E C O N O M I C S B U L L E T I N

Habit Persistence and Beliefs Based Liquidity Effect

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Abstract

The paper introduces habit persistence in consumption decisions in an infinitely−lived agents monetary model with a cash−in−advance constraint. We show that strong enough habit persistence yields indeterminate equilibria. However, real indeterminacy is not per se sufficient to obtain a liquidity effect. The form of the beliefs matters.

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Introduction

The empirical literature — in particular that using VAR models — that has studied the short–run non–neutrality property of money usually reports three main stylized facts: following a contractionary monetary policy, *(i)* there is a persistent decline in real GDP; *(ii)* prices are almost non responsive in the very short–run but decrease and *(iii)* the nominal interest rate rises. These results seem to be robust across different identification schemes (see e.g. Sims [1992], Leeper, Sims and Zha [1996], Christiano, Eichenbaum and Evans [1999]). Consequently, any structural model that could plausibly be used for monetary policy analysis should be able to account for these facts. This paper focuses on facts *(i)* and *(iii)* — which define the so–called liquidity effect — and aims at proposing a mechanism that account for both.

Standard flexible price monetary models, relying either on a cash–in–advance constraint or a money in the utility function specification, predict that output drops and the nominal interest rate rises following a persistent increase in the money supply. Indeed, these models generate an inflation tax that leads the individuals to substitute leisure for consumption, as a way to avoid paying the tax. Therefore, as labor supply drop, so does output. Further, since households postpone consumption and save more, and because inflation is expected to rise (to go back to the steady state), the nominal interest rate increases, which is at odds with the empirical evidence.

Recently, Matheny [1998] or Benhabib and Farmer [2000] have developed[1] monetary models which — while keeping with *ex–ante* prices flexibility and complete information assumptions — generate real indeterminacy and have the potential of generating a liquidity effect.[2] In this paper, we investigate the role of intertemporal substitution in the propagation of monetary shocks — as most of the negative effects associated with the inflation tax can largely be explained by the intertemporal substitution motives in consumption — in a cash–in–advance economy. To do so, we introduce intertemporal complementarities in consumption decisions. This is achieved by considering that households' preferences are characterized by habit persistence. The later assumption is a convenient way of introducing time non–separability in consumption decisions and has proven to be relevant for understanding puzzles related to the permanent income model, solving the equity premium puzzle, and improving the ability of business cycle models to account for aggregate fluctuations.

[1]Another route that has been pursued is to assume limited participation in the model (see e.g. Lucas [1990], Christiano [1991], Fuerst [1992]), implying that households cannot adjust their behavior to any changes in financial market circumstances. However, as noticed by Christiano [1991], the liquidity effect is not robust to a persistent money injection as the one found in the data.

[2]Matheny [1998] considers the potential of Pareto substitutability between consumption and leisure, while Benhabib and Farmer [2000] allow for positive transaction externalities.

Our results indicate that high enough habit persistence generates real indeterminacy in our monetary economy. It stems from the interplay between habit persistence and the cash–in–advance constraint, given a specific environment on the labor and asset markets. However, we show that real indeterminacy is not sufficient *per se* to generate the liquidity effect we are mainly interested in, the form of the beliefs matters. Indeed, when beliefs are not correlated with money injection the model generates perfect price flexibility and money is neutral. Conversely, when beliefs are sufficiently correlated with money injections the model displays a positive response of output and a negative response of the interest rate, therefore generating a liquidity effect.

The paper is organized as follows. A first section presents our benchmark model economy. Section 2 characterizes the local dynamic properties of the model. Section 3 investigates the nominal interest rate behavior and evaluates the ability of our mechanisms to generate a liquidity effect. A last section offers some concluding remarks.

1 The model economy

This section describes the main ingredients that characterize our model economy.

