E C O N O M I C S B U L L E T I N

Multi−stage financing and the winner's curse

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Abstract

Raising cash in multiple stages can lower financing costs for an entrepreneur faced with multiple, differentially informed, strategic investors. By affecting investor incentives to participate in different rounds, the winner's curse problem can be partially alleviated. The results offer insight into the choice of the relative size of an IPO versus a SEO, and into gradual and partial privatization strategies.

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1. Introduction

We consider the problem of an entrepreneur trying to raise funds for an investment project by offering equity shares to multiple potential investors. The investors are differentially informed about project prospects and face a winner's curse problem when subscribing to the equity offer. Consequently, the entrepreneur has to incur financing costs in the form of a discount on the shares offered. The discount reflects the informational rents that he has to leave to the investors.

We show that the entrepreneur can lower these financing costs by raising the funds in two stages from the same set of investors, compared to doing so in one stage. By designing the two stages suitably, he can alter the incentives of different classes of investors to participate in each stage, lowering his overall financing costs. In fact, the entrepreneur has recourse to two fundamentally different multi-stage financing strategies that differ in the incentives they provide for investor participation and in the sources of their benefits. We call the first strategy a 'revelation strategy' and the second, an 'exclusion strategy'.

With the revelation strategy, the entrepreneur chooses the size of the first issue to be 'large' enough, inducing informed (and uninformed) investors to participate in it. The outcome of the first stage is informative, providing either favorable or unfavorable information before offers are submitted in the second stage. On average, the effect of revelation of information from the first stage is beneficial, strictly lowering the total financing costs of the entrepreneur.

With the exclusion strategy, in contrast, the entrepreneur chooses the size of the first issue to be 'small' enough, inducing informed investors to *not* participate in the first issue. The outcome of the first stage thus has no information content. Nevertheless, since informed investors do not participate, uninformed investors who do participate in the first stage do not fear a winner's curse problem and bid aggressively. Consequently, the entrepreneur is able to raise part of the funds at zero cost, strictly benefitting in comparison to one-stage financing.

Finally, we show that the exclusion strategy dominates the revelation strategy when the uncertainty about the project prospects is large enough, or when the competition between informed investors is low. The model is related to the large literature on common value auctions and the results offer insight into the choice of the relative size of an initial public offer versus a seasoned equity offer, the prevalence of multi-stage financing in venture capital contexts, as well as gradual and partial privatization strategies for state owned enterprises.[1]

2. The Model and One–Stage Financing

An entrepreneur has encountered an investment project requiring an outlay of \$1. The project yields revenues of X_ω which depend on the state of the world $\omega \in \{H, L\}$ with $X_H > X_L$. Let $\lambda \in (0,1)$ be the prior probability that $\omega = H$ and $X_\lambda \equiv \lambda X_H + (1-\lambda)X_L$ be the expected cash flows from the project. The entrepreneur is risk–neutral and uninformed about ω. He needs to raise the required \$1 from risk–neutral outside investors, who are potentially informed about ω, in return for ownership shares in the project.[2] We normalize the riskless rate to zero and assume

[1] For reasons of space we omit a detailed review of the related literature, referring the reader instead to Chakraborty and Yilmaz (2002).

[2] Our qualitative results extend to the case where the entrepreneur offers other kinds of securities to finance the project. For an analysis of the choice of securities, see Chakraborty and Yilmaz (2002).

that regardless of ω, the project has positive NPV, i.e., $X_L > 1$. Therefore, in a first best world without informational asymmetries, the entrepreneur should invest regardless of ω, e.g., by offering an equity share $\frac{1}{X_\lambda}$.

The set of investors are heterogeneously informed about the project. In particular, there are $m \geq 2$ uninformed investors, indexed by k, who do not possess any private information about ω. In addition, there are potentially $n \geq 1$ informed investors, indexed by i, who know the state of the world ω. All investors and the entrepreneur are uncertain about the number of potential informed investors and assign probability $\beta \in (0, 1)$ to any particular informed investor being present in the market, independently across informed investors and independent of ω. All investors must earn non–negative expected profits in order to be willing to provide funds.

