E C O N O M I C S   B U L L E T I N

The effect of aggregation on the magnitude of behavioral heterogeneity.

Ralf Wilke
University of Dortmund

Abstract

Recent literature has shown that the magnitude of behavioral heterogeneity of a population has an impact on the structure of mean demand. This paper investigates the effect of aggregation on the magnitude of behavioral heterogeneity if we aggregate disjoint subpopulations. Using the Hildenbrand and Kneip (1999) framework of behavioral heterogeneity, we show: (i) aggregation cannot decrease the degree of behavioral heterogeneity; (ii) conditions under which aggregation increases the degree of behavioral heterogeneity are derived; (iii) aggregation weakly increases the degree of behavioral heterogeneity.

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1 Introduction

In this note we consider the aggregation of disjoint subpopulations and its effect on the magnitude of the degree of behavioral heterogeneity as given by Hildenbrand and Kneip (1999). Similar results might be obtained using the other frameworks of behavioral heterogeneity. We consider here the Hildenbrand and Kneip (1999) framework because it directly applies to the Jacobian of mean demand rather to its Slutsky decomposition terms as in Hildenbrand (1993) and Kneip (1999). Moreover, their framework is a generalization of Grandmont's (1992) economy and the degree of behavioral heterogeneity, as defined by Hildenbrand and Kneip, can only assume values between zero and one.

Our analysis relates the degree of behavioral heterogeneity of arbitrary composed disjoint subpopulations to the one of the entire population, i.e. we investigate whether (dis-)aggregation may affect the structural properties of aggregate demand. This question is important for empirical analysis, since demand systems are often estimated using homogenous samples of households. We obtain three results: Firstly, aggregation never decreases the degree of behavioral heterogeneity. In other words, the degree of behavioral heterogeneity at the aggregate level is higher than the lowest degree of behavioral heterogeneity of arbitrary disjoint subpopulation. Secondly, we show that the degree of behavioral heterogeneity at the aggregate level can be either greater or smaller than the maximal degree of heterogeneity of all subpopulations. We derive the conditions for generating behavioral heterogeneity due to aggregation. Thirdly, and finally, we show that aggregation weakly generates heterogeneity. In other words, aggregation leads to a higher degree of behavioral heterogeneity at the aggregate level when compared to the weighted average of arbitrary disjoint subpopulations. We conclude that a decomposition of a heterogenous population into homogenous subpopulations might destroy mathematical properties of mean demand which are induced by behavioral heterogeneity.

We use the model and the notation of Hildenbrand and Kneip (1999).

2 Behavioral Heterogeneity and Aggregation

Consider $m = 1, \ldots, k$ nonempty subpopulations of H with $\dot{\bigcup}_{m=1}^{k} H^m = H$, where $\dot{\bigcup}$ denotes the union of disjoint sets. The disjoint subpopulations are allowed to be of arbitrary size and arbitrary composition.

Definition 1 *Aggregation reduces heterogeneity as measured by* γ*, if*

$$\gamma(H) < inf_m \gamma(H^m)$$

is true.

Definition 2 *Aggregation increases heterogeneity as measured by* γ*, if*

$$\gamma(H) \geq sup_m \gamma(H^m)$$

is true.

Proposition 1 *Aggregation cannot reduce the degree of behavioral heterogeneity as measured by γ, i.e. for every H and $\{H^m\}_{m=1,\ldots,k}$ such that $\dot{\bigcup}_{m=1}^{k} H^m = H$ it follows*

$$\gamma(H) \geq inf_m \gamma(H^m).$$

Proof. It suffices to prove the proposition for $k = 2$, since $H_k \cup \left(\dot{\bigcup}_{m=1}^{k-1} H^m\right) = H$. Suppose we have two subpopulations m and n and assume without loss of generality that $\gamma(H^m) \leq \gamma(H^n)$. Then,

$$\begin{aligned} I_{ij}^{\epsilon}(p) &= \frac{1}{\#H^m + \#H^n} \#\{h \in H^m \cup H^n | p \in A_{ij}^{\epsilon}(w^h, x^h)\} \\ &= \frac{1}{\#H^m + \#H^n} \left(\#H^m I_{ij}^{\epsilon m}(p) + \#H^n I_{ij}^{\epsilon n}(p)\right) \qquad (1) \\ &\leq \sup\{I_{ij}^{\epsilon m}(p), I_{ij}^{\epsilon n}(p)\} \end{aligned}$$

for $\epsilon \in [0, 1]$, where

$$I_{ij}^{\epsilon m}(p) := \frac{1}{\#H^m} \#\{h \in H^m | p \in A_{ij}^{\epsilon}(w^h, x^h)\}.$$