Households

The economy is comprised of a unit mass continuum of identical infinitely lived agents, so that we will assume that there exists a representative household in the economy. Households enter period t with real balances M_t/P_t carried over the previous period and nominal bonds B_t. The household supplies her hours h_t on the labor market at the real wage w_t. During the period, the household also receives a lump–sum transfer from the monetary authorities in the form of cash equal to N_t/P_t and interest rate payments from bond holdings $((R_{t-1}-1)B_t/P_t)$. All these revenues are then used to purchase a consumption bundle c_t, money balances and nominal bonds for the next period. Therefore, the budget constraint simply writes as

$$\frac{B_{t+1}}{P_t}+\frac{M_{t+1}}{P_t}+c_t=w_th_t+R_{t-1}\frac{B_t}{P_t}+\frac{M_t}{P_t}+\frac{N_t}{P_t} \tag{1}$$

Money is held because the household must carry cash in order to purchase goods. She therefore faces a cash–in–advance (CIA hereafter) constraint of the form

$$c_t \leqslant \frac{M_t+N_t+R_{t-1}B_t-B_{t+1}}{P_t} \tag{2}$$

Each household has preferences over consumption and leisure represented by the following utility function:

$$E_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left[\log(c_\tau-\theta c_{\tau-1})-h_\tau\right] \tag{3}$$

where $\beta \in (0,1)$ is the discount factor. We only depart from the standard cash–in–advance model, in that we allow for habit persistence in the consumption behavior. $\theta \in (0,1)$ is the habit persistence parameter. Note that following Constantidines and Ferson [1991], Braun, Constantidines and Ferson [1993], the specification of habit persistence is taken in difference and involves only one lag. Further, it will be fully internalized by the household.[3] It is also noteworthy that setting θ to zero allows to retrieve the standard cash–in–advance model. The household decides on her optimal consumption/saving, labor supply and money and bond holdings plans maximizing (3) subject to (1)–(2). Using the first order condition on consumption and money holdings, the labor supply decision is determined by

$$\frac{1}{w_t} = \beta E_t \frac{P_t}{P_{t+1}} \left[\frac{1}{c_{t+1} - \theta c_t} - \frac{\beta\theta}{c_{t+2} - \theta c_{t+1}} \right] \tag{4}$$

Likewise, asset holdings decision is determined by the standard non–arbitrage condition

$$R_t = \left[\frac{1}{c_t - \theta c_{t-1}} - E_t \frac{\beta\theta}{c_{t+1} - \theta c_t} \right] w_t \tag{5}$$

Firms

The technology is described by the constant return to scale production function $Y_t = h_t$, such that in equilibrium the real wage is $w_t = 1$.

Money Supply and Government Budget Constraint

Money is exogenously supplied according to the following money growth rule $M_{t+1} = g_t M_t$ where g_t follows an AR(1) process :

$$\log(g_t) = \rho_g \log(g_{t-1}) + (1 - \rho_g)\log(\bar{g}) + \sigma_{\varepsilon^g} \varepsilon_t^g$$

ε_t^g is a white noise with unit variance, $\sigma_\varepsilon^g > 0$ and $|\rho_g| < 1$. The government issues nominal bonds B_t to finance open market operations.[4] The government budget constraint is $M_{t+1} + B_{t+1} = M_t + R_{t-1}B_t + N_t$ with M_0 and B_0 given.

Equilibrium

An equilibrium is a sequence of prices and allocations, such that given prices, allocation maximizes profits and maximizes utility, and all markets clear (implying $c_t = y_t = h_t$).

2 Habit persistence and real indeterminacy

The dynamic properties of output are strongly related to a perfect foresight version of the model economy. First of all, note that the deterministic steady state value of output – or

[3]Habit persistence actually raises three important modeling issues: *(i)* the speed with which habit reacts to consumption, *(ii)* internal vs external habit and *(iii)* the functional form of habit formation (ratio vs. difference). Auray, Collard and Fève [2001] study the dynamic implications of various forms of intertemporal complementarities.

[4]These nominal bonds could also be used to finance government consumption. Nevertheless, this issue is beyond the scope of the paper.

identically consumption – is uniquely determined by $y^\star = \beta(1-\beta\theta)/(\overline{g}(1-\theta))$. Holding the rate of growth of money supply constant, the log–linear approximation of output dynamics around $y^\star$ yields the following linear second order finite difference equation:

