E C O N O M I C S B U L L E T I N

Modelling the misalignments of the Dollar−Sterling real exchange rate: A nonlinear cointegration perspective

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Abstract

This paper proposes a comparison of three nonlinear error−correction models to account for the asymmetric and slow adjustment dynamics of the Dollar−Sterling real exchange rate over a long period (1957−2002). We conclude that two NEC models adequately describe the nonlinear mean−reverting mechanism: smooth transition and rational polynomial NEC models.

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1 Introduction

This paper is a further contribution to the growing literature on the nonlinear adjustment of the exchange rates towards their long-run equilibrium value. Such a literature, based on nonlinear cointegration analysis, is viewed as a response to the failure to find support for purchasing power parity or interest rates parity and as an answer to the unsuccessful validation of long-run models of exchange rate determination. There are many circumstances in which the exchange rates may evolve nonlinearly. Some factors that are usually evoked are: the transaction costs (Dumas, 1992 and O'Connel and Wei, 1997), the presence of heterogenous agents (Taylor and Allen, 1992), target zones (Krugman, 1991), abrupt changes due to noisy traders (De Long, Summers, Shleifer and Waldman, 1990). Once nonlinearities are at play in the adjustment dynamics of the exchange rates, the usual cointegration framework becomes inappropriate. This has motivated new econometric models based on nonlinear cointegration[1].

This paper proposes three versions of a nonlinear error-correction model for the Dollar-Sterling real exchange rate over a long period ranging from 1957 to 2002. The main contributions of the paper are the following.

(i) As indicated before, there are many possible explanations of the nonlinear misalignments of the exchange rates. So, unless one has a theoretical *a priori* on the factors causing the nonlinear mean-reversion mechanisms, several specifications must be used for purpose of comparison. Contrary to many papers that concentrate on threshold models (the main argument being the presence of transaction costs causing "band effects"), we compare three types of nonlinear adjustments. The first two specifications rely upon cubic and rational polynomial functions. Their framework allows taking into account several potential sources of nonlinearities: abrupt changes in adjustment speeds, multiple long-run attractors, compensation mechanisms between positive and negative shocks, differing adjustment according to the size and sign of the misalignments. We also consider a third specification based on an exponential smooth transition model (ESTAR), as is usually the case in the empirical literature. This specification helps modelling the asymmetries inherent to the misalignment dynamics. The asymmetries are useful in the sense that they may explain, for instance, the unequal durations of overvaluations and undervaluations.

(ii) In spite of modelling the nonlinear misalignment solely (this approach is common to a majority of papers), it is interesting to examine its impact on the short-run relationship between the exchange rate and its main determinants. This implies incorporating the nonlinear adjustment mechanism in an error-correction model.

The plan of the paper is as follows. Section 2 considers a modified monetarist model for which the hypothesis of linear cointegration between the Dollar-Sterling exchange rate and its macroeconomic determinants is rejected. Section 3 compares three nonlinear error-correction models and shows that both the ESTAR and rational polynomial specifications yield satisfactory results. Section 4 concludes.

2 Rejection of linear cointegration from a monetarist model

We consider a model based on the monetarist interpretation of the exchange rate determination. Although several variants are available, the common specification includes the following explanatory variables for domestic and foreign countries: the stock of money, the real GDP, the inflation rate, and the short-term and long-term interest rates. Our analysis is based upon the following specification:

$$\log(S_t) = a_0 + a_1 \left[\log(Y_t) - \log(Y_t^*)\right] + a_2(i_t - i_t^*) + a_3(\pi_t - \pi_t^*) + z_t, \tag{1}$$

where z_t is a stationary process representing the misalignments of the exchange rate from its long-run value. S_t is the USD/BP real exchange rate defined as the ratio of the price index of the

[1] The following papers give an overview of some aspects of the current empirical literature: Michaël, Nobay and Peel (1997), Ma and Kanas (2000), Chen and Wu (2000), Taylor, Peel and Sarno (2001), Baum, Barkoulas and Caglayan (2001), Dufrénot and Mignon (2002).