Applying this inequality gives

$$\begin{aligned} 1 - \gamma_{ij}(p) = \int_0^1 I_{ij}^{\epsilon}(p) d\epsilon &\leq \int_0^1 \sup\{I_{ij}^{\epsilon m}(p), I_{ij}^{\epsilon n}(p)\} d\epsilon \leq 1 - \inf\{\gamma_{ij}^m(p), \gamma_{ij}^n(p)\} \\ \Leftrightarrow \gamma_{ij}(p) &\geq \inf\{\gamma_{ij}^m(p), \gamma_{ij}^n(p)\} \end{aligned}$$

for all i, j and $p \in (0, \infty)^l$, which proves Proposition 1. ■

Note, however, that we can neither infer from Proposition 1 that an expansion of the population by an additional household does not lead to a decrease in the index of heterogeneity nor that $\gamma(H) < \sup_m \gamma(H^m)$ holds. More generally, we ask whether $\gamma(H) \geq \sup_m \gamma(H^m)$ may occur. We show that this inequality does not hold in general: because of equation (1) one can infer $I_{ij}^{\epsilon}(p) \geq \inf\{I_{ij}^{\epsilon m}(p), I_{ij}^{\epsilon n}(p)\}$ (see Figure 1). Therefore

$$1 - \gamma_{ij}(p) = \int_0^1 I_{ij}^{\epsilon}(p) d\epsilon \geq \int_0^1 \inf\{I_{ij}^{\epsilon m}(p), I_{ij}^{\epsilon n}(p)\} d\epsilon \leq 1 - \sup\{\gamma_{ij}^m(p), \gamma_{ij}^n(p)\},$$

which is not a unique relation.

The next propositions shed light on this point. Proposition 2 looks at an expansion of the original population by an additional household, while Proposition 3 considers the general case when aggregating subpopulations of arbitrary size.

The Intruder's Influence. We are looking at the degree of heterogeneity while expanding the original population H by one additional household. Let $H^+ = H \cup H^1$ where H^1 consists of one household only, the intruder. Let $I\!I$ denote the set of (i, j, p) such that $\gamma(H) = 1 - \int_0^1 I_{ij}^{\epsilon}(p) d\epsilon$. Note that $I\!I$ may contain of more than one element.

Proposition 2 *Increasing the size of the population by one additional household leads to $\gamma(H^+) \geq \gamma(H)$ if*
1)

$$c^1_{ij}(p) := \frac{p_j|\partial_{p_j} w^1_i(p,x^1)|}{d^1_{ij}} \leq \int_0^1 I^\epsilon_{ij}(p)d\epsilon \quad \text{for all } (i,j,p) \in I\!I$$

and
2)

$$\int_0^1 I^\epsilon_{ij}(p)d\epsilon \geq \frac{1}{(1+\#H)} + \int_0^1 I^\epsilon_{\tilde{i}\tilde{j}}(\tilde{p})d\epsilon \quad \text{for all } (\tilde{i},\tilde{j},\tilde{p}) \in CI\!I,$$

where $CI\!I$ denotes the complementary set of $I\!I$.