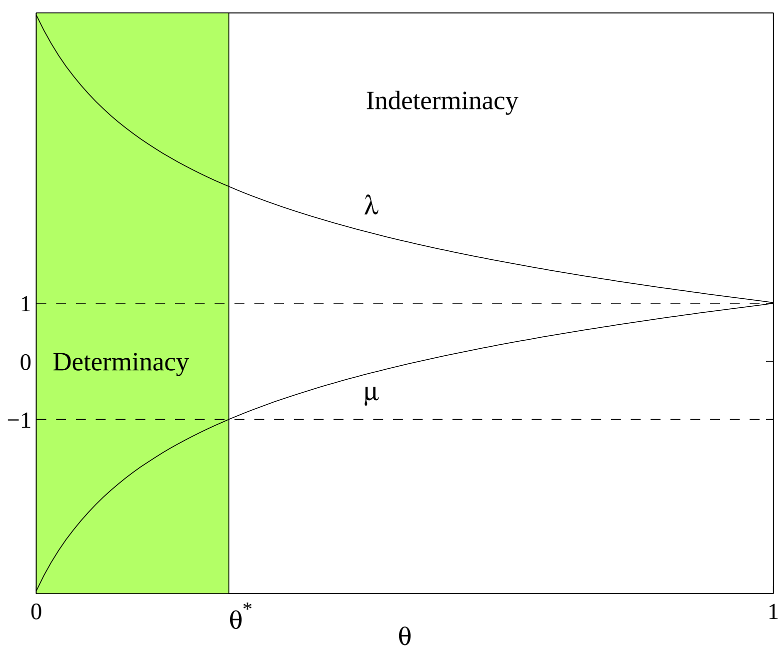
$$\widehat{y}_{t+2} + \frac{(1-\theta)(1-\beta\theta)-(1+\beta\theta^2)}{\beta\theta}\widehat{y}_{t+1} + \left[\frac{1}{\beta} - \frac{(1-\theta)(1-\theta\beta)}{\beta\theta}\right]\widehat{y}_t = 0 \qquad (6)$$

where $\widehat{y}_t = \log(y_t/y^\star)$. Equation (6) can then be expressed in the more compact form $(1-\lambda L)(1-\mu L)\widehat{y}_{t+2} = 0$, where L denotes the lag operator. The local dynamic properties of the economy then depend on the position of λ and μ around the unit circle. In particular, if at least one of the eigenvalues lies inside the unit circle the equilibrium is locally indeterminate, *i.e.* there exists a continuum of equilibrium paths all converging to the steady state. Note that both μ and λ are real[5] therefore ruling out the possibility of persistent deterministic oscillations in output dynamics. Besides, we have the following proposition.

Proposition 1 *There exists $\theta^\star \in (0,1)$ such that for all $\theta \geqslant \theta^\star$ one and only one eigenvalue lies inside the unit circle.*

Proposition 1 establishes that — although the money supply is exogenous[6] — there exists a value of θ above which the equilibrium is locally indeterminate. Figure 1 illustrates this

Figure 1: Roots of the characteristic polynomial



proposition. The two curves represent the two roots of the characteristic polynomial as a

[5]The discriminant of the characteristic polynomial associated to (6) is positive, provided $\theta \in (0,1)$.

[6]Carlstrom and Fuerst [2000] have shown that the exogeneity of the money growth rule is a sufficient condition for saddle path in in a plausibly calibrated monetary model without habit persistence.

function of the habit persistence parameter. The shaded area reports values of θ for which the equilibrium is saddle path. Above $\theta^\star$, the lower root lies within the unit circle while the other one remains greater than 1, and the equilibrium becomes indeterminate. It is worth noting that as θ tends to 1, the stable root tends to one. This can be easily checked by setting $\theta = 1$ in the characteristic polynomial associated with equation (6). The two roots are then $1/\beta$ (the explosive root) and 1 (the formerly stable root). More interestingly, figure 1 indicates that the stable root is positive for high level of habit persistence as established in the following proposition.

Proposition 2 *There exists a unique $\widetilde{\theta} \in (\theta^\star, 1)$ such that for any $\theta \in (\widetilde{\theta}, 1)$ the stable root is strictly positive.*

An implication of this result is that strong enough habit persistence parameter leads to positive serial correlation in output dynamics, which is supported by the empirical evidence. This contrasts with the standard CIA model that generates no persistence, when the money growth process — if assumed to be exogenous — is serially uncorrelated. In other words, proposition 2 establishes that the CIA model, when coupled with habit persistence, possesses internal propagation mechanisms strong enough to generate persistence. This proposition shows that persistence comes together with real indeterminacy as $\widetilde{\theta} > \theta^\star$.[7]