domestic country relative to the foreign country, Y_t and Y_t^* are the domestic and foreign GDP's, i_i and i_t^* are the domestic and foreign long-term interest rates (10 year government bond yields), π_t and π_t^* are the domestic and foreign inflation rates. In the sequel, the domestic country is the UK and the foreign country the US. Note that, given the presence of the inflation rates in the equation, we omit the money stock variables. Both variables indeed reflect the monetary policy, but we choose the inflation rate in regard to the fact that the latter has become a key objective of the monetary authorities that enters in their reaction function. Moreover, given the long period under consideration, we assume a convergence of the economic structures of both countries, thereby implying equal values of the elasticities and semi-elasticities.

The cointegration analysis can be used to see whether the above relationship holds in the long-run. The original data consist of quarterly series ranging over the period 1957:01 - 2002:03. The source is the OECD database.

Table 1 shows the results of usual unit root tests (Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP)) on individual series. As is seen, all the series are $I(1)$ except the inflation rate. However, the results relative to the GDP and interest rates differences are more ambiguous, since the null hypothesis of unit root is accepted by the ADF test while it is rejected by the PP semiparametric test. Applying the ADF and PP tests to the residuals $\hat{z}_t$ of the estimated static equation, we obtain the following results:

$$\begin{aligned} \log(S_t) &= -0.0365 - 1.586\,[\log(Y_t) - \log(Y_t^*)] + 0.0167(i_t - i_t^*) \\ &\quad + 0.2318(\pi_t - \pi_t^*), \\ ADF &= -2.02, \quad PP = -1.98. \end{aligned} \tag{2}$$

Comparing ADF and PP values with the Engle and Yoo (1987) critical values for $T = 200$ yields to retain the null hypothesis of no-cointegration.

The rejection of the cointegration hypothesis may have several causes: misspecification, low power of the tests used, measurement errors in the series. While not rejecting these potential causes, we want to test another hypothesis, namely that the deviations from the long-run value follow a nonlinear dynamics, which means that the linear framework for testing cointegration is inappropriate.

3 Nonlinear cointegration and NEC versions of the monetarist model

3.1 The framework of nonlinear cointegration analysis

The Engle-Granger cointegration approach performs poorly when used in a nonlinear context. Several alternative definitions can be proposed[2]. Our approach here is based on "mixing processes", a concept that helps characterizing the short-range dependence in time series following nonlinear processes.

Definition 1 *Define a probability space (Ω, K, P) where Ω is a sample space, K is an algebra and P is a probability function with domain K. Consider $\{X_h\}_{h>1}$ a sequence of random variables on (Ω, K, P) and $F_m^h = \sigma(X_t : m \leq t \leq h)$. Then, $\{X_t\}$ is strictly mixing (or $\phi-$mixing) if*

$$\phi(h) = \sup_{m \geq 1} \phi(F_1^m, F_{m+h}^\infty) \underset{h\to\infty}{\to} 0.$$

Heuristically, a process is mixing when it is short-range dependent, meaning that the dependence between past and future events becomes negligeable when the time span between the two

[2]The recent book by Dufrénot and Mignon (2002) gives an overview of the statistical apparatus needed to deal with nonlinear cointegration and nonlinear error-correction (NEC) models.

events increases. $\phi(h)$ measures the speed of mixing between two events separated by h time periods. Several tests are available to empirically test mixing conditions. We use here three tests that are simple to apply and that have been extensively studied in the literature: the R/S test (Lo, 1991), the $KPSS$ test (Kwiatkowski, Phillips, Schmidt and Shin, 1992), the mutual information test (Granger and Lin, 1994 and Escribano and Mira, 1997).

Using this concept of mixing, it is possible to extend the Engle-Granger theorem to the nonlinear case. The following definition and theorem give some conditions for nonlinear cointegration and NEC (Nonlinear Error-Correction) representation.

