E C O N O M I C S   B U L L E T I N

Investment efficiency and intergenerational income distribution: a paradoxical result

Akiomi Kitagawa
Tohoku University

Ryo Horii
Osaka University

Koichi Futagami
Osaka University

Abstract

Using a simple overlapping generations model, this note shows that an improvement in the efficiency of human capital investment decreases the net income of the young household while increasing that of the old. Without compensating redistribution, it deteriorates lifetime utilities of all generations except for the initial old households.

We are grateful for helpful comments and suggestions from an anonymous referee. All remaining errors are naturally ours.
Citation: Kitagawa, Akiomi, Ryo Horii, and Koichi Futagami, (2004) "Investment efficiency and intergenerational income distribution: a paradoxical result." *Economics Bulletin,* Vol. 15, No. 2 pp. 1−6
Submitted: October 4, 2004. **Accepted:** November 11, 2004.
URL: http://www.economicsbulletin.com/2004/volume15/EB−04O40005A.pdf

1. Introduction

In the literature of endogenous growth, it is well known that the efficiency of human capital investment critically affects the long-term path of aggregate output.[1] This note demonstrates that the investment efficiency also has a nontrivial effect on the intergenerational distribution of income, affecting adversely the welfare of households in the long run. We present a simple overlapping generations model in which young households invest in the process of human capital production (namely, education). With a higher investment efficiency, each young household increases the supply of human capital. The increased aggregate supply of human capital, on one hand, raises the productivity of physical capital and the interest rate, thereby raising the income of old households. On the other hand, the net income of young households is reduced because the increased aggregate supply lowers the price of their innate human capital (i.e., wages) while the revenue from selling extra human capital is offset by the interest payments on investment expenses.[2] Although the initial old households unilaterally benefit from increased interest earnings, the overall welfare effect on subsequent generations is shown to be negative under plausible parameter values.

2. Model

Production Technology. A version of Diamond's (1965) overlapping generations model is considered, in which time is divided into periods $t = 1, 2, \cdots, \infty$. In each period, a single final good, denoted by Y_t, is competitively produced from physical capital K_t and human capital H_t by a Cobb-Douglas technology. The production function is $Y_t = AK_t^{\alpha} H_t^{1-\alpha}$, where $A > 0$ and $\alpha \in (0, 1)$, respectively, represent total factor productivity and the share of physical capital. Factor markets are perfectly competitive, so that the market price of physical capital r_t and that of human capital w_t, in terms of the final good, are determined by their marginal

[1] In a model with a representative agent, Lucas (1988) and many others have shown that the long-term rate of economic growth is an increasing function of the efficiency of human capital investment. Our separate paper (Kitagawa, Horii and Futagami 2003) shows in an overlapping generations setting that an expansion in the maximum rate at which human capital can be accumulated (e.g., a greater availability of higher education) has non-monotonic effects on the long-term rate of growth.

[2] In a static model of child labor, Basu and Van (1998) also showed that an exogenous expansion of labor-supply capacity does not necessarily benefit its suppliers.

productivities:

$$r_t = A\alpha(K_t/H_t)^{\alpha-1}, \quad w_t = A(1-\alpha)(K_t/H_t)^{\alpha}. \tag{1}$$

The final goods produced in a certain period can either be consumed in that period or be saved for production in the next period. Once saved, the good can either be used as an input to human capital production or be used directly as physical capital, which implies that the price of the saved good (i.e. the interest rate) is r_t. Human and physical capital depreciates within one period and therefore cannot be carried over to subsequent periods.

Households. At each period, there are two generations of households, which we call the young and the old. Each generation contains a unit mass of households and lives for two periods. The objective of the generation t households (those born at period t) is to maximize their lifetime utility

$$u_t = (1-\beta)\ln c_{1t} + \beta \ln c_{2\,t+1}, \tag{2}$$

where $\beta \in (0,1)$ is a parameter specifying the patience of agents and c_{1t} and $c_{2\,t+1}$ represent their consumption in youth and old age. Each young household is endowed with $\delta > 0$ units of human capital, which are either possessed innately or obtained through home education without explicit expenditure.[3] In addition, they can augment their human capital through investment (i.e., higher education), which must be financed by borrowing from the old. Let $e_t \geq 0$ be the amount of saved goods borrowed from the old generation to invest in this process. Then the total amount of their human capital is