The first condition ensures that $1-\int_0^1 I^{\epsilon+}_{ij}(p)d\epsilon \geq 1-\int_0^1 I^\epsilon_{ij}(p)d\epsilon$ for all $(i,j,p) \in I\!I$. The intuition is that the inequality is more likely to be satisfied if the original population is homogeneous or if $c^1_{ij}(p)$ is small, meaning that the variability of the intruder's budget share is small at $(i,j,p) \in I\!I$. The second condition ensures firstly that $\gamma(H^+) = 1-\int_0^1 I^{\epsilon+}_{ij}(p)d\epsilon \leq 1-\int_0^1 I^{\epsilon+}_{\tilde{i}\tilde{j}}(\tilde{p})d\epsilon$ for all $(\tilde{i},\tilde{j},\tilde{p}) \in CI\!I$ and secondly that at least one original element of $(i,j,p) \in I\!I$ remains in this set after the expansion of the population, i.e. we have the maximal area under the step function $I^\epsilon_{ij}(p)$. Obviously, this condition is likely to be satisfied for large populations. However, the condition is stronger than required, because if for all $(\tilde{i},\tilde{j},\tilde{p}) \in CI\!I$: $c^1_{\tilde{i}\tilde{j}}(\tilde{p}) \leq \int_0^1 I^\epsilon_{\tilde{i}\tilde{j}}(\tilde{p})d\epsilon$, then $\int_0^1 I^{\epsilon+}_{\tilde{i}\tilde{j}}(\tilde{p})d\epsilon \leq \int_0^1 I^\epsilon_{\tilde{i}\tilde{j}}(\tilde{p})d\epsilon$. In those cases we do not require the second condition. However, we use the stronger version of the second condition.

The first condition is also satisfied if

$$c^1_{ij}(p) \leq \inf_{h\in H} \frac{p_j|\partial_{p_j} w^h_i(p,x^h)|}{d^h_{ij}},$$

meaning that the intruder needs to have less relative variability of the budget share for $(i,j,p) \in I\!I$ than every household of the original population. In fact, this condition is stronger than the first one.

Let us prove Proposition 2 and illustrate it with the help of three examples.

Proof. The inequality $\gamma(H^+) \geq \gamma(H)$ corresponds to $\sup_{i,j,p}\int_0^1 I^{\epsilon+}_{ij}(p) \leq \sup_{i,j,p}\int_0^1 I^\epsilon_{ij}(p)$. In the Part A we prove the proposition for the strong version of the first condition. In Part B we show the general result.

A Since $G^\epsilon_{ij}(p)$ is the cumulative distribution function of $|s_i(p,x^h)|/d^h_{ij}$, we have for all $(i,j,p) \in I\!I$ that $G^{\epsilon+}_{ij}(p) \geq G^\epsilon_{ij}(p)$ for $\epsilon \in [0,1]$, if $c^1_{ij}(p) \leq \inf_{h\in H} p_j|\partial_{p_j} w^h_i(p,x^h)|/d^h_{ij}$ and therefore $I^{\epsilon+}_{ij}(p) - I^\epsilon_{ij}(p) \leq 0$. Using the properties of first order stochastic dominance (Lemma 2 in the appendix) leads to $\gamma^+_{ij}(p) \geq \gamma_{ij}(p)$ for all $(i,j,p) \in I\!I$. In order to ensure $\int_0^1 I^\epsilon_{ij}(p)d\epsilon \geq \int_0^1 I^{\epsilon+}_{\tilde{i}\tilde{j}}(\tilde{p})d\epsilon$ for all $(i,j,p) \in I\!I$ and all $(\tilde{i},\tilde{j},\tilde{p}) \in CI\!I$ we need the second condition, since $1/(1+\#H) \geq \sup_{i,j,p}\left(\int_0^1 I^{\epsilon+}_{ij}(p)d\epsilon - \int_0^1 I^\epsilon_{ij}(p)d\epsilon\right)$ for all $(i,j,p) \in I\!I \cup CI\!I$.

B One can show that

$$I^{\epsilon+}_{ij}(p) - I^\epsilon_{ij}(p) = \begin{cases} \frac{1}{1+\#H}\left(1-I^\epsilon_{ij}(p)\right) & \text{if } \frac{|w^1_i(p,x^1)|}{d^1_{ij}} \geq \epsilon \\ \frac{-I^\epsilon_{ij}(p)}{1+\#H} & \text{otherwise} \end{cases}$$