Definition 2 *Consider two processes* $\{X_t\}_{t=1}^{\infty}$ *and* $\{Y_t\}_{t=1}^{\infty}$ *that are* $I(1)$ *and suppose that there exists a measurable nonlinear function* $g(X_t, Y_t, \theta)$ *such that the sequence* $\{g(X_t, Y_t, \theta)\}_{t=1}^{\infty}$ *is mixing for* $\theta = \theta^*$ *and non-mixing for* $\theta \neq \theta^*$. *Then, we say that* X_t *and* Y_t *are nonlinearly cointegrated with cointegrating function* g.

Theorem 1 (Escribano and Mira, 1998) *Consider two sets of* $I(1)$ *variables: an endogenous variable* y_t *and a vector* X_t *of* K *explanatory variables (all the variables in* X_t *need not to be* $I(1)$*). A nonlinear error-correction model is written as follows:*

$$\begin{aligned} \Delta y_t &= \textstyle\sum_{i=1}^{q} \gamma_i^{'} \Delta X_{t-i} + \sum_{j=1}^{p} \delta_i \Delta y_{t-j} + \lambda_1 z_{t-1} + \lambda_2 f(z_{t-1}, \theta) + u_t, \\ \Delta X_t &= v_t, \\ z_t &= y_t - \beta' X_t, \end{aligned} \tag{3}$$

where the $\gamma_i^{\prime}s$ *and the* $\delta_i^{\prime}s$ *are vectors of parameters. Assume that:*

(i) u_t *and* v_t *are mixing processes with finite second-order moments and cross-moments;*

(ii) f *is a nonlinear function that is continuously differentiable and that satisfies some regularity conditions:*

$$-2 < \frac{\partial f(z_{t-1}, \theta)}{\partial z_{t-1}} < 0; \tag{4}$$

(iii) the roots of

$$\left| 1 - \sum_{i=1}^{p} \delta_i L^p \right| = 0, \tag{5}$$

all lie outside the unit circle;

(iv) u_t *is a martingale difference process with zero mean and constant variance. Under these assumptions,* z_t *is NED (near epoch dependent) and* y_t *and* X_t *are cointegrated with cointegrating vector* $(1, -\beta^{'})$.

The NED (Near Epoch Dependence) assumption on z_t is weaker than mixing but can be tested in the same way.

3.2 Application to the monetarist model

The first step is to see whether the residuals of the estimated linear model are mixing. If this is the case, then this would imply the presence of a mean-reverting process in the exchange rate dynamics. As previously mentioned, in order to test this possibility, we apply the $KPSS$ test, the R/S test and the mutual information test. The results are reported in tables 2 and 3. For the $KPSS$ test, we use the values recommended by Schwert (1989) for truncation parameter $l_4 = int\left[4\left(\frac{T}{100}\right)^{1/4}\right]$ and $l_{12} = int\left[12\left(\frac{T}{100}\right)^{1/4}\right]$, where T denotes the number of observations.

For a time series $\{X_t\}_{t=1}^{T}$, with mean $E[X_t] = \overline{X}$, the R/S statistic is expressed as[3]:

$$R/S = \frac{1}{\widehat{\sigma}_T(q)} \left[\underset{1\leq\ i\ \leq T}{Max} \sum_{j=1}^{i} (X_j - \overline{X}) - \underset{1\leq\ i\ \leq T}{Min} \sum_{j=1}^{i} (X_j - \overline{X}) \right], \tag{6}$$

where

$$\widehat{\sigma}_T^2(q) = \frac{1}{T} \sum_{j=1}^{T} (X_j - \overline{X})^2 + \frac{2}{T} \sum_{j=1}^{q} \omega_j(q) \left\{ \sum_{k=j+1}^{T} (X_k - \overline{X})(X_{k-j} - \overline{X}) \right\}. \tag{7}$$