$$H_t = \delta + \gamma e_t, \tag{3}$$

where parameter $\gamma \geq 0$ represents the efficiency of human capital investment, affected by such factors as the quality of the education environment in the economy. They sell off their human capital at market price w_t and in return receive $w_t(\delta+\gamma e_t)$ units of the final good. After repaying $r_t e_t$ units of the final good, they consume part of their net income at the end of that period and save the remainder for consumption in their old age. The intertemporal budget constraint is

$$c_{1t} + c_{2\,t+1}/r_{t+1} = w_t(\delta + \gamma e_t) - r_t e_t. \tag{4}$$

[3]It is natural to assume that individuals have some ability to work even without no formal education (e.g., child labor in least developed countries). Assumption $\delta > 0$ is also required to obtain sensible results; if we set $\delta = 0$, then equation (7) below implies that both consumption and saving are always zero.

For every $t \geq 0$, the generation t households choose e_t, c_{1t} and $c_{2\,t+1}$ so as to maximize (2) under constraint (4). Net income on the right hand side of (4) is maximized by choosing

$$e_t \begin{cases} = 0 & \text{if } \gamma w_t - r_t < 0; \\ \in [0, +\infty) & \text{if } \gamma w_t - r_t = 0; \\ = +\infty & \text{if } \gamma w_t - r_t > 0. \end{cases} \tag{5}$$

Since human capital investment must be finite in equilibrium, condition (5) implies that

$$\gamma w_t - r_t \leq 0 \text{ with equality whenever } e_t > 0. \tag{6}$$

From (6), we see that the maximized net income is δw_t. Since the equilibrium rate of return from human capital investment is zero, the net income of the young household is simply the market value of endowed human capital. Then, from (2) and (4), the consumption and savings of generation $t \geq 0$ households in equilibrium are written in terms of factor prices:

$$c_{1t} = (1-\beta)\delta w_t, \;\; c_{2\,t+1} = \beta\delta r_{t+1} w_t, \;\; S_t = \beta\delta w_t. \tag{7}$$

At period 0, the initial old (generation -1) households are endowed with $S_0 > 0$ units of saved goods and consume $r_0 S_0$ units of final goods in exchange for their endowment.

Equilibrium. Substituting (1) and (3) into condition (6) gives a relation between two kinds of capital,

$$H_t = \max\left\{\gamma \frac{1-\alpha}{\alpha} K_t, \delta\right\}. \tag{8}$$

Aggregate demand for the saved good consists of demand for physical capital K_t and demand for input to human capital investment. From (3), the latter is $\gamma^{-1}(H_t - \delta)$. Thus, the market-clearing condition for saved goods is

$$K_t + \gamma^{-1}(H_t - \delta) = S_{t-1}, \tag{9}$$

where S_{t-1} is the savings of generation $t-1$ households. Given S_{t-1}, (8) and (9) determine the equilibrium (K_t, H_t) pair,

$$\big(K_t, H_t\big) = \begin{cases} \big(S_{t-1}, \delta\big) & \text{if } S_{t-1} \leq \widehat{S}(\gamma); \\ \big(\alpha(S_{t-1} + \delta/\gamma), (1-\alpha)(\gamma S_{t-1} + \delta)\big) & \text{if } S_{t-1} > \widehat{S}(\gamma), \end{cases} \tag{10}$$

