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Constructing a measure of industry−specific human capital using Tobin's q theory

Kam−Ki Tang
School of Economics, University of Queensland

Yi−Ping Tseng
Melbourne Institute of Applied Economic and Social Research, University of Melbourne

Abstract

In this paper, we develop a measure of industry specific human capital using the Tobin's q theory. The measure is derived from a structural model of heterogeneous knowledge labor, which is homogeneous physical labor embodied with industry specific human capital.

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1. Introduction

Increasingly, "human capital" has assumed a central place in various level of analysis of economic growth and development. At an individual level, human capital is influential in determining a person's income, employment status and labor mobility (e.g., Heckman, Lochner and Taber 1998). Macroeconomists have also identified human capital's important role in enhancing a country's overall economic performance (Barro 1999). In turn, the development and promotion of human capital has become a central feature of international policy discussions (e.g., OECD 1994).

Accurate measures of human capital are an essential prerequisite for the scientific discussion of growth and economic performance, as well as for the proper evaluation of education, training, and industry policies. However, there are some practical complications. The term human capital covers a wide range of elements including: knowledge accumulated via education; skills acquired by training; experience gained during employment; ideas and inventions developed in research, or even personal networks established in the workplace. This conceptual depth imposes obvious limitations on the definition of a single summary measure of human capital.

This dilemma has lead researchers to employ a range of indicators for human capital. For example, the most commonly used measure of general human capital is educational attainment. Both Barro and Lee (2000) and de la Fuente and Demenech (2000) have made concentrated efforts to build accurate, cross-country databases for educational attainment. The specific dimensions of human capital are then accounted for by other indicators such as: average years of schooling; test scores; labor market experience; and cognitive ability. However, integrated approaches to measuring human capital are becoming more common. Fernandez and Mauro (2000) draw on Jorgenson's seminal work to devise a human capital index that weights labor inputs by educational attainment and wages. Hanushek and Kim (2000) also accounts for the heterogeneity of labor quality at the cross-country level.

In comparison, quantitative assessment of industry-specific human capital (ISHC) is far less developed, despite empirical evidence indicating that it is an important determinant of inter-industry wage differential and labor mobility (Neal 1995; Kim 1997). The difficulty with measuring industry-specific human capital is the lack of a good proxy for the effects of experience and benefits of training.

Against this background, this paper aims at developing a measure of ISHC from a structural model of knowledge labor. The essence of the paper is to establish the theoretical linkage between unobserved ISHC and observed return to industry experience. The link is the Tobin's q of ISHC. In our model, ISHC is interpreted as experience and know-how, in contrast to the interpretation of idea and invention in the growth literature, but compatible to the interpretation of skill in labor economics. Our method thus provides a complementary measure of human capital to the traditional method of using education attainment.

Tobin's q theory has been widely used in examining physical capital investment and firm performance. However, little attempt has made to apply the Tobin's q theory in human capital assessment. Therefore, this paper can be viewed as a new application of the Tobin's q theory.

2. Theoretical Model

Our model distinguishes between knowledge labor and physical labor. Production of an industry requires knowledge labor specific to that industry. A heterogeneous knowledge labor is a homogeneous physical labor that has accumulated ISHC. Labor accumulates ISHC through a learning-by-doing process, which incurs an implicit learning cost, akin to the installation cost of physical capital (Lucas 1967; Hayashi 1982).

The objective of a representative household is to maximize its intertemporal utility function

$$\int_t^{\infty} U\left(C, \tilde{L}\right) e^{-\mu\tau} d\tau \text{ subject to}$$

$$\sum_i W_i^Z L_i^Z + rA = PC + \dot{A} \text{ (financial budget constraint)} \qquad (1)$$

$$1 = \sum_i L_i + \tilde{L} \text{ (labor supply constraint)} \qquad (2)$$

$$L_i^Z = Z_i^{\beta_i} L_i^{1-\beta_i} \text{ (composition of knowledge labor)} \qquad (3)$$

$$L_i = J_i^Z \left(1 + \frac{\phi_i^Z}{2} \frac{J_i^Z}{Z_i}\right) \text{ (gross investment in ISHC)} \qquad (4)$$

$$\dot{Z}_i = J_i^Z - \delta_i^Z Z_i \text{ (accumulation of ISHC)} \qquad (5)$$

where μ is the subjective discount rate; C is consumption; P is a price index; $\tilde{L}$ is leisure hours; A is financial asset holding; L_i and L_i^Z are the physical and knowledge labor hours deployed in industry i, respectively; J_i^Z is fixed formation of ISHC − Z_i; and δ_i^Z and ϕ_i^Z are the depreciation rate and learning cost coefficient of ISHC, respectively. Time subscript t is omitted for notational cleanness.