The term between brackets in (6) expresses the range of the time series. The higher the value of the R/S statistic, the higher the probability of strong dependence in the memory of a time series[4]. The formula (7) is the long-term variance of the series, which includes both the short-term variance and the short-term autocovariances. The latter are weighted by standard spectral windows $(\omega_j(q))$. Concerning the choice of q, we consider the following values: $q = 1, 5$ and a value corresponding to Andrews (1991) formula:

$$q = [k_T] \text{ where } k_T = \left(\frac{3T}{2}\right)^{\frac{1}{3}} \left(\frac{2\hat{\rho}}{1-\hat{\rho}^2}\right)^{\frac{2}{3}} \tag{8}$$

with $[k_T] = int\,(k_T)$ and $\hat{\rho}$ the estimate of the first-order autocorrelation coefficient.

The mutual information test allows us to consider the issue of dependence in time series in terms of entropy (see, among others, Escribano and Aparicio, 1997). Let $\{X_t\}$ and $\{X_{t\pm\tau}\}$ be two stochastic processes with a joint density function $f(X_t, X_{t\pm\tau})$ and marginal density functions $f_t(X_t)$ and $f_\tau(X_{t\pm\tau})$. We define the mutual information between X_t and $X_{t\pm\tau}$ as:

$$I(X_t, X_{t\pm\tau}) = E\left[\ln\left(\frac{f(X_t, X_{t\pm\tau})}{f_t(X_t) f_\tau(X_{t\pm\tau})}\right)\right]. \tag{9}$$

This expression can also be written as:

$$I(X_t, X_{t\pm\tau}) = H_t(X_t) + H_\tau(X_{t\pm\tau}) - H(X_t, X_{t\pm\tau}) \tag{10}$$

where $H(\bullet)$ is the Shannon entropy and is helpful to see wether the information inherent to $X_{t+\tau}$ helps reducing the uncertainty present in X_t. Given $\sigma_{t,\tau}^2/\sigma_t\sigma_\tau = \rho_{t,\tau}^2$, where $\sigma_t = V[X_t]$, $\sigma_\tau = V[X_{t\pm\tau}]$, $\sigma_{t,\tau} = cov[X_t, X_{t\pm\tau}]$, it is possible to show that (see Dufrénot and Mignon, 2002, for details):

$$\rho_{t,\tau} = [1 - \exp(-2I(X_t, X_{t\pm\tau}))]^{1/2}. \tag{11}$$

Thus, $I(X_t, X_{t\pm\tau}) = 0$ (strict independence) implies that $\rho_{t,\tau} = 0$. Moreover, $I(X_t, X_{t\pm\tau}) \to +\infty$ (strong dependence) implies that $\rho_{t,\tau} = 1$. To find a consistent estimator of $I(X_t, X_{t\pm\tau})$ one usually uses Kernel densities to approximate the functions $f_t(X_t)$, $f_\tau(X_{t\pm\tau})$ and $f(X_t, X_{t\pm\tau})$. The interest of mutual information criterion is that it can be used to study the rate of decline of $\rho_{t,\tau}$ for a variety of nonlinear models.

[3]Authors in the literature usually use the term "modified" R/S when they refer to this statistic. The reason is that it includes the empirical autocovariances and differs from the formulation originally suggested by Hurst (1951), which only considered the empirical variance in the denominator.

[4]Critical values for $V = (R/S)/\sqrt{T}$ have been tabulated by Lo (1991). It is thus possible to test the null hypothesis of null or short-range dependence (mixing) against the alternative of long-range dependence (non-mixing).