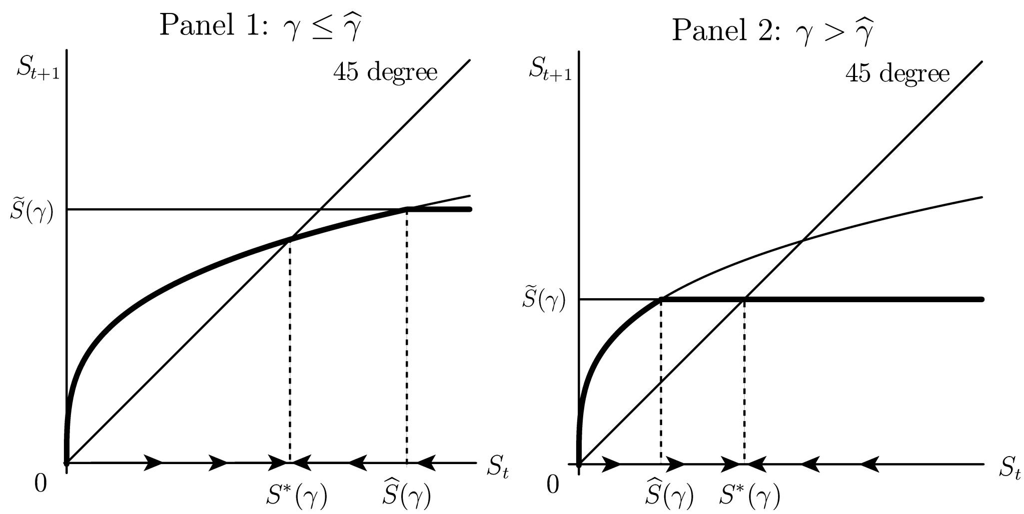

Figure 1: Saving Dynamics. The saving locus is first increasing and then becomes flat at $\widetilde{S}(\gamma) \equiv A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\beta\delta\gamma^{-\alpha}$. The horizontal line at $\widetilde{S}(\gamma)$ shifts down as γ increases.

where $\widehat{S}(\gamma) \equiv \alpha\delta/((1-\alpha)\gamma)$. Equation (10) shows that the young households invest in human capital if and only if S_{t-1} is larger than $\widehat{S}(\gamma)$.

Substituting (1) and (10) into (7) yields the saving dynamics:

$$S_t = \begin{cases} A(1-\alpha)\beta\delta^{1-\alpha}S_{t-1}^{\alpha} & \text{if } S_{t-1} \leq \widehat{S}(\gamma); \\ A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\beta\delta\gamma^{-\alpha} & \text{if } S_{t-1} > \widehat{S}(\gamma). \end{cases} \tag{11}$$

Given initial $S_0 > 0$, equation (11) generates the equilibrium sequence of aggregate saving. As shown by Figure 1, the sequence monotonically converges to

$$S^*(\gamma) = \begin{cases} (A(1-\alpha)\beta)^{1/(1-\alpha)}\delta \leq \widehat{S}(\gamma) & \text{if } \gamma \leq \widehat{\gamma}; \\ A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\beta\delta\gamma^{-\alpha} > \widehat{S}(\gamma) & \text{if } \gamma > \widehat{\gamma}, \end{cases} \tag{12}$$

where $\widehat{\gamma} \equiv \alpha\left(\beta A(1-\alpha)^{2-\alpha}\right)^{-1/(1-\alpha)}$. The pair of factor prices in the steady state is obtained by substituting (12) for (1):

$$\big(w^*(\gamma), r^*(\gamma)\big) = \begin{cases} \left((A(1-\alpha)\beta^{\alpha})^{1/(1-\alpha)}, \alpha/((1-\alpha)\beta)\right) & \text{if } \gamma \leq \widehat{\gamma}; \\ \left(A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\gamma^{-\alpha}, A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\gamma^{1-\alpha}\right) & \text{if } \gamma > \widehat{\gamma}. \end{cases} \tag{13}$$

3. Implications of a Higher Investment Efficiency

Intergenerational Income Distribution. Equation (12) implies that in the long run households invest in human capital only if $\gamma > \widehat{\gamma}$. When γ is in this range, (13) shows that a higher efficiency of human capital investment increases the interest rate but reduces the wage rate in the steady state. Then, from (7), consumption of young households decreases, whereas that of old households tends to increase. The intuition behind this redistributional effect is as follows. Since the net income of young households is the market value of their endowed human capital, δw_t, the increased aggregate supply of human capital reduces the market value of their endowment and therefore their net income. It decreases their young-age consumption because they always find it optimal to consume a constant fraction of the net income. Their savings are also low, but the old-age consumption can be higher because the increased supply of human capital boosts the interest rate. Specifically, when $\gamma > \widehat{\gamma}$, consumption of old households is $(A\alpha^{\alpha}(1-\alpha)^{1-\alpha})^2\beta\delta\gamma^{1-2\alpha}$, which is increasing in γ given that $\alpha < 1/2$. We assume, reasonably, that $\alpha < 1/2$ because α is the share of physical (non human) capital.