The entrepreneur offers an ownership share that guarantees a fraction $\alpha \in [0, 1]$ of the cash flows to the investors, with the fraction $1 - \alpha$ being his own share. In this section we restrict attention to the benchmark case where the entrepreneur aims to raise all of the necessary funds at one go. In the next section, who compare this with multi–stage financing. With one stage financing, after observing the equity share α offered by the entrepreneur, each uninformed investor offers to buy a fraction $y_k \in [0, 1]$ of the shares offered and each (present) informed investor offers to buy a fraction $y_i(\omega) \in [0, 1]$, as a function of ω, in return from providing the same fraction of funds. Let y be the vector of offers from informed and uninformed investors.

When the offers made by all investors add up to an amount greater than 1, the entrepreneur has to employ a quantity rationing rule. We assume that he has to provide preferential treatment to informed investors. Specifically, each informed investor obtains what he offers, with proportional prorating if the sum of informed offers is greater than the available amount. If there are any shares left over after informed demands are met, then these are allocated to uninformed investors, with proportional prorating among uninformed investors in case the sum of uninformed demands is greater than the amount available for them.[3] Let $z_k(y)$ be the *actual* allocation to the kth uninformed investor, and $z_i(y)$ be that to the ith informed investor, with $z(y)$ being the vector. Then $z_i(y) = \frac{y_i}{\sum_{i'} y_{i'}}$ if $\sum_{i'} y_{i'} > 1$, and equal to y_i otherwise; while, $z_k(y) = 0$ if $\sum_{i'} y_{i'} \geq 1$, equal to $\frac{y_k}{\sum_{k'} y_{k'}}(1 - \sum_{i'} y_{i'})$ if $\sum_{k'} y_{k'} \geq 1 - \sum_{i'} y_{i'} > 0$, and equal to y_k otherwise. Given a share α, offers y and state ω, the payoff to uninformed investor k is $z_k(y)[\alpha X_\omega - 1]$ and that to informed investor i is $z_i(y)[\alpha X_\omega - 1]$.

The possible presence of informed investors creates a winner's curse problem for uninformed investors. When uninformed investors get to invest in the project they know that either informed investors do not exist or chose not to invest. The latter case is a negative signal about the value of the project. Uninformed investors will take this into account when forming their expectations about ω, and so will be unwilling to subscribe unless the issue is offered at a discount relative to the symmetric information world.

Let $W(\alpha)$ be the ex–ante expected payoff to the entrepreneur as a function of α, where we have suppressed its dependence on the offers y to avoid notational clutter. Given α, we will focus on

[3]While this allocation rule has close parallels to those used in prectice in the context of IPOs in the U.S., we use it entirely for reasons of analytical simplicity. Our qualitative results on the costs and benefits of multi–stage financing extend to other allocation rules (e.g., the uniform rule where the entrepreneur allocates the shares proportionately to *all* investors), without the benefit of providing any additional insight.

symmetric Bayesian Nash equilibrium where all informed investors submit identical offers $y_I(\omega)$ and all uninformed investors also submit identical offers y_U. Using the equilibrium of the bidding game between investors, in our first result we characterize the optimal equity share α^* and the associated investment policy, in the benchmark one–stage financing case. The following condition on parameters will aid in stating the result concisely:

$$\frac{\lambda(X_H - X_L)}{X_L} < (1-\lambda)(X_L - 1). \tag{1}$$

Proposition 1 *(One-Stage Financing) (a) If (1) holds, the entrepreneur sets* $\alpha^* = \frac{1}{X_q}$ *where* $X_q \equiv qX_H + (1-q)X_L$ *and*

$$q = \frac{(1-\beta)^n\lambda}{(1-\beta)^n\lambda + 1 - \lambda} \in (0, \lambda). \tag{2}$$

