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Unit root cycles in the US unemployment rate

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Abstract

The annual structure of the U.S. unemployment rate is examined in this article by means of new statistical techniques developed by Robinson (1994), which permit us to test unit root cycles in raw time series. The tests have standard null and local limit distributions and unlike other procedures, they allow us to determine the number of periods per cycle. The results show that the cycles in the U.S. unemployment seem to occur approximately every four or five years.

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1. Introduction

It is a well-known stylised fact that many macroeconomic time series contain cyclical components. However, there is still little consensus about which may be the best statistical way of modelling this behaviour. Initially, deterministic approaches based on trigonometric functions of time were proposed, but they were shown to be inappropriate in many series. Then, stochastic approaches based on autoregressive (AR) processes (see, e.g. Harvey, 1985) were considered. These models assume that the time series is stationary, (or at least that it has been previously transformed via first or second differences). However, we know that most of economic and financial time series are nonstationary, and first (or second) differences may not be the appropriate transformation. Unit root cycles have been studied in Ahtola and Tiao (1987) and Gray et al. (1989, 1994). In the first of these articles, they propose tests for unit root cycles, which are embedded in an AR(2) process of form:

$$x_t \;=\; \phi_1\, x_{t-1} \;+\; \phi_2\, x_{t-2} \;+\; u_t \qquad (1)$$

which, under the null hypothesis H_o: $|\phi_1| < 2$ and $\phi_2 = -1$, it becomes the cyclical unit root model. Gray et al. (1989, 1994) also propose unit root cycles, but unlike Ahtola and Tiao (1987), they are not nested in an AR model but in a fractional structure of form:

$$(1 \;-\; 2\mu L \;+\; L^2)^d\, x_t \;=\; u_t, \quad t \;=\; 1, 2, ..., \qquad (2)$$

where L is the lag-operator (i.e., $Lx_t = x_{t-1}$), and where the unit root null corresponds to d = 1.[1] Robinson (1994) examined tests of this hypothesis along with other real values of d. These tests have standard null and local limit distributions, and they permit us to test for unit root cycles at any frequency of the spectrum and thus, we are able to approximate the number of periods per cycle.[2] In a recent article, Gil-Alana (2001) shows that the tests of Robinson (1994) outperform Ahtola and Tiao (1984) in a number of cases.

In this article we use a version of the tests of Robinson (1994) for testing unit root cycles in the US unemployment rate. This is a new alternative way of modelling this series and unlike most of the empirical work, we do not assume first differences on the original data but test for first differences on its cyclical behaviour. The structure of the paper is as follows: In Section 2 we briefly describe the version of the tests of Robinson (1994) used in the article. In Section 3, we apply the tests to the US unemployment rate, while Section 4 contains some concluding comments.

2. Testing for unit root cycles with the tests of Robinson (1994)

Robinson (1994) considers the following regression model,

$$y_t \;=\; \beta' z_t \;+\; x_t, \quad t = 1, 2, ..., \qquad (3)$$

where y_t is the raw time series we observe; β is a (kx1) vector of unknown parameters; z_t is a (kx1) vector of exogenous regressors, and the regression errors x_t are such that:

[1] Unit root cycles were also examined among others by Chan and Wei (1988) and Gregoir (1999a, b).
[2] These tests impose the number of periods per cycle to be a given number and thus, they permit us to test unit root cycles for different periods per cycle.

$$(1 - 2\cos w_r\, L + L^2)^d \; x_t \; = \; u_t, \qquad t \; = \; 1, 2, ...,, \tag{4}$$

with I(0) u_t,[3] and $w_r = 2\pi/r$, r being the number of periods per cycle. A Lagrange Multiplier (LM) test of the null hypothesis:

$$H_o: \; d \; = \; 1, \tag{5}$$

in (3) and (4) is then given by:

$$\hat{R} \; = \; \hat{r}^2; \quad \hat{r} \; = \; \frac{T^{1/2}}{\hat{\sigma}^2} \hat{A}^{-1/2} \, \hat{a} \tag{6}$$