3 Beliefs and the liquidity effect

In this section, we present some quantitative implications of our simple model, which illustrates its ability to account for a liquidity effect. Since the liquidity effect is strongly related to the from of the belief function, we focus on real indeterminate solutions of the form:[8]

$$\widehat{y}_t = \mu \widehat{y}_{t-1} - \frac{\rho_g}{\lambda - \rho_g} \frac{(1-\theta)(1-\theta\beta)}{\beta\theta} \widehat{g}_{t-1} + \varepsilon_t^y$$

where ε_t^y denotes a martingale difference sequence that can be related to fundamental shocks (money shocks), depending on individuals' beliefs about monetary policy. Hence, ε_t^y writes

$$\varepsilon_t^y = b\varepsilon_t^g + \nu_t \tag{7}$$

with $E_{t-1}\nu_t = 0$ and $|b| < \infty$. ν_t denotes purely extrinsic beliefs that are unrelated to fundamentals. The extrinsic parameter b determines the correlation between agents' beliefs and fundamentals. Like in Benhabib and Farmer [2000] and Matheny [1998], the value of b

[7]Interestingly, it also establishes that oscillatory sunspot equilibria may also occur for lower habit persistence ($\theta \in (\theta^\star, \widetilde{\theta})$) since then the stable eigenvalue is strictly negative. But in this case, output is negatively serially correlated.

[8]Real indeterminacy occurs rather easily in this economy, as for β close to unity, $\theta^\star \approx 0.17$ and $\widetilde{\theta} \approx 0.38$. Remarkably, the model generates positive serial correlation in output dynamics with a value of θ which is close to existing point estimates (see Constantidines and Ferson [1991], Braun et al. [1993] on macro data and Naik and Moore [1996] on micro data.)

is critical for the properties of the equilibrium. Given (7), output and nominal interest rate dynamics are given by

$$\widehat{y}_t = \mu\widehat{y}_{t-1} - \frac{\rho_g}{\lambda-\rho_g}\frac{(1-\theta)(1-\theta\beta)}{\beta\theta}\widehat{g}_{t-1} + b\varepsilon_t^g + \nu_t \tag{8}$$

$$\begin{aligned}\widehat{R}_t &= \frac{\theta(1+\beta\mu^2)-\mu(1+\beta\theta^2)}{(1-\theta)(1-\beta\theta)}\widehat{y}_{t-1} - \frac{\rho_g}{\lambda-\rho_g}\widehat{g}_t \\ &\quad -\frac{\beta\theta\mu-1-\beta\theta^2}{\beta\theta}\frac{\rho_g}{\lambda-\rho_g}\widehat{g}_{t-1} + \frac{\beta\theta\mu-1-\beta\theta^2}{\beta\theta}(b\varepsilon_t^g+\nu_t)\end{aligned} \tag{9}$$

To keep the exposition simple, let us focus on $\rho_g = 0$ and $\nu_t = 0$, $\forall t$. Then, (8) and (9) reduce to

$$\widehat{y}_t = \mu\widehat{y}_{t-1} + b\varepsilon_t^g \tag{10}$$

$$\widehat{R}_t = \frac{\theta(1+\beta\mu^2)-\mu(1+\beta\theta^2)}{(1-\theta)(1-\beta\theta)}\widehat{y}_{t-1} + b\frac{\beta\theta\mu-1-\beta\theta^2}{(1-\theta)(1-\beta\theta)}\varepsilon_t^g \tag{11}$$

Provided $\mu > 0$ ($\theta > \widetilde{\theta}$), the model generates persistence. Note that (11) clearly shows that real indeterminacy is not *per se* sufficient to generate a liquidity effect. Additional assumptions have to be placed on individuals' belief function, as we now illustrate. Let us first consider the case where b=0, such that the above system reduces to

$$\begin{aligned}\widehat{y}_t &= \mu\widehat{y}_{t-1} \\ \widehat{R}_t &= \frac{\theta(1+\beta\mu^2)-\mu(1+\beta\theta^2)}{(1-\theta)(1-\beta\theta)}\widehat{y}_{t-1}\end{aligned}$$