Tables 2 and 3 show contradictory results. As is seen, the R/S statistic yields to reject the null of mixing, while the latter is accepted in regard to the $KPSS$ test. Considering the entropy based test, the mixing hypothesis is also accepted, but for high lags. This would mean that, given the long period under consideration, the speed of adjustment towards the fundamental value of the Dollar-Sterling real exchange rate might be very slow. Despite this contradiction, we retain the mixing hypothesis for the following reason. The $KPSS$ and entropy based tests are nonparametric tests, whilst the R/S test is parametric. When applying the latter, the null hypothesis is a white noise process or an AR process with very few lags. In our case, if the mean-reversion is slow, it is likely that the residuals are conveniently described by an AR process with high lags, or even an $ARFIMA$ (*AutoRegressive Fractionally Integrated Moving Average*) model. Consequently, even if there exists an error-correcting mechanism in the misalignments of the exchange rate, the latter is likely not to be detected by the R/S approach. In this case, it seems better to use nonparametric approaches.

To see whether the slow adjustment of the Dollar-Sterling exchange rate can be satisfactorily represented by a nonlinear error-correction model, we consider the following three formulations for the function f.

- *A cubic polynomial function*:

$$f(z_{t-1}) = \alpha_1 z_{t-1} + \alpha_2 z_{t-1}^2 + \alpha_3 z_{t-1}^3. \tag{12}$$

  This function allows for the possibility of multiple equilibria and several threshold points when the adjustment is asymmetric.

- *A rational polynomial function (RPF)*:

$$f(z_{t-1}) = \frac{(z_{t-1} + \gamma_1)^3 + \gamma_2}{(z_{t-1} + \gamma_3)^2 + \gamma_4}. \tag{13}$$

  Such a function has proven to be a useful flexible parametric approximation to unknown functional forms (the arguments are based on convergence of Padé approximants (see Escribano, 1997). An important advantage of RPF is that they allow modelling asymmetric dynamics in a more flexible way than threshold models.

- *An exponential smooth transition function (ESTAR)*:

$$f(z_{t-1}) = 1 - \exp\left[-\delta_1(z_{t-1}^2 - \delta_2)\right]. \tag{14}$$

  This function is common in the literature on nonlinear mean-reversion and allows modelling smooth and gradual changes during the successive periods of over- and under-valuations of the exchange rates.

The results of the estimations are reported in table 4. Tables 5 and 6 report the results of the mixing tests on the residuals of the NEC models. The estimates are based on nonlinear least squares and rely upon the statistical inference analysis for NEC models developed by Escribano (1997) and Escribano and Mira (1998). For purpose of clarity in the interpretations of the results, a remark is in order. Our formulation of the monetarist model allows one cointegration relationship, so that we are implicitly assuming that the fundamental value of the Dollar-Sterling real exchange rate is unique in the long-run. The use of NEC models based on an $ESTAR$ transition function allows us to study the extent to which this long-run equilibrium is compatible with the presence of many intermediate "states" that are visited by the exchange rate in a context of non constant adjustment. $ESTAR$ functions are useful to detect temporal paths governed by smooth changing regimes. NEC models based on RPF or cubic polynomial transition functions are used in order to highlight asymmetric dynamics between the overvaluation and undervaluation regimes. Note that if the true misalignment involves several equilibria, then this may imply hysteresis. In this case, the function $f(z_{t-1})$ would yield a non-mixing dynamics, but this question is beyond the scope of the paper.

Comparing the three models, it is seen that the NEC models based on *ESTAR* and *RPF* specifications yield satisfactory results.

For the *ESTAR-NEC* model, the parameter λ_2 in table 4 is negative and statistically significant, meaning that the nonlinear adjustment dynamics is error-correcting. From tables 5 and 6 we see that the residuals of the *ESTAR-NEC* model are, satisfactorily, mixing. More interestingly, this model corroborates a feature that has been previously observed in the literature. Indeed, the parameter λ_1 in table 4 is *not* statistically significant, meaning that the linear component of the adjustment dynamics is not mean-reverting. One explanation is that the exchange rate tends to move back to its equilibrium only for large deviations and it is possible to observe a divergent behavior for small deviations.