where T is the sample size and

$$\hat{a} \; = \; \frac{-2\pi}{T} \sum_{j=1}^{*} \psi(\lambda_j)\, g(\lambda_j; \hat{\tau})^{-1} \, I(\lambda_j); \qquad \hat{\sigma}^2 \; = \; \sigma^2(\hat{\tau}) \; = \; \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j; \hat{\tau})^{-1} \, I(\lambda_j);$$

$$\hat{A} = \; \frac{2}{T} \left(\sum_{s=1}^{*} \psi(\lambda_s)^2 \; - \; \sum_{s=1}^{*} \psi(\lambda_s)\, \hat{\varepsilon}(\lambda_s)' \left(\sum_{s=1}^{*} \hat{\varepsilon}(\lambda_s)\, \hat{\varepsilon}(\lambda_s)' \right)^{-1} \sum_{s=1}^{*} \hat{\varepsilon}(\lambda_s)\, \psi(\lambda_s) \right),$$

$$\psi(\lambda_s) \; = \; \log \left| 2 \left(\cos \lambda_s \; - \cos w_r \right) \right|; \qquad \hat{\varepsilon}(\lambda_s) \; = \; \frac{\partial}{\partial \tau} \log \; g(\lambda_s; \hat{\tau}); \qquad \lambda_s \; = \; \frac{2\, \pi \, s}{T}.$$

and the summation over * in the above expressions refers to all the unbounded discrete frequencies λ_s. $\hat{\tau} \; = \; \arg\min_{\tau \in T^*} \sigma^2(\tau)$, where T^* is a suitable compact subset of the R^q Euclidean space, and $I(\lambda_s)$ is the periodogram of $\hat{u}_t \; = \; (1 - 2\cos w_r\, L + L^2)\, y_t \; - \; \hat{\beta}' w_t$, where

$$\hat{\beta} \; = \; \left(\sum_{t=1}^{n} s_t \, s_t' \right)^{-1} \sum_{t=1}^{n} s_t \, (1 - 2\cos w_r \, L + L^2)\, y_t \,, \; \text{ with } \quad s_t \; = \; (1 - 2\cos w_r\, L + L^2)\, z_t,$$

and the function g above is a known function coming from the spectral density function of u_t,

$$f(\lambda; \tau) \; = \; \frac{\sigma^2}{2\pi} g(\lambda; \tau), \qquad\qquad -\pi \; < \; \lambda \; \leq \; \pi.$$

Note that these tests are purely parametric and therefore, they require specific modelling assumptions to be made regarding the short memory specification of u_t. Thus, for example, if u_t is white noise, $g \equiv 1$ and, if u_t is an AR process of form: $\phi_p(L)\, u_t = \varepsilon_t$, $g = |\phi_p(e^{i\lambda})|^{-2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficients are functions of τ.

Based on H_o (5), Robinson (1994) established that under certain regularity conditions,[4]

[3] An I(0) process is defined as a covariance stationary process with spectral density function that is positive and finite at any frequency on the spectrum.

[4] These conditions are very mild concerning technical assumptions, which are satisfied by model (3) and (4).

$$\hat{R} \rightarrow_d \chi_1^{\,2} \quad as \quad T \rightarrow \infty, \tag{7}$$

and also the Pitman efficiency theory of the test against local departures from the null. Thus, we are in a classical large sample testing situation by reasons described in Robinson (1994). Because $\hat{R}$ involves a ratio of quadratic forms, its exact null distribution can be calculated under Gaussianity via Imhof's algorithm. However, a simple test is approximately valid under much wider distributional assumptions: An approximate 100α% level test of (5) against the alternative H_a: d ≠ 1 rejects H_o (5) if $\hat{R} > \chi^2_{1,\alpha}$, where Prob ($\chi^2_1 > \chi^2_{1,\alpha}$) = α.

Other versions of the tests of Robinson (1994) have been successfully applied to economic time series in Gil-Alana and Robinson (1997, 2001) and Gil-Alana (1999, 2000), testing for I(d) processes with the roots occurring at zero and at zero and the seasonal frequencies. However, testing cyclical I(1) models with the tests of Robinson (1994), there are few empirical studies, and one by-product of this work is its emergence as a credible alternative to the usual I(1) specification, which have become conventional in parametric modelling of macroeconomic time series.

