E C O N O M I C S   B U L L E T I N

Testing for no autocorrelation using a modified Lobato test

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Abstract

This paper suggests modifying the Lobato test for no autocorrelation by using the bandwidth parameter (M) of the covariance estimator as a fixed proportion of the sample size (T): M=bT, where b (0,1] is a constant. It is shown by means of simulations that the modified test has good control over size regardless the choice of b and a higher testing power can be achieved if a mall b is chosen.

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1 Introduction

The Box-Pierce (1970) Q statistic, defined as the product of the sample size and the sum of the squares of the first K sample autocorrelations, is often used to test the null hypothesis that the first K autocorrelations of a time series are zero (Campbell et al. 1997). The Q test is considered as a test of no autocorrelation, but its asymptotic distribution (chi-square with K degrees of freedom) under the null actually assumes independence. For many financial and economic time series, the independent assumption is questionable; among them, the widely-used GARCH model for financial returns is a leading example. When the null hypothesis is true but the time series is statistically dependent, since the asymptotic covariance of the sample autocorrelations is no longer the identity matrix, the Q test is not valid and can produce serious misleading inferences (Romano and Thombs, 1996).

Several modified Q tests that are robust to statistical dependence have been proposed in the literature. In this paper, we are particularly interested in two of these tests. The first test of interest is the test of Lobato, Nankervis, and Savin (2002; hereafter, LNS). The LNS test is constructed using a consistent estimator of the asymptotic covariance matrix of the sample autocorrelations and is advantageous to have a limiting chi-square distribution under the null hypothesis. However, to estimate the asymptotic covariance consistently, the test requires the selection of a user-chosen parameter (the bandwidth parameter) and statistical inference can be sensitive to that. We are also interested in the test proposed by Lobato (2001). Unlike the LNS test, the Lobato test does not utilize a consistent estimator of the covariance of the sample autocorrelations and is free from the bandwidth selection. Even though the covariance estimator itself is inconsistent the Lobato test statistic is shown to be asymptotically pivotal (nuisance parameter free).

Interestingly, despite their apparent difference, the two modified Q test statistics can actually be related in a special way. Based on the work of Kiefer and Vogelsang (2002), it can be shown that the Lobato statistic is exactly equivalent to using the full-bandwidth (i.e. the bandwidth equals to the sample size) Bartlett covariance estimator in the construction of the LNS statistic. Theoretical evidence shows that the full-bandwidth covariance estimates based tests are more accurate to approximate the limiting null distribution than those built on consistent estimates (Jansson, 2004; Kiefer and Vogelsang, 2003). In fact, this asymptotic result has been reflected in finite-sample studies: the Lobato test provides a better control over size than the LNS test. However, the good size property of the Lobato test comes at the cost that the test is generally less powerful than the LNS test. See Lobato (2001) for details.

This paper is motivated by Kiefer and Vogelsang (2003) in studying finite sample properties of the Lobato test when the test is modified by setting the bandwidth parameter (M) of the covariance estimator as a fixed proportion of the sample size (T). More specifically, we consider the bandwidth parameter: M=bT where $b \in (0,1]$ is a constant. When b=1 is chosen, the modified Lobato test coincides with the Lobato test. In contrast, since the LNS test requires estimating the asymptotic covariance consistently, the bandwidth has to grow at a slower rate than the sample size (that is, b must go to zero as T increases), so the LNS test does not belong to the family of our modified test. Finite sample properties of the modified test are studied using Monte Carlo simulations. In general, our simulation results show that the modified Lobato test has surprisingly good control over size regardless the choice of b and a higher testing power can be achieved if a small b is used.

2 Tests for No Autocorrelation

Let $\{y_t\}_{t=1}^{T}$ be a covariance stationary time series with mean µ and the jth-lag autocovariance $g_j = E(y_t - m)(y_{t-j} - m)$. We are interested in testing the null hypothesis that y_t is uncorrelated up to order K,

$$H_0 : g_1 = \ ... \ = g_K = 0, \tag{1}$$

against the alternative that some of the first K autocorrelations in y_t are correlated,

$$H_1 : g_j \neq 0 \text{ for some j, } \quad \text{j=1,...,K.} \tag{2}$$

Consider that a sample of y_t for t=1,...,T is observed. The jth-lag sample autocovariance is given by $\hat{g}_j = T^{-1}\sum_{t=j+1}^{T}(y_t - \bar{y})(y_{t-j} - \bar{y})$ where $\bar{y} = T^{-1}\sum_{t=1}^{T} y_t$ is the sample mean. Let $\hat{C}_K = (\hat{g}_1,...,\hat{g}_K)'$ be the vector of sample autocovariances. Also, let $Z_t = (z_{1,t},..., z_{K,t})'$ and $z_{j,t} = (y_t - \bar{y})(y_{t-j} - \bar{y})$. Following Lobato (2001) and LNS (2002), we assume covariance stationary and the concept of near epoch dependence (NED) on a mixing process is used.

