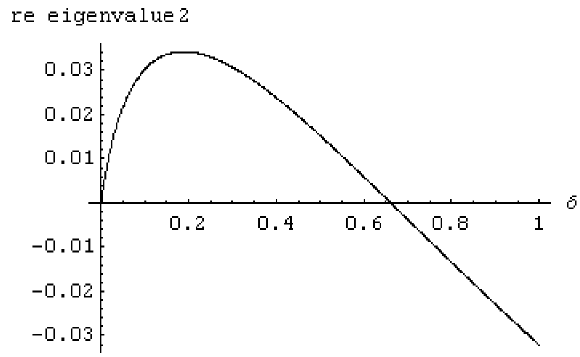

Figure I: Real part of the pair of complex eigenvalues as δ is varied. The function g is (9a), parameter values are $c = 1$, $d = .5$, $b = .1$,$\theta = 1$, $\bar{L} = .5$. The Hopf bifurcation value δ_B is approximately 0.6612.

Simulations also suggest that, for all relevant parameter values, the Hopf bifurcation is supercritical for each of the three functional forms for g that we considered (9a-9c). Thus, for each of these cases, the closed orbits occur for a range of values of δ which are less than δ_B, and these closed orbits are locally attracting, while the steady state is locally repelling. Thus, each case admits stable limit cycles. An example is given in Figure II below, which shows the evolution of employment L_t and the supply shifter $\bar{w}$ over 1000 periods. As in Figure I, the function g is linear and parameter values are $c = 1$, $d = .5$, $\theta = 1$, $b = .1$. If we set δ above the Hopf bifurcation value of $\delta_B \approx 0.66$, the steady state is asymptotically stable. If we set δ somewhat below δ_B, the steady state is repelling and the limit cycle absorbing.

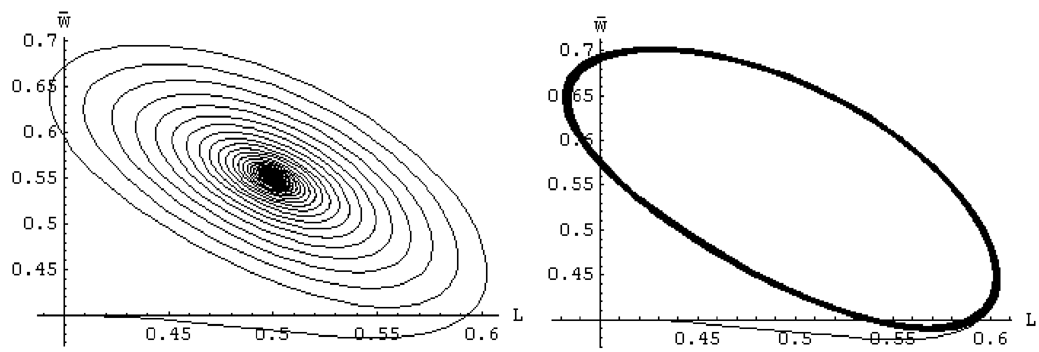

Figure II: Evolution of employment L_t and the supply shifter $\bar{w}_t$ over 1000 periods. As in Figure I, the function g is (9a) and parameter values are $c = 1$, $d = .5$, $\theta = 1$, $b = .1$. At left, $\delta = .8 > \delta_B$. At right, $\delta = .65 < \delta_B$. Initial conditions are $L_0 = .4$, $\bar{w}_0 = .4$, $\dot{\bar{w}}_0 = 0$.

3 Other Specifications

The qualitative results from the above simulations can be replicated under a number of other specifications of the demand for labor. For example, they hold for a Cobb Douglas specification of the production function with the capital stock either held fixed or driven by profit maximizing investment dynamics under quadratic adjustment costs. [5] The source of the cycles continues to be the dependence of the efficiency wage on both the employment history and expected employment growth. Thus, these cycles differ fundamentally from Goodwin's (1967) growth cycle which is driven by the interaction of wage and investment dynamics.

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[5] E.g., if production is Cobb Douglas and investment in the capital stock depends on profit maximization under quadratic costs of adjustment, then there are five eigenvalues. Three are real, and one of these is positive. A transversality condition requires that the initial conditions be selected so that the capital stock remains bounded. The other two may admit a Hopf bifurcation as the rate of time preference δ and/or other parameters (such as the Cobb Douglas elasticity of output with respect to capital, α) are varied.