3. Testing for unit root cycles in the US unemployment

Although the study of unemployment behaviour has been a major preoccupation for macroeconomists and labour market economists there is a general view that it is still not well understood. Recent contributions echoing this pessimistic conclusion are found in Carruth, Hooker and Oswald (1998) in a study of US unemployment, and Bean (1994) and Nickell (1997) in their general surveys of unemployment models. Most of the discussion here concerns about the degree of integration of the series, usually 0 (and thus stationary) or 1 (a unit root). Funke (1999) used a fractional model and he concludes that the US unemployment has an order of integration between 0 and 1. In this paper, we take a completely different approach and, instead of looking at the long run or zero frequency, we concentrate on its cyclical structure. A similar approach is adopted in Bierens (2001) when modelling the UK monthly unemployment.

The time series data analysed in this section correspond to the US unemployment rate, annually, for the time period 1960 - 1999, obtained from the website: 'http://www.fgn.unisg. ch/euromacro'.

Figure 1 displays plots of the original series along with the correlogram and the periodogram. A visual inspection of the series clearly shows that there is a cyclical component and this is substantiated by both the correlogram (with significant values at some lags relatively far away from zero), and the periodogram (with a large peak at zero but also at other frequencies away from zero).[5]

Denoting unemployment by y_t, we employ throughout the model given by (3) and (4) with $z_t = (1,t)'$, $t \geq 1$, $z_t = (0,0)'$ otherwise, i.e.

$$y_t = \beta_1 + \beta_2 t + x_t, \qquad t = 1, 2, \ldots \tag{8}$$

[5] The periodogram is an estimate of the spectral density function. Thus, if there are unit root cycles in the data, the periodogram should show some peaks at some frequencies on the spectrum.

$$(1 - 2\cos w_r\, L + L^2)^d \; x_t \; = \; u_t, \quad w_r \; = \; 2\pi / r, \qquad t \; = \; 1, 2, \ldots, \tag{9}$$

testing the null H_o (5) for values of r = 2, (1), 20,[6] i.e., allowing unit root cycles occurring at 2, (1), 20 periods (years) respectively. We treat separately the cases $\beta_1 = \beta_2 = 0$ a priori, β_1 unknown and $\beta_2 = 0$ a priori, and (β_1, β_2) unknown, that is, studying the cases of no regressors, an intercept, and an intercept and a linear time trend respectively, and model the I(0) disturbances u_t to be both white noise and to have parametric autocorrelation.

The test statistic reported in Table 1 corresponds to the test statistic given by $\hat{R}$ in (6), and spite of the small number of observations (T = 40), we rely on the asymptotic critical values of the χ_1^2 distribution. In fact, Gil-Alana (2001) shows that the tests perform relatively well even with such a small number of observations. Starting with the case of white noise disturbances, we see that H_o (5) cannot be rejected when r is between 4 and 9, and this happens for the three cases of no regressors, an intercept, and an intercept and a linear time trend. Imposing AR(1) or AR(2) disturbances, the results are more conclusive and the unit root null hypothesis cannot be rejected when r = 4 and 5, implying that, if this model specification is correct, the cycles in the US unemployment occur every four or five years, which is consistent with most of the empirical literature that says that cycles in economics occur approximately every five years.

4. Concluding comments

In this article we have examined the annual structure of the US unemployment by means of new statistical techniques for testing unit root cycles. This is a new alternative way of modelling cycles, which is not based on first differences (or alternatively, ARMA models), but on fractional structures directly applied to the cyclical component. Using a version of the tests of Robinson (1994) that permits us to test this type of hypotheses, the results show that the cycles in the US unemployment seem to occur every four or five years. This has important implications in terms of economic policy. The fact that unit root cycles cannot be rejected in this series implies that cycles in the US unemployment are highly persistent, and shocks affecting them will have permanent effects. Thus, strong policy actions should be required to bring the variable back to its original level.

An argument that can be employed against this type of models is that, contrary to seasonal cycles, business cycles are typically weak and irregular and are spread evenly over a range of frequencies rather than peaked at a specific value. However, contrary to that argument, we can explain that, in spite of the fixed frequencies used in this specification, the flexibility can be achieved throughout the first differencing polynomial, the ARMA components and the error term.[7] In that respect, the results presented here lead us to unambiguous conclusions and they are completely in line with the literature on business cycle duration that says that cycles take place with a periodicity constrained between 3 and 6 years. Another drawback of the present work might be the bounded nature of the unemployment rate, which is theoretically inconsistent with the presence of unit roots. This

[6] Note that in case of r = 1, the model reduces to a classical I(d) hypothesis, with a peak ocurring exclusively at the long run or zero frequency.