In order to obtain

$$\int_0^1 I_{ij}^{\epsilon+}(p) - I_{ij}^{\epsilon}(p)d\epsilon \leq 0,$$

we need

$$\frac{1}{1+\#H}\left(\int_0^{c_{ij}^1(p)} 1d\epsilon - \int_0^{c_{ij}^1(p)} I_{ij}^{\epsilon}d\epsilon - \int_{c_{ij}^1(p)}^1 I_{ij}^{\epsilon}(p)d\epsilon\right) \leq 0$$

and therefore

$$\frac{1}{1+\#H}\left(c_{ij}^1(p) - \int_0^1 I_{ij}^{\epsilon}(p)d\epsilon\right) \leq 0$$

for all $(i,j,p) \in I\!I$ such that $\gamma(H) = 1 - \int_0^1 I_{ij}^{\epsilon}(p)d\epsilon$. Using $\int_0^1 I_{ij}^{\epsilon}(p)d\epsilon - \int_0^1 I_{\tilde{i}\tilde{j}}^{\epsilon}(\tilde{p})d\epsilon \geq 1/(1+\#H)$ for all $(\tilde{i},\tilde{j},\tilde{p}) \in CI\!I$ proves the proposition. ■

Increasing Heterogeneity Due to Aggregation Suppose $H = \dot{\bigcup}_{m=1}^k H^m$ and let $\sup_m \gamma(H^m) =: \gamma(H^n)$. In addition, let $I\!I$ be the set of (i,j,p) such that $\gamma(H^n) = 1 - \int_0^1 I_{ij}^{\epsilon n}(p)d\epsilon$.

Proposition 3 *Aggregation increases the degree of behavioral heterogeneity as measured by γ, i.e. $\gamma(H) \geq sup_m \gamma(H^m)$, if the following conditions hold:*
1)

$$\int_0^1 I_{ij}^{\epsilon m}(p)d\epsilon \leq \int_0^1 I_{ij}^{\epsilon n}(p)d\epsilon$$

for all $(i,j,p) \in I\!I$ and $m = 1,\ldots,k$ and
2)

$$\int_0^1 I_{ij}^{\epsilon n}(p)d\epsilon - \int_0^1 I_{\tilde{i}\tilde{j}}^{\epsilon n}(\tilde{p})d\epsilon \geq \frac{\#H - \#H^n}{\#H}$$

for all $(i,j,p) \in I\!I$ and for all $(\tilde{i},\tilde{j},\tilde{p}) \in CI\!I$ such that

$$\int_0^1 I_{\tilde{i}\tilde{j}}^{\epsilon m}(\tilde{p})d\epsilon \geq \int_0^1 I_{\tilde{i}\tilde{j}}^{\epsilon n}(\tilde{p})d\epsilon.$$

Proof. The proof follows the same reasoning as in the proof of Proposition 2. The first condition implies

$$\int_0^1 I_{ij}^{\epsilon}(p)d\epsilon \leq \int_0^1 I_{ij}^{\epsilon n}(p)d\epsilon$$

for all $(i,j,p) \in I\!I$. Let $CI\!I = \cup_{i=1}^2 CI\!I_i$, where $(\tilde{i},\tilde{j},\tilde{p}) \in CI\!I_1$ if $\int_0^1 I_{\tilde{i}\tilde{j}}^{\epsilon m}(\tilde{p})d\epsilon \leq I_{\tilde{i}\tilde{j}}^{\epsilon m}(\tilde{p})d\epsilon \leq \int_0^1 I_{ij}^{\epsilon n}(p)d\epsilon$ and $(\tilde{i},\tilde{j},\tilde{p}) \in CI\!I_2$ if $\int_0^1 I_{\tilde{i}\tilde{j}}^{\epsilon m}(\tilde{p})d\epsilon \geq I_{\tilde{i}\tilde{j}}^{\epsilon m}(\tilde{p})d\epsilon \leq \int_0^1 I_{ij}^{\epsilon n}(p)d\epsilon$. Therefore we have

$$\int_0^1 I_{\tilde{i}\tilde{j}}^{\epsilon}(\tilde{p})d\epsilon \leq \int_0^1 I_{\tilde{i}\tilde{j}}^{\epsilon m}(\tilde{p})d\epsilon$$

for all $(\tilde{i},\tilde{j},\tilde{p}) \in CI\!I_1$ and

$$\int_0^1 I_{\tilde{i}\tilde{j}}^{\epsilon}(\tilde{p})d\epsilon \geq \int_0^1 I_{\tilde{i}\tilde{j}}^{\epsilon m}(\tilde{p})d\epsilon$$