The results obtained for the *RPF-NEC* model show other interesting features of the misalignment dynamics. Facing some difficulties in obtaining convergent estimates for all the parameters of the *RPF*, we estimate the *RPF-NEC* model under the following restrictions $\gamma_1 = \gamma_3 = \gamma_4 = 1$ and $\gamma_2 = 0$. Figure 1 shows the histogram of the rational polynomial function for the different values of $\{\widehat{z}_{t-1}\}$. The graph shows a bimodal density with two modes of unequal heights. These reflect the extreme misalignments corresponding respectively to the regimes of undervaluation and overvaluation (the values of $\widehat{z}_{t-1}$ — not shown here — corresponding to the function f in the interval $[0.30,\ 0.45]$ are negative, while they are positive for $f \in [0.46,\ 0.7]$). Around the modes, it is seen that the values of the *RPF* are distributed asymmetrically. In the undervaluation regime the distribution is skewed to the left, while it is skewed to the right in the overvaluation regime. This illustrates a kind of persistent dynamics: the probability of overvaluation (resp. undervaluation) in a given period increases if an overvaluation (resp. an undervaluation) was observed in the preceding period. This is confirmed by the estimation in table 4 where the parameter λ_2 is not statistically significant, thereby implying that the nonlinear adjustment mechanism is not mean reverting. One caveat of this RPF concerns the regularity conditions on the function $f(z_{t-1})$. This implies that even if the assumptions *(i)*, *(ii)* and *(iv)* in the theorem 1 are satisfied, we have no guarantee that the NEC model will be mixing. To answer, we must look at the conclusions of the tests. As is seen, two tests (the two nonparametric procedures) yield to accept the null hypothesis of mixing on the residuals of the estimated *RPF-NEC* model (see tables 5 and 6).

In terms of estimation, the NEC model based on the cubic polynomial function yields results that are similar to those obtained for the *RPF-NEC* model. However, this model fails to capture the asymmetries in the misalignment dynamic. As appears in figure 2, the graph is symmetric around $z_{t-1} = 0$.

4 Conclusion

This paper has proposed a comparative analysis of three nonlinear error-correction models to account for the misalignment mechanism of the Dollar-Sterling exchange rate over the period 1957-2002. Using the notion of mixing, it has been shown that such a mechanism can be adequately represented by a smooth transition function or by a rational polynomial function. The former accounts for a slow adjustment mechanism, while the latter reproduces the asymmetries inherent to the misalignment dynamics. A natural extension would be the study of the possibility of multiple equilibria. Such a question requires a more general framework where the above results extend to the multivariate case.

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Table 1: Unit root tests on individual series

	ADF		*PP*	
	Level	Δ	Level	Δ
$\log(S_t)$	-1.06 (1)	-3.80* (1)	-0.76 (1)	-11.76* (1)
$\log(Y_t) - \log(Y_t^*)$	-1.72 (1)	-14.29* (1)	-2.75* (1)	-14.38* (1)
$i_t - i_t^*$	-1.33 (1)	-11.03* (1)	-3.77* (2)	-14.52* (1)
$\pi_t - \pi_t^*$	-3.24* (1)	-8.49* (1)	-11.82* (2)	-37.82* (1)

*: Rejection of the null hypothesis at the 5% significance level. (1): model without constant, nor deterministic trend. (2): model with constant, but without trend.

Table 2: Mixing tests on estimated error term

	KPSS		*Lo (R/S)*		
	l_4	l_{12}	*Andrews*	$q = 1$	$q = 5$
Residuals	0.33	0.14	3.31*	3.30*	1.97*

*: Rejection of the null hypothesis at the 5% significance level.

Table 3: Entropy based test on estimated error term

Lag	*Test value*
1	0.91*
2	0.87*
3	0.84*
4	0.86*
5	0.81*
6	0.77*
7	0.80*
8	0.71*
9	0.58*
10	0.61*
20	0
30	0.27

*: Significant coefficient at the 5% significance level.