The equilibrium offers are $y_I(H) = 1$, $y_I(L) = 0$ *and* $y_U = 1$. *The entrepreneur's expected payoff is* $W(\alpha^*) = (1-\alpha^*)X_\lambda$ *and he obtains the necessary funds regardless of* ω. *Uninformed investors make zero profits and informed investors make strictly positive profits.*

(b) If (1) does not hold, then there exists $\overline{\beta} \in (0,1)$ *such that the outcome is identical to that for case (a) when* $\beta \leq \overline{\beta}$. *For* $\beta > \overline{\beta}$, *the entrepreneur sets* $\alpha^* = \frac{1}{X_H}$, *equilibrium offers are* $y_I(H) = 1$, $y_I(L) = 0 = y_U$, *the entrepreneur's expected payoff is* $W(\alpha^*) = \lambda(1-(1-\beta)^n)(X_H - 1)$ *and he obtains the necessary funds only when* $\omega = H$ *and at least one informed investor is present. Uninformed investors make zero profits and informed investors make strictly positive profits.*

Proof. Consider first the case where the equity share α is such that the entrepreneur obtains the necessary funds regardless of ω. If uninformed investors conjecture that informed investors participate only when $\omega = H$, (i.e., $y_I(H) = 1, y_I(L) = 0$) then they know that they will obtain any shares (i.e., $z_k(y) > 0$) only when $\omega = H$ and no informed investor is present or when $\omega = L$. Hence, an uninformed investor will ascribe a probability q, given by (2), to the state $\omega = H$, conditional on being allocated any shares. As a result, they will be willing to subscribe (i.e., earn non–negative expected profits) only if $\alpha^* X_q \geq 1$. The lowest such value of α maximizes the entrepreneur's payoff, implying $\alpha^* = \frac{1}{X_q}$. For such α, it is straightforward to check that informed investors will subscribe iff $\omega = H$ and earn positive profits, and uninformed investors will earn zero profits. By design, the entrepreneur will obtain financing regardless of ω.

The entrepreneur may also choose to set α low enough so that he obtains financing only from informed investors and that too when $\omega = H$. The optimal such α is equal to $\frac{1}{X_H}$ and the equilibrium behavior and payoffs are as given by part (b) above. Comparing the two expressions for $W(\alpha^*)$ in cases (a) and (b), straightforward algebra yields that the entrepreneur would like to implement the outcome of part (a) for all β, if (1) holds, or, provided β is lower than a cutoff, if (1) does not, completing the proof.■

The intuition is as follows. Since uninformed investors get shares allocated to them only when informed investors are either absent or do not submit offers, they face a winner's curse problem that is captured by the probability q. As a result, they will not submit offers unless the shares are offered at a discount. Equivalently, the entrepreneur has to offer a higher share $\alpha^* = \frac{1}{X_q}$ of the cash flows than the share $\frac{1}{X_\lambda}$ he would offer in the symmetric information world. As part (b) shows, this may lead the entrepreneur to actually inefficiently underinvest.

To minimize the number of cases to consider, in the rest of this paper we will assume that (PR1) holds. As we show next, by financing the investment in two stages, the entrepreneur will lower the cost of raising capital. Thus, he will find it profitable invest efficiently with multi–stage financing even when he does not find it profitable to do so with one–stage financing.

3. Multi–Stage Financing

We now consider the case where the entrepreneur finances the project in two stages by choosing a fraction x of the \$1 to raise in the first stage and a fraction $1-x$ in the second stage. The same set of informed and uninformed investors are present in both stages and strategically take into account the effect of their behavior in the first stage on the second stage game. The first stage outcome, including the bids, allocations and the amount actually raised, is publicly observed. Thus, the second stage equity share offered by the entrepreneur as well as the bids may depend on the first stage outcome. We assume that if the entrepreneur cannot raise the \$1 after the two stages he has to forego the investment opportunity.