for all $(\tilde{i},\tilde{j},\tilde{p}) \in C\mathbb{I}_2$. Thus, the second condition ensures $\int_0^1 I_{ij}^{\epsilon}(p)d\epsilon \geq I_{\tilde{i}\tilde{j}}^{\epsilon}(\tilde{p})d\epsilon$ for all $(i,j,p) \in \mathbb{I}$ and all $(\tilde{i},\tilde{j},\tilde{p}) \in C\mathbb{I}_2$, since $\sup_{i,j,p}\left(\int_0^1 I_{ij}^{\epsilon}(p) - I_{ij}^{\epsilon m}(p)d\epsilon\right) \leq (\#H - \#H^m)/\#H$ for all m, i, j and $p \in (0,\infty)^l$. Hence the set of (i,j,p) such that $\gamma(H) = 1 - \int_0^1 I_{ij}^{\epsilon}d\epsilon$ might consist of elements of $\mathbb{I}$, $C\mathbb{I}_1$ and $C\mathbb{I}_2$. ■

The first condition of Proposition 3 implies that for all $(i,j,p) \in \mathbb{I}$ the heterogeneity of subpopulation n has to be the lowest. The second condition says that for subpopulation n the largest expanding area below the step function has to be smaller than the largest diminishing area minus the largest possible size of variation. The second condition is more likely to be satisfied if $\#H^n$ is large compared to the rest of the population.

Weakly Increasing Heterogeneity. Now, we look at a weaker definition of increasing heterogeneity. Since Proposition 2 and Proposition 3 involve complicated conditions, this may allow for more intuitive results. We use a concept that compares the degree of heterogeneity on average.

Definition 3 *Aggregation weakly increases heterogeneity, as measured by γ, if*

$$\gamma(H) \geq \sum_{m=1}^{k} \frac{\#H^m}{\#H}\gamma(H^m)$$

is true.

Before presenting the result by a proposition, we state a lemma.

Lemma 1 *For all i,j and $p \in (0,\infty)^l$, $\gamma_{ij}(p)$ is an element of a convex set. The lower bound is $inf_m \gamma_{ij}^m(p)$ and the upper bound is $sup_m \gamma_{ij}^m(p)$, where all nonempty subpopulations H^m are disjoint and $H = \dot{\bigcup}_{m=1}^{k} H^m$ for every positive integer $k \leq \#H$.*

Proof. We have to prove that $\gamma_{ij}(p) \in [\inf_m \gamma_{ij}^m(p), \sup_m \gamma_{ij}^m(p)]$ for all i,j and $p \in (0,\infty)^l$. For a fixed ϵ, one can infer from the definition of $I_{ij}^{\epsilon}(p)$ that

$$I_{ij}^{\epsilon}(p) = \sum_{m=1}^{k} \frac{\#H^m}{\#H} I_{ij}^{\epsilon m}(p),$$

which is a convex combination of $I_{ij}^{\epsilon m}(p)$ over $m = 1,\ldots,k$. By rearranging, it follows immediately that

$$\gamma_{ij}(p) := 1 - \int_0^1 I_{ij}^{\epsilon}(p)d\epsilon = 1 - \sum_{m=1}^{k} \frac{\#H^m}{\#H} \int_0^1 I_{ij}^{\epsilon m}(p)d\epsilon,$$

which is evidently a convex combination of $1 - \sup_m \int_0^1 I_{ij}^{\epsilon m}(p)d\epsilon$ and $1 - \inf_m \int_0^1 I_{ij}^{\epsilon m}(p)d\epsilon$. ■

Now we ask whether $\gamma(H) \geq \sum_{m=1}^{k}(\#H^m/\#H)\gamma(H^m)$ holds. Preliminarily, this inequality is likely to be satisfied if all $\gamma(H^m)$ are very small, which corresponds to very homogeneous subpopulations, or if $\gamma(H)$ is close to one. The next proposition provides an unambiguous answer.

Proposition 4 *Aggregation weakly generates heterogeneity as measured by γ.*

Before we provide some intuition, let us prove the proposition.