Welfare effects. We now examine how the change in the intergenerational income distribution affects the overall welfare of households. Substituting (7) and (13) into the utility function (2) gives the lifetime utility of consumers in the steady state:

$$\begin{aligned} u^*(\gamma) &= \text{constant} + \ln w^*(\gamma) + \beta \ln r^*(\gamma) \\ &= \text{constant} + (\beta(1-\alpha) - \alpha) \ln \gamma. \end{aligned} \tag{14}$$

Lifetime utility is decreasing in γ if (and only if)

$$\beta < \alpha/(1-\alpha). \tag{15}$$

Recall that α is the share of physical capital while β is the young agents' propensity to save. Using a conventional value of 0.3 for α, condition (15) becomes $\beta < 0.428$, which is met under plausible values for β. In addition, (15) coincides with the condition for the economy to be dynamically efficient for all γ in the steady state because applying (15) for (13) gives $r^*(\gamma) \geq \alpha/((1-\alpha)\beta) > 1$.[4] Therefore, given that parameters are within an empirically plausible range or in a range that guarantees the economy's dynamic efficiency, the utility loss among young households dominates the (discounted) utility gain that can be enjoyed later when they become old.

Who benefits? One may wonder why relaxing one of the resource constraints in the economy results in an adverse consequence. To be precise, the economy with a

[4] Since population is constant over time, dynamic efficiency requires the gross interest rate to be higher than 1.

high γ is *not* Pareto inferior to the economy with a low γ, because consumption of the initial old households is higher in the economy with a high γ.[5] Contrary to the usual perception, a higher investment efficiency actually benefit the old generation who has already finished the investment process while thrusting future generations into an 'educational rat race'.

Compensating Policies. When combined with appropriate redistributional policies, however, a better environment for human capital investment can has a positive effect on the welfare of all generations. Substituting (10) into the production function and then differentiating it with respect to γ yields

$$\frac{\partial Y_t}{\partial \gamma} = \begin{cases} 0 & \text{if } S_{t-1} \leq \widehat{S}(\gamma). \\ A\alpha^{\alpha}\left((1-\alpha)\gamma\right)^{-\alpha}\left(S_{t-1} - \widehat{S}(\gamma)\right) > 0 & \text{if } S_{t-1} > \widehat{S}(\gamma). \end{cases} \tag{16}$$

Given S_{t-1}, (16) shows that aggregate output is increasing in γ whenever young households invest in human capital. Thus, old households benefit from a larger γ even when the authority implements a lump-sum redistribution policy that transfers income from the old to the young so that the income of young households (and therefore their savings) are unaffected by the increase in γ. When continued forever, this combination of a larger γ and the intergenerational transfer benefits all generations because they can enjoy more consumption when old while consumption in their youth is unchanged. In an economy with a highly developed education system, this argument legitimizes income transfers from old to young in the forms of grants and scholarships funded by taxes on the elder generation.[6]

References

Basu, K. and P. H. Van (1998) "The economics of child labor" *American Economic Review* 88, 412-427.

[5]Recall that initial old households consume $r_0 S_0$ units of goods, where r_0 is (weakly) increasing in γ and S_0 is historically given.

[6]In the U.S., the percentage of students receiving grants and the average amounts received by students with grant aid have increased between 1990 and 2000, which seems to be mitigating the problem. (Choy, 2004). By contrast, the Japanese government has recently abolished a scholarship loan forgiveness program so that all scholarships received by Japanese residents must be repaid after graduation. Although the supply of scholarship loans has been increased in compensation, our model predicts that this change will aggravate intergenerational income inequality in the long run.

Choy, S. P. (2004) "Paying for college: changes between 1990 and 2000 for full-time dependent undergraduates" NCES 2004-075, U.S. Department of Education, National Center for Education Statistics.

Diamond, P. A. (1965) "National debt in a neoclassical growth model" *American Economic Review* 55, 1126-1150.

Kitagawa, A.; R. Horii; and K. Futagami (2003) "Availability of higher education and long-term economic growth" Discussion Papers in Economics and Business 03-14, Osaka University.

Lucas, R. E. (1988) "On the mechanics of economic development" *Journal of Monetary Economics* 22, 3-42.