Let α_t denote the 'per-dollar raised' equity share offered in stage $t = 1, 2$, so that $\alpha_t x$ is the total equity share offered in stage 1 and $\alpha_2(1-x)$ that offered in stage 2. As before, we will consider symmetric strategies for the bidding game between the investors, denoting by $y_{t,U}$ the offer made by each uninformed investor and by $y_{t,I}(\omega)$ the offer made by each informed investor, in stage $t = 1, 2$, in equilibrium *and on the path of play.*

We first characterize the optimal exclusion strategy, followed by the optimal revelation strategy, concluding with the characterization of the optimal multi–stage financing strategy.

3.1. The Exclusion Strategy

With the exclusion strategy, the entrepreneur chooses α_1, α_2 and x such that informed investors do not find it in their interest to participate in the first stage. Specifically, x is chosen to be 'small' enough so that it is not profitable for any informed investor to submit an offer in stage 1, revealing his information and eliminating the possibility of second stage profits. Consequently, uninformed investors, when submitting their offers in the first stage, do not suffer from a winner's curse problem and are willing to subscribe to the first issue even without a discount. In contrast, all investors participate in the second issue and so the second issue must be offered at a discount.[4]

Let α_t^E be the optimal per–dollar equity share offered by the entrepreneur under the exclusion strategy in stage t, and let x^E be the dollar amount that he raises in stage 1 with $1 - x^E$ the amount raised in stage 2. Since only uninformed investors participate in stage 1, and all investors participate in stage 2, we are looking for the optimal choice of $x^E$and α_t^E, $t = 1, 2$, such that $y_{t,U} = 1$ for all t, $y_{1,I}(\omega) = 0$ for all ω, $y_{2,I}(H) = 1$ and $y_{2,I}(L) = 0$, in the candidate equilibrium of the bidding game between investors. Since uninformed investors do not face a winner's curse

[4] In all cases, exclusion or revelation, all types of investors will participate in the second stage in equilibrium. Note that we are implicitly assuming that the entrepreneur cannot impose outright 'bans' on any type of investor. If he could do so, he would ban the informed investors and the problem would be uninteresting.

problem, they will participate in the first stage if $\alpha_1^E \geq \frac{1}{X_\lambda}$. On the other hand since all investors participate in the second stage, the entrepreneur must offer a discount in the second stage, in order to guarantee success in fund raising. That is, on the path of play (i.e., given no informed investor has participated in the first stage), we must have $\alpha_2^E \geq \frac{1}{X_q}$, for reasons analogous to those underlying Proposition 1. Clearly, given x^E, the lowest possible values $\frac{1}{X_\lambda}$ and $\frac{1}{X_q}$ of α_1^E and α_2^E respectively are optimal for the entrepreneur.

It remains to show that x^E is chosen such that no informed investor finds it in his interest to participate in the first stage. Since an informed investor has an incentive to do so only when $\omega = H$, we assume that if a bid from an informed investor is observed in the first stage, the entrepreneur and all uninformed investors attach probability 1 to this state. Consequently, after such a first stage deviation by an informed investor, the entrepreneur will set the second stage share equal to $\frac{1}{X_H}$ and obtain financing (at least) from the uninformed investors. Thus, the incentive constraint guaranteeing non–participation from an informed investor in the first stage can be written as:

$$\xi(1 - x^E)(\frac{X_H}{X_q} - 1) \geq x^E(\frac{X_H}{X_\lambda} - 1), \tag{3}$$

where $\xi \equiv \sum_{i=0}^{n-1} \binom{n-1}{i} \beta^i (1-\beta)^{n-1-i} \frac{1}{i+1}$ is the expected second period allocation to an informed investor if he follows his candidate equilibrium strategy. The right–hand side of (3) is the deviation profit to an informed investor from submitting an offer in the first stage, upon observing $\omega = H$. By doing so he foregoes any second period profit, as then the second period share equals $\frac{1}{X_H}$. The left–hand side is the expected profit from behaving as specified in the candidate equilibrium and submitting an order only in the second stage. Since the entrepreneur does not have to offer a discount in the first stage, he will try to set x^E as large as possible subject to (3), so that (3) must indeed bind for the optimal exclusion strategy. Using this, after some manipulation we obtain that the expected payoff to the entrepreneur from the optimal exclusion strategy is

$$W^E \equiv (1 - \frac{1}{X_\lambda}x^E - \frac{1}{X_q}(1 - x^E))X_\lambda = W(\alpha^*) + S^E, \tag{4}$$

where $W(\alpha^*)$ is his payoff from one–stage financing and

$$S^E = x^E(\frac{1}{X_q} - \frac{1}{X_\lambda})X_\lambda > 0, \tag{5}$$

is the expected gain in payoffs (i.e., saving in financing costs) from the exclusion strategy over one–stage financing. We have our next result.