Table 4: Estimation of NEC models

	Cubic	*Rational*	*ESTAR*
Constant	-0.0008 (-0.74)	0.0166 (0.06)	0.0055 (1.37)
$\Delta\log(S_{t-1})$	0.3938 (4.33)	0.3924 (4.33)	0.3456 (3.71)
$\Delta\left(\log(Y_{t-1}) - \log(Y_{t-1}^*)\right)$	0.0209 (0.30)	0.0103 (0.15)	0.0300 (0.45)
$\Delta\left(i_{t-1} - i_{t-1}^*\right)$	0.0018 (2.45)	0.0019 (2.56)	0.0016 (2.17)
$\Delta\left(\pi_{t-1} - \pi_{t-1}^*\right)$	-0.2146 (-2.91)	-0.2125 (-2.90)	-0.1820 (-2.44)
λ_1	0.0034 (0.16)	0.0269 (0.06)	-0.0081 (-0.77)
λ_2	$\underset{(-0.14)}{-0.0223}z_{t-1}^2 - \underset{(-0.61)}{0.8704}z_{t-1}^3$	-0.0347 (-0.07)	-0.0071 (-1.99)

t-statistics of the coefficients are given in parentheses.

Table 5: Mixing tests on NECM residual series

	KPSS			*Lo (R/S)*	
	l_4	l_{12}	*Andrews*	$q = 1$	$q = 5$
Cubic	0.37	0.24	0.94	1.63*	1.72*
Rational	0.38	0.25	0.92	1.60*	1.69*
ESTAR	0.50*	0.33	1.74*	1.79*	1.58

*: Rejection of the null hypothesis at the 5% significance level.

Table 6: Entropy based test on NECM residual series

Lag	*Cubic*	*Rational*	*ESTAR*
1	0	0	0
2	0.15	0	0
3	0	0	0
4	0.40*	0.37	0
5	0	0	0
6	0	0	0
7	0	0	0
8	0.31	0.32	0
9	0	0	0.18
10	0	0	0.22
20	0.41*	0.68*	0.09
30	0.19	0	0.16

*: Significant coefficient at the 5% significance level.

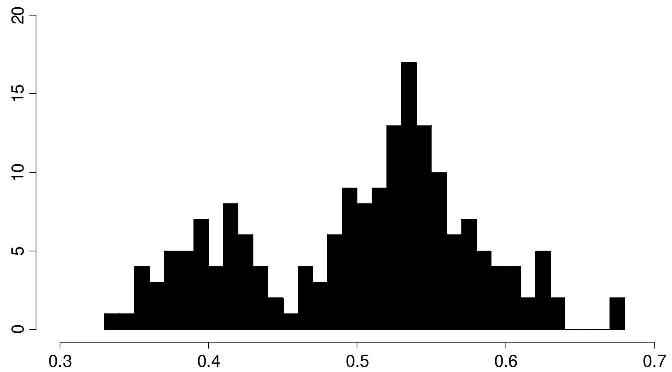

Figure 1: Rational polynomial function. Histogram

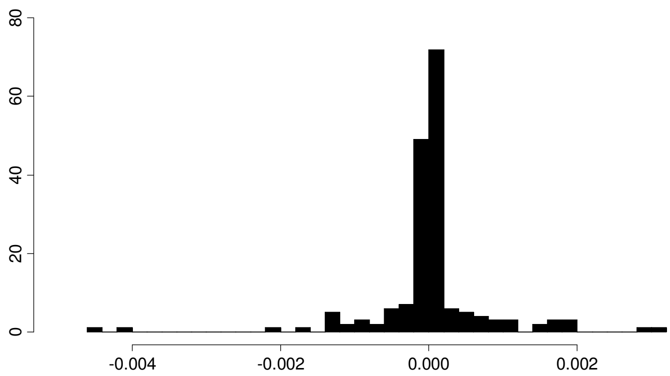

Figure 2: Cubic function. Histogram