Proof.

$$\begin{aligned}
\inf_{i,j,p}\gamma_{ij}(p) &\geq 1 - \sup_{i,j,p}\sum_{m=1}^{k}\frac{\#H^m}{\#H}\int_0^1 I_{ij}^{\epsilon m}(p)d\epsilon \\
&\geq 1 - \sum_{m=1}^{k}\frac{\#H^m}{\#H}\sup_{i,j,p}\int_0^1 I_{ij}^{\epsilon m}(p)d\epsilon \\
&= \sum_{m=1}^{k}\frac{\#H^m}{\#H}\left(1 - \sup_{i,j,p}\int_0^1 I_{ij}^{\epsilon m}(p)d\epsilon\right) \\
&= \sum_{m=1}^{k}\frac{\#H^m}{\#H}\inf_{i,j,p}\gamma_{ij}^m(p)
\end{aligned}$$

We remark that the first inequality is due to Lemma 1. ■

Intuitively, $\gamma(H)$ is the smallest weighted average over all $\gamma_{ij}^m(p)$ with respect to (i,j,p) due to $\gamma(H) := \inf_{i,j,p}\gamma_{ij}(p)$, while $\sum_{m=1}^{k}(\#H^m/\#H)\gamma(H^m)$ is the weighted average over $\inf_{i,j,p}\gamma_{ij}^m(p)$. Note, the fact that Proposition 4 includes Proposition 1 as a weak increase in heterogeneity rules out a decrease in heterogeneity as defined in Definition 1.

One can infer that the separation of the entire population into homogeneous subgroups changes the structural properties of mean demand. Then we have on average less behavioral heterogeneity and we therefore may lose for example the monotonicity property of mean demand.

Proposition 1 and Proposition 4 are in accordance with results from Kneip (1999) for the coefficient of sensitivity, a measure of structural stability of a population.

3 Conclusion

We derive sufficient conditions for generating behavioral heterogeneity due to aggregation and we show that aggregation weakly generates behavioral heterogeneity. We conclude that restricting attention to homogeneous subgroups of households may not allow one to capture the impacts of behavioral heterogeneity on aggregate values, such as mean demand.

Appendix

Lemma 2 *Consider two populations H and H^* such that $I_{ij}^{\epsilon *}(p) - I_{ij}^{\epsilon}(p) \leq 0$ for given i,j and $\epsilon \in [0,1]$. Then we have*

$$\gamma_{ij}^*(p) - \gamma_{ij}(p) \geq 0.$$

Proof. For $\epsilon \in [0,1]$ we have

$$I_{ij}^{\epsilon *}(p) \leq I_{ij}^{\epsilon}(p) \;\Leftrightarrow\; G_{ij}^{\epsilon *}(p) \geq G_{ij}^{\epsilon}(p).$$

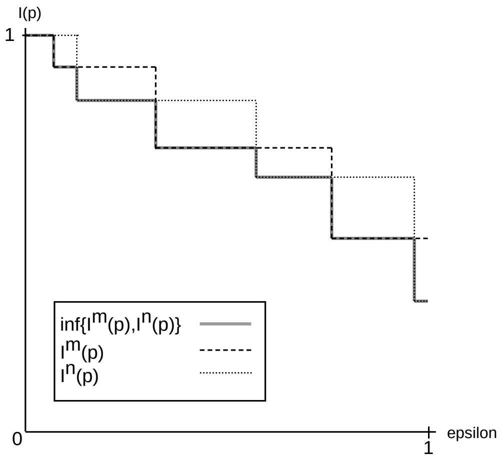

Figure 1: $\int_0^1 \inf\{I_{ij}^{\epsilon m}(p), I_{ij}^{\epsilon n}(p)\}d\epsilon \leq 1 - \sup\{\gamma_{ij}^m(p), \gamma_{ij}^n(p)\}$.

We know that $G_{ij}^{\epsilon}(p) = 1 - I_{ij}^{\epsilon}(p)$ is the cumulative distribution function of $\frac{|s_{ij}^h(p,x^h)|}{d_{ij}^h}$, so $G_{ij}^{\epsilon}(p)$ first order stochastically dominates $G_{ij}^{\epsilon *}(p)$. By definition of first order stochastic dominance one yields

$$\int \epsilon dG_{ij}^{\epsilon}(p) \geq \int \epsilon dG_{ij}^{\epsilon *}(p)$$

which is equivalent to

$$1 - \gamma_{ij}(p) \geq 1 - \gamma_{ij}^*(p).$$

If this inequality holds for all p, i, j, one obtains $\gamma(H^*) \geq \gamma(H)$ by definition of $\gamma(H)$. ■

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