[7] Bierens (2001) also use a model of this sort, to test for the presence of business cycles in the annual change of monthly unemployment in the UK.

may be sorted out, for example, by using a logistic transformation of the data.[8] However, in spite of that, the existence of unit roots (at the zero frequency) has been widely assumed in the empirical work on this variable (see, e.g., Mitchell, 1993; Breitung, 1994; Hatanaka, 1996; Carruth et al., 1998) and the same can be done with respect to the cyclical part.

Several other lines of research are under way which should prove relevant to the analysis of these and other macroeconomic or financial data. A natural following-up step would be to test fractional cycles, i.e., allowing d_o in (7) to be a real number rather than 1. Of course, it would also be of interest in this context to estimate the order of integration of the cyclical component of the series. There exist several procedures for estimating the fractional differencing parameter in seasonal and cyclical contexts, (e.g., Ooms, 1995; Arteche and Robinson, 1999, 2000; etc.), however, they are not only computationally more expensive, but it is then in any case confidence intervals rather than point estimates which should be stressed.

[8] See Wallis (1987) for a justification based on the logistic transformation being defined between $\pm\infty$ so that standard distributions apply.

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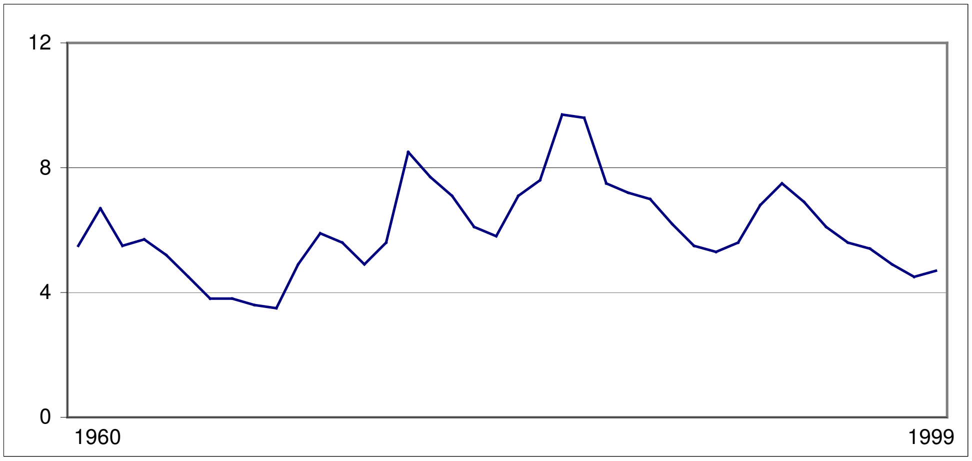
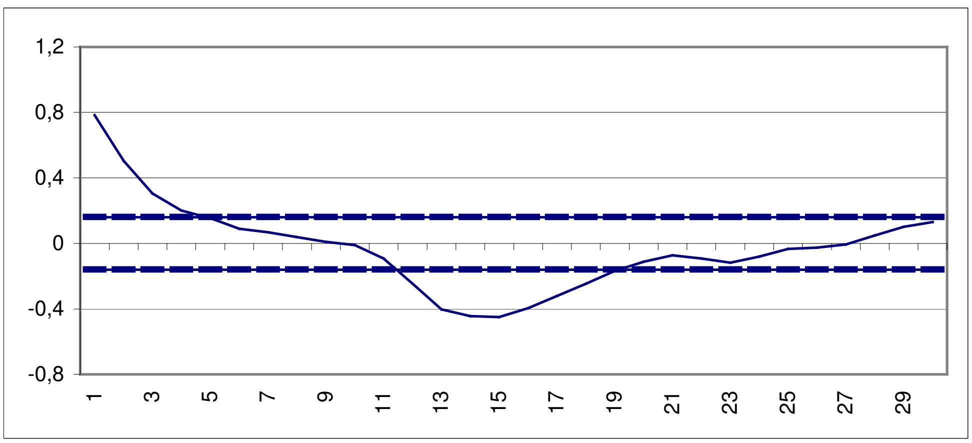
FIGURE 1