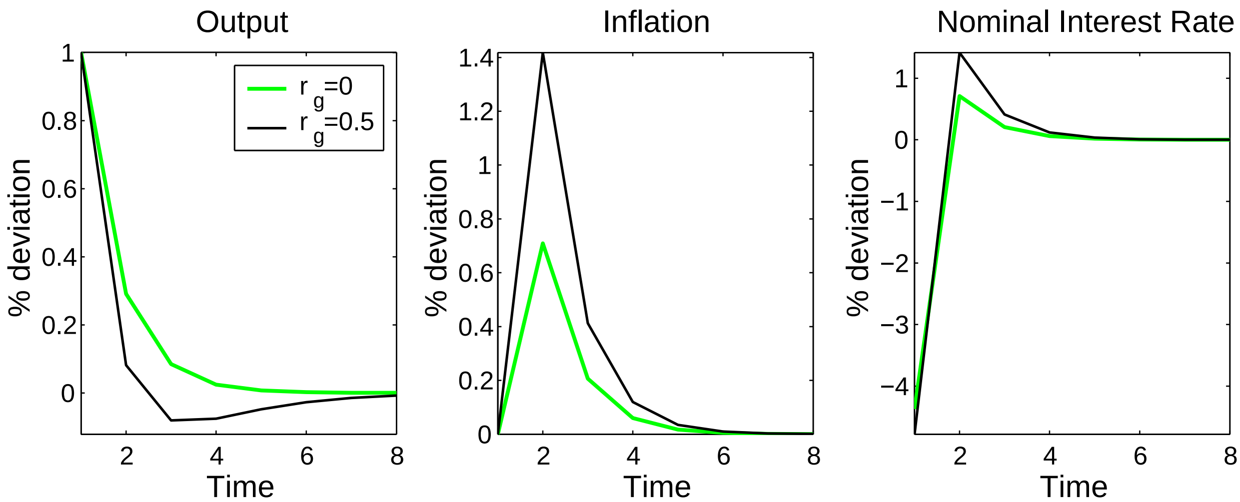
The output and the nominal interest rate are left unaffected. As a matter of fact, when agents beliefs are uncorrelated with money injections, prices are fully flexible and we retrieve the quantitative theory of money. Money is neutral.

We now investigate a situation where individuals' beliefs are positively correlated with the money supply shock, say b=1. The output/nominal interest rate dynamics rewrites

$$\begin{aligned}\widehat{y}_t &= \mu\widehat{y}_{t-1} + \varepsilon_t^g \\ \widehat{R}_t &= \frac{\theta(1+\beta\mu^2)-\mu(1+\beta\theta^2)}{(1-\theta)(1-\beta\theta)}\widehat{y}_{t-1} + \frac{\beta\theta\mu-1-\beta\theta^2}{(1-\theta)(1-\beta\theta)}\varepsilon_t^g\end{aligned}$$

Following a positive transient money injection, output – or consumption – instantaneously responds one for one. Since $|\mu| < 1$ (the stable root), the nominal interest rate drops in face a positive money shock. The model therefore generates a liquidity effect. This is illustrated by figure 2, which reports the impulse response functions of output, inflation and the nominal interest rate to money injection. It is worth noting that inflation does not instantaneously respond implying that the model generates endogenous price sluggishness. Interestingly, contrary to the standard limited participation model (see Christiano [1991]), the liquidity effect

Figure 2: The Liquidity effect



Note : These figures are drawn for $\beta = 0.99$, $\theta = 0.5$ and $b = 1$.

is robust to higher serial correlation ($\rho_g = 0.5$) in this model. When beliefs are positively related to money growth, a money injection leads individuals to increase their current consumption because prices do not respond. The intertemporal complementarity generated by habit persistence induces sustained higher consumption in the next period. The households is therefore willing to transfer wealth toward the future in order to support future consumption. This can be achieved either by increasing future money holdings and/or by purchasing bonds. However, because of the CIA constraint, purchasing nominal bonds reduces today's cash–in–hand available for current consumption. It is then optimal for the household to substitute money for bonds, which puts downward pressure on the nominal interest rate; therefore generating the observed liquidity effect. This is this interplay between the cash–in–advance constraint and the habit persistence which lies at the core of the mechanism. The liquidity effect would not occur in an economy with no habit persistence, as money injection would then trigger a cut in consumption purchases, lowering household's willingness to hold money. She would then essentially rely on bonds to transfer wealth intertemporally, putting upward pressure on the nominal interest rate.