Equation (1) is the financial budget constraint of the household, who distribute physical labor hours amongst various industries. Equation (2) is the total time constraint, which is normalized to one. Equation (3) indicates that knowledge labor is a Cobb-Douglas combination of both ISHC and physical labor.[1] The multiplicative functional form of L_i^Z implies that ISHC has to be coded into a physical labor for it to be operational. Equation (4) indicates that more ISHC will be acquired as a result of longer accumulated working hours in an industry. The inputs of physical labor in production and human capital accumulation are not exclusive, signifying that the latter is

[1] With a Cobb-Douglas function, allowing β_i to differ across industries will not change the equilibrium ISHC intensity of an industry.

a learning-by-doing process.[2] Gross investment in ISHC is partially consumed as learning costs – $\frac{\phi_i^Z}{2}\frac{\left(J_i^Z\right)^2}{Z_i}$, which is implicit due to the learning-by-doing mechanism and, thus, does not enter the household's budget constraint. Since the learning cost is a quadratic function of J_i^Z, investment in ISHC is irreversible.

The Lagrangian of the optimal control problem is

$$h = U(C,\tilde{L}) + \lambda^A\left(\sum_i W_i^Z Z_i^{\beta} L_i^{1-\beta} - PC + rA\right) + \lambda^L\left(1 - \sum_i L_i - \tilde{L}\right)$$
$$+\sum_i\left\{\lambda_i^Z\left(J_i^Z - \delta_i^Z Z_i\right) + \lambda_i^J\left[L_i - J_i^Z\left(1 + \frac{\phi_i^Z}{2}\frac{J_i^Z}{Z_i}\right)\right]\right\} .$$

Solving the optimization problem, we obtain, amongst other first order conditions,

$$\frac{\lambda_i^Z}{\lambda_i^J} = 1 + \phi_i^Z \frac{J_i^Z}{Z_i} \qquad (6)$$

where λ_i^Z is the shadow value of ISHC, and λ_i^J is the shadow cost of gross investment in ISHC. The first order condition in relation to $\dot{\lambda}_i^Z$ can be integrated to give:

$$\lambda_i^Z = \int_{s=t}^{\infty}\left[\lambda^A \beta_i W_i^Z\left(\frac{L_i}{Z_i}\right)^{1-\beta_i} + \lambda_i^J\left(\frac{\phi_i^Z}{2}\right)\left(\frac{J_i^Z}{Z_i}\right)^2\right]e^{-(\mu+\delta_i^Z)(s-t)}ds . \qquad (7)$$

Equation (7) states that the value of a unit of ISHC at a given time equals the discounted valued of its future marginal value. The first term inside the bracket is the value of the financial reward of increasing the supply of knowledge labor. The second term is the value of the reduction of learning cost in ISHC accumulation. Therefore, the ratio of λ_i^Z to λ_i^J can be naturally interpreted as the Tobin's *q* of ISHC:

$$q_i^Z \equiv \frac{\lambda_i^Z}{\lambda_i^L} . \qquad (8)$$

[2] The process can be modified into learning-by-training by specifying a share of physical labor hours spent on human capital investment. As long as the share is constant, it will not change the properties of the model.

In the case of physical capital, Tobin's q can be measured by the market value of outstanding securities of a firm divided by the replacement cost of its net assets.[3] Nevertheless, the market value and replacement cost of ISHC are not directly measurable. Therefore, proxies have to be called in for empirical work. The closest proxy for q_i^Z is probably the wage differentials between staffs with industry experience and those without, after controlling for other characteristics:

$$q_i^Z - 1 = \frac{\text{wage rate of experienced staff}}{\text{wage rate of inexperienced staff}} - 1 = \text{marginal return to industry experience.} \quad (9)$$

The imputed value of q_i^Z has a central role in the estimation of ISHC, as shown below.