U.S. unemployment rate with the corresponding correlogram and periodogram

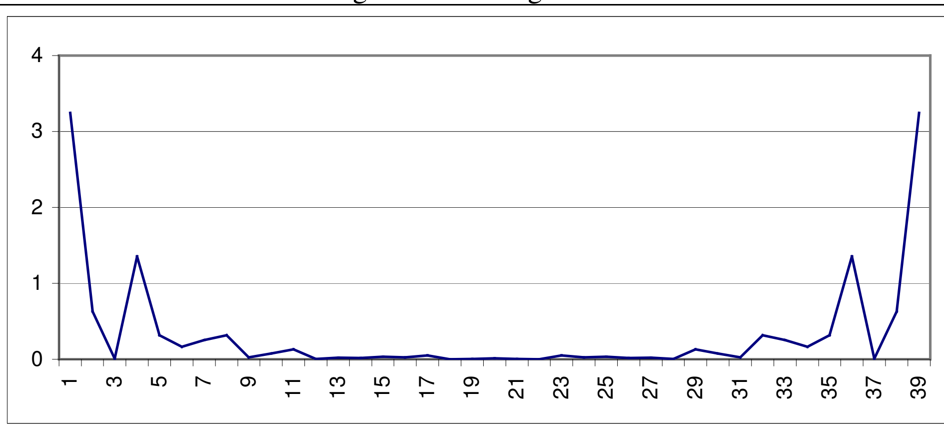
Original time series



First 30 sample autocorrelation values of the original time series



Periodogram of the original time series



The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.158.

TABLE I

Testing H_o (5) in (3) and (4) with $\hat{R}$ given by (6)

Disturbances	White noise			AR (1)			AR (2)		
Periods / Cycle	$z_t = 0$	$z_t = 1$	$z_t = (1,t)'$	$z_t = 0$	$z_t = 1$	$z_t = (1,t)'$	$z_t = 0$	$z_t = 1$	$z_t = (1,t)'$
1	6.008	6.228	6.002	8.453	8.420	8.451	278.571	112.514	106.185
2	16.462	18.981	18.207	4.419	12.565	10.422	75.860	123.919	118.595
3	20.436	23.835	22.305	1.365	13.338	10.024	20.328	28.414	25.562
4	**0.724**	**1.025**	**0.867**	**0.228**	**0.304**	**0.143**	**0.046**	**0.401**	**0.281**
5	**0.061**	**0.218**	**0.134**	**0.003**	**0.753**	**0.469**	**0.253**	**0.747**	**0.662**
6	**0.019**	**0.335**	**0.146**	5.846	7.682	5.985	98.504	11.301	12.613
7	**0.210**	**0.541**	**0.380**	9.072	7.410	5.659	82.472	6.008	5.363
8	**0.160**	**0.333**	**0.288**	9.974	10.315	6.224	12.644	10.281	10.173
9	**2.424**	**3.402**	**3.356**	14.001	13.953	4.011	134.828	7.757	6.108
10	4.699	4.045	4.005	4.653	5.687	4.360	13.844	10.406	10.439
11	4.050	5.432	5.359	33.295	5.954	5.299	139.539	15.644	15.494
12	4.428	5.791	5.677	67.201	5.164	5.838	136.814	22.411	21.559
13	3.581	4.625	4.505	34.147	4.735	4.008	47.242	10.590	10.982
14	4.277	5.317	5.202	43.332	6.740	5.326	73.106	19.422	19.374
15	4.977	6.103	5.995	40.049	7.285	7.019	152.646	48.116	45.258
16	5.184	6.237	6.143	34.552	7.141	7.499	201.984	77.311	72.510
17	5.109	6.053	5.973	22.856	6.484	7.289	180.672	82.184	81.049
18	4.810	5.628	5.562	15.203	5.284	6.579	115.273	58.599	60.658
19	4.220	4.891	4.838	9.696	10.002	5.334	54.931	30.021	31.506
20	3.087	4.264	4.422	10.559	10.817	10.817	11.798	11.058	11.081

In bold, the non-rejection values at the 5% significance level