Proposition 2 *(Multi–Stage Financing: Exclusion) With the optimal exclusion strategy,* $\alpha_1^E = \frac{1}{X_\lambda}$, $\alpha_2^E = \frac{1}{X_q}$, x^E *solves (3),* $y_{t,U} = 1$ *for all* t, $y_{1,I}(\omega) = 0$ *for all* ω, $y_{2,I}(H) = 1$ *and* $y_{2,I}(L) = 0$. *The entrepreneur is better off compared to one–stage financing, with the saving in financing costs being given by (5).*

Proof. Follows from the discussion above.■

The intuition behind this result is clear. By designing the sizes of the successive issues so that informed investors do not find it in their interest to participate in the first issue and reveal their information, the entrepreneur can raise part of the $1 from uninformed investors without paying financing costs.

3.2. The Revelation Strategy

We now consider the revelation strategy, where the entrepreneur designs the two issues so that informed investors participate in the first issue and reveal their information, enabling him to raise money in the second issue at lower ex–ante expected cost. While all types of investors will participate in the second stage, the entrepreneur can design the issue so that either (i) only informed investors participate in the first stage, or (ii) both types of investors participate in the first stage. We show below that, for the optimal revelation strategy, case (i) must arise.

Let α_t^R be the stage t per dollar shares and x^R be the dollar amount raised in stage 1, for the optimal revelation strategy. Consider first case (i), a revelation strategy where both types of investors participate in the first stage, i.e., $y_{t,U} = 1 = y_{t,I}(H)$ and $y_{t,I}(L) = 0$ for all t. Given that informed investors are participating at $t = 1$, to guarantee participation from uninformed investors, the entrepreneur must choose $\alpha_1^R \geq \frac{1}{X_q}$; indeed, for the optimal such strategy, the inequality must hold as an equality making uninformed investors exactly indifferent. Given our rationing rule, if any informed investor participates in the first stage, the entrepreneur will conclude that $\omega = H$ and accordingly he will set $\alpha_2^R = \frac{1}{X_H}$ in such cases. On the other hand, if no such investor participates in the first stage, he will attach probability q, given by (2), to $\omega = H$, and will set $\alpha_2^R = \frac{1}{X_q}$. Finally, to give each informed investor the incentive to participate in the first stage, he must choose x^R to satisfy:

$$\xi x^R(\frac{X_H}{X_q} - 1) \geq (1-\beta)^{n-1}(1-x^R)(\frac{X_H}{X_q} - 1). \tag{6}$$

The left–hand side of the inequality is the expected profit to an informed investor in state $\omega = H$, from behaving as specified and submitting a demand in the first round, whereas the right–hand side is his expected profit from not doing so and submitting an offer only in the second stage. Since the expected per–dollar share raised in the second stage is lower than that in the first stage, the entrepreneur's payoff is higher the lower is x^R, so that (6) must hold with equality. Thus, the entrepreneur's expected payoff from this revelation strategy can be written as $W^R = W(\alpha^*) + S^R$ where

$$S^R = \lambda(1-(1-\beta)^n)(1-x^R)(\frac{1}{X_q} - \frac{1}{X_H}))X_H > 0, \tag{7}$$

is the expected saving from the revelation strategy over one–stage financing.