3. Industry-specific Human Capital Index

Using (4) – (8), it can be deduced that at steady state

$$ISHC_i \equiv \frac{Z_i}{L_i} = \frac{2}{\delta_i^Z (2 + \phi_i^Z \delta_i^Z)} \quad (10)$$

$$q_i^Z = 1 + \phi_i^Z \delta_i^Z . \quad (11)$$

Here we define $\frac{Z_i}{L_i}$ as industry-specific human capital intensity $- ISHC_i$.[4] To empirically construct $ISHC_i$, we need to know the value of either δ_i^Z or ϕ_i^Z. However, the values of both parameters are unobservable. A solution is to impose a theoretical relationship between the two parameters.

Firstly, for a given finite value of ϕ_i^Z, as δ_i^Z tends to zero, Z_i will tend to infinitely large. Therefore, industry *i* will dominate the economy; this thus violates our competitive market framework. To exclude this scenario, it is necessary to specify that a sector that can accumulate more ISHC (i.e. a small δ_i^Z) incurs greater learning costs (i.e. a large ϕ_i^Z). Secondly, the

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[3] Under the assumptions of competitive output and capital markets, and linear homogenous production and adjustment cost functions, marginal Tobin's q is equivalent to average Tobin's q (Hayashi 1982). While the former is not observable or measurable, the latter is. In our model, the labor market is competitive and the learning cost function is linearly homogenous.

[4] In the case of heterogeneous labor input, (10) can be generalized to $ISHC_i \equiv \sum_j \theta_{ij} ISHC_{ij}$ where θ_{ij} is the weighting of the *j*-th type workers in industry *i*. θ_{ij} can be measured by the share of wage bills: $\theta_{ij} = W_{ij}^Z L_{ij}^Z / \sum_j W_{ij}^Z L_{ij}^Z$.

following inequalities must prevail: $1 \geq \delta_i^Z \geq 0$; $\infty > \phi_i^Z \geq 0$; and $\infty > q_i^Z \geq 1$.[5] To satisfy the above inequalities, as well as capture the negative relationship between δ_i^Z and ϕ_i^Z, we postulate that

$$\delta_i^Z = \frac{(\bar{q}-1)}{(\bar{q}-1)+\phi_i^Z}; \; \bar{q} > 1 . \tag{12}$$

Substituting (12) into (11), we get

$$q_i^Z = \bar{q} - (\bar{q}-1)\delta_i^Z . \tag{13}$$

$\bar{q}$ is the upper bound of q_i^Z. This can be seen from that if $\delta_i^Z \to 0$, then $\phi_i^Z \to \infty$ and $q_i^Z \to \bar{q}$. On the other hand, if $\delta_i^Z \to 1$, then $\phi_i^Z \to 0$ and $q_i^Z \to 1$. That is, if ISHC is costless to accumulate but quick to depreciate, industry experience will not be valued at all.

Substituting (12) and (13) into (10), we obtain a measure of ISHC intensity:

$$ISHC_i = \frac{2(\bar{q}-1)}{(\bar{q}-q_i^Z)(1+q_i^Z)} . \tag{14}$$

A condition for $ISHC_i$ to be strictly monotonic increasing in q_i^Z is $3 > \bar{q} > 1$.[6] The value of $\bar{q}$ can be specified arbitrarily (within the inequality constraint). Choosing different values for $\bar{q}$ will affect the scaling of the measurement but not the ordering. To guarantee $\bar{q}$ being the upper bound limit, we can choose a value according to the greatest *observed* value of q_i^Z. Econometric estimations of wage equations suggest that return to experience typically is only about a few percentage points per annum; see, e.g., Preston (1997) and Chang and Miller (1996). Therefore, using this specification method, it is likely to assign $\bar{q}$ a value between 1 and 1.1, and the inequality $3 > \bar{q} > 1$ will be comfortably satisfied.

4. Conclusions

In this paper, we integrate together two prominent theories in economics – Tobin's *q* theory and human capital theory – to develop a measure of ISHC. The measure is derived from a dynamic optimization model of heterogeneous knowledge labor. Making use of the concept of Tobin's *q*, we construct a measure of ISHC in terms of marginal return to industry experience. Lastly, while

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[5] These imply $\infty > ISHC_i \geq 1$.

[6] This is because $q_i^Z \geq 1$ and $\frac{\partial ISHC_i}{\partial q_i^Z} = \frac{-2(\bar{q}-1)(\bar{q}-1-2q_i^Z)}{(\bar{q}-q_i^Z)^2(1+q_i^Z)^2}$.

the model is constructed in terms of ISHC, the whole framework is equally applicable to firm specific human capital. The only modification will be to measure Tobin's *q* in terms of marginal returns to firm experience.

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