4 Concluding remarks

This paper introduces time non–separability in consumption decisions in an infinitely–lived agents monetary model with a cash–in–advance constraint. We show that high enough habit persistence yields real indeterminacy. Further, we find that — depending on the form of the beliefs — the model can generate either money neutrality or a liquidity effect. Several issues may be worth considering. First, the robustness of our results to other specifications of the money demand and market arrangements may be checked. Second, one may wonder whether other monetary policy rules could rule out real indeterminacy.

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Appendix

Proof (proposition 1): Real indeterminacy occurs as at least one eigenvalue lies inside the unit circle. We therefore seek conditions for which $|\lambda| = 1$, where λ is an eigenvalue of the log–linearized dynamic equation, holds. The characteristic polynomial associated with the log–linearized version of the economy is given by:

$$P(\lambda) = \lambda^2 - \frac{1+\beta}{\beta}\lambda + \left[\frac{1}{\beta} - \frac{(1-\theta)(1-\theta\beta)}{\theta\beta}\right]$$

and satisfies

$$\begin{aligned} P(1) &= -\frac{(1-\theta)(1-\theta\beta)}{\theta\beta} \\ P(-1) &= \frac{2(1+\beta)}{\beta} - \frac{(1-\theta)(1-\theta\beta)}{\theta\beta} \end{aligned}$$

As can be easily checked, only two values of θ are compatible with $P(1) = 0$: $\theta = 1$ and $\theta = 1/\beta$. As we are interested in situations where $\theta \in (0,1)$ none of them is relevant for our purpose.

We now study the possibility for real indeterminacy to be associated with $P(-1) = 0$. Solving this condition for θ amounts to solve

$$Q(\theta) \equiv \theta^2 - 3\frac{1+\beta}{\beta}\theta + \frac{1}{\beta} = 0$$

First note that:

$$Q'(\theta) < 0 \text{ for } \theta < \theta_0 \equiv 3\frac{1+\beta}{2\beta}$$

Then, as $\beta \in (0,1)$, $\theta_0 > 1$. This implies that $Q(\theta)$ is strictly decreasing for $\theta \in (0,1)$. Further, $Q(0) = 1/\beta > 0$ and $Q(1) = -2(1+\beta)/\beta < 0$. There exists a unique value $\theta^\star \in (0,1)$ such that $Q(\theta) = 0$. Finally, since $Q(\theta)$ is strictly decreasing, $Q(\theta) < 0$ (identically $P(-1) > 0$) for all $\theta > \theta^\star$.

□

Proof (proposition 2): Determining conditions for which the "stable" eigenvalue is positive essentially amounts to find conditions on θ for $P(0) = 0$ to hold. This is actually equivalent to study:

$$R(\theta) \equiv \theta^2 - \frac{2+\beta}{\beta}\theta + \frac{1}{\beta} = 0$$

First note that $R'(\theta) < 0$ for $\theta < \theta_1 \equiv (2+\beta)/(2\beta)$. Then, as $\beta \in (0,1)$, $\theta_1 > 1$. This implies that $R(\theta)$ is strictly decreasing for $\theta \in (0,1)$. Further, $R(0) = 1/\beta > 0$ and $R(1) = -1/\beta < 0$ such that there exists a unique value $\widetilde{\theta} \in (0,1)$ such that $R(\theta) = 0$. Finally, as $R(\theta)$ is strictly decreasing, $R(\theta) < 0$ (identically $P(0) > 0$) for all $\theta > \widetilde{\theta}$. This establishes the existence of $\widetilde{\theta}$.

In order to verify that $\widetilde{\theta} \geqslant \theta^\star$, we study the sign of $R(\theta) - Q(\theta)$. Recall that $Q(\theta)$ is strictly decreasing for all $\theta > \theta^\star$ (see proof of proposition 1) and that $R(\theta)$ is strictly decreasing for all $\theta > \widetilde{\theta}$. Then, $R(\theta) - Q(\theta) = \theta(1+2\beta)/\beta$, which is strictly positive. Now since both $R(\theta)$ and $Q(\theta)$ are decreasing, we necessarily have $\widetilde{\theta} > \theta^\star$.

□