Consider next case (ii), a revelation strategy where the entrepreneur induces uninformed investors not to submit offers in the first period, i.e., $y_{1,U} = 0 = y_{2,I}(L)$ and $y_{2,U} = 1 = y_{t,I}(H)$ for $t = 1, 2$. Let $\widehat{\alpha}_t^R$, and $\widehat{x}^R$ denote the entrepreneur's choices. It is enough to set $\widehat{\alpha}_1^R \in [\frac{1}{X_H}, \frac{1}{X_q})$ in order to he get offers only from informed investors, and only when $\omega = H$. When the first stage succeeds, he sets $\widehat{\alpha}_2^R = \frac{1}{X_H}$ in the second stage and obtains the remaining funds. If it fails however, he has to raise the entire \$1 in the second stage, by offering a share $\widehat{\alpha}_2^R = \frac{1}{X_q}$. For such a strategy the relevant incentive constraint for informed investors to participate in stage 1, given $\omega = H$, is

$$\xi \widehat{x}^R(\widehat{\alpha}_1^R X_H - 1) \geq (1-\beta)^{n-1}(\frac{X_H}{X_q} - 1), \tag{8}$$

while the payoff to the entrepreneur from such revelation strategy is $W = W(\alpha^*) + \widehat{S}$, where

$$\widehat{S} = \lambda(1-(1-\beta)^n)(\frac{1}{X_q} - \widehat{\alpha}_1^R \widehat{x}^R - \frac{1}{X_H}(1-\widehat{x}^R))X_H. \tag{9}$$

Comparing with the expression for S^R in (7) we see that $S^R < \widehat{S}$ if and only if $x^R(\frac{X_H}{X_q} - 1) > \widehat{x}^R(\widehat{\alpha}_1^R X_H - 1)$. But since $x^R < 1$, from (8) and (6) (which binds), one obtains a contradiction. Hence the optimal revelation strategy must induce participation from all investors in both the stages.

Proposition 3 *(Multi–Stage Financing: Revelation) With the optimal revelation strategy $\alpha_1^R = \frac{1}{X_q}$ and $\alpha_2^R = \frac{1}{X_q}$ if informed investors do not bid in stage 1, and equal to $\frac{1}{X_H}$ otherwise. Further, x^R solves (6) and $y_{t,U} = 1 = y_{t,I}(H)$ and $y_{t,I}(L) = 0$ for all t.The entrepreneur is strictly better off compared to one–stage financing, with the saving in financing costs being given by (7).*

Proof. Follows from the discussion above.■

By inducing informed participation in stage 1, the entrepreneur is able to ensure information revelation and able to sell the shares at a lower cost in the second stage. Furthermore, he also induces uninformed participation in stage 1. If instead he excludes the latter, he worsens the incentive constraint for informed investors— when uninformed investors are not present to soak up any excess supply in stage 1, the expected deviation gain to an informed investor from not submitting an early offer is higher. To preserve incentives, the entrepreneur has to raise a larger dollar amount in stage 1, making him worse off.

3.3. Optimal Financing

Proposition 4 *(Optimal Financing) Exclusion dominates revelation iff $x^E > 1 - x^R$ which occurs iff:*

$$\frac{\lambda X_H}{(1-\lambda)X_L} > \frac{1-(1-\beta)^{n-1}}{\beta(1-\beta)^{n-1}} \tag{10}$$

Proof. Straightforward manipulation of the definitions of S^R and S^E in (7) and (5), using (2), yields $S^E > S^R$ iff $x^E > 1 - x^R$. Solving for x^E and x^R, using (3) and (6) (which hold with equality), and comparing, we then obtain (10).■

For the exclusion strategy, the funds raised in the first stage have lower costs than the funds raised in the second stage. The converse is true for the revelation strategy. Proposition 4 shows that whichever strategy allows the larger amount to be raised at lower cost is optimal. In terms of parameters, as n becomes large, x^E must become smaller in order to preserve incentives for informed investors to avoid stage 1. However, for the same reason, x^R also becomes smaller, making the revelation strategy more attractive. Asymptotically, as n grows to infinity, the revelation strategy yields the symmetric information payoffs to the entrepreneur.

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