Assumption 1. *(1) Let y_t be a covariance stationary process that satisfies $E|y_t|^s < \infty$ for some $s > 4$ and all t, and is L_2-NED of size -1/2 on a process V_t where V_t is an a-mixing sequence of size –s/(s-4). (2) The null hypothesis (1) is satisfied.*

Lobato (2001) suggests the following test statistic

$$\hat{Q}_K^L = T\,\hat{C}_K'\hat{\Omega}_K^{-1}\,\hat{C}_K, \tag{3}$$

where $\hat{\Omega}_K = T^{-2}\sum_{t=1}^{T} S_t S_t'$ with $S_t = \sum_{j=1}^{t}(Z_j - \hat{C}_K)$. According to Lobato (2001), although $\hat{\Omega}_K$ is not a consistent estimator of the asymptotic covariance of $\hat{C}_K$, $\hat{Q}_K^L$ is asymptotically pivotal under Assumption 1 and has the following limiting null distribution:

$$\hat{Q}_K^L \Rightarrow B_K(1)\Xi_K^{-1}B_K(1)'. \quad (4)$$

In (4), "$\Rightarrow$" denotes weak convergence, $\Xi_K = \int_0^1 \bar{B}_K(r)\bar{B}_K(r)'dr$, where $B_K(r)$ is a standard K-vector Brownian motions, and $\bar{B}(r) = B_K(r) - rB_K(1)$ is a K-vector of Brownian bridges.

Alternatively, LNS (2002) suggest a modified Q test:

$$\hat{Q}_K^{LNS} = T\,\hat{C}_K'\hat{\Phi}_{K,M}^{-1}\,\hat{C}_K, \quad (5)$$

where $\hat{\Phi}_{K,M}$ be a kernel-based nonparametric estimator of the asymptotic covariance of $\hat{C}_K$, defined by

$$\hat{\Phi}_{K,M} = \sum_{j=-T+1}^{T-1} k\left(\frac{j}{M}\right)\hat{\Gamma}(j) \quad (6)$$

and

$$\hat{\Gamma}(j) = \begin{cases} T^{-1}\sum_{t=1}^{T-j} Z_{t+j}Z_t' & for\ j \geq 0 \\ T^{-1}\sum_{t=-j+1}^{T} Z_{t+j}Z_t' & for\ j < 0. \end{cases} \quad (7)$$

In (6), $k(\bullet)$ is a kernel function and M is a bandwidth parameter. Consistency of $\hat{\Phi}_{K,M}$ requires "$M \to \infty$ and $M/T \to 0$ as $T \to \infty$". LNS (2002) show that $\hat{Q}_K^{LNS}$ converges to a chi-square distribution under the null hypothesis, provided that $\hat{\Phi}_{K,M}$ is a consistent estimator of the asymptotic covariance of $\hat{C}_K$. Despite the apparent difference between $\hat{\Omega}_k$ and $\hat{\Phi}_{K,M}$, these two covariance estimators can actually be related in a special way. According to Kiefer and Vogelsang (2002), if $k(\bullet)$ is the Bartlett kernel (i.e. $k(x) = 1 - |x|^{-1}\ for\ |x| < 1$ and 0, otherwise) and the bandwidth is set equal to the sample size (M=T), then $\hat{\Omega}_k = \hat{\Phi}_{K,M}/2$. Therefore, in such an occasion, $\hat{Q}_K^L = 2\hat{Q}_K^{LNS}$.

Recently, Kiefer and Vogelsang (2003) suggest setting the bandwidth parameter as a fixed proportion of the sample size (i.e. M=bT, where $b \in (0,1]$ is a constant) in defining heteroskedasticity-autocorrelation robust estimators in regression models. See Kiefer and Vogelsang (2003) for the motivation behind. Following Kiefer and Vogelsang (2003), we define $\hat{Q}_K^L(b)$, the modified Lobato test, as follows:

$$\hat{Q}_K^L(b) = T\,\hat{C}_K'\,\hat{\Phi}_{K,M=bT}^{-1}\,\hat{C}_K\,. \quad (8)$$

In this paper we assume that the kernel function $k(\bullet)$ using in $\hat{\Phi}_{K,M=bT}$ is the Bartlett kernel. Under Assumption 1, we are able to copy the proof of Theorem 1 in Kiefer and Vogelsang (2003) to show that

$$\hat{Q}_K(b) \Rightarrow B_K(1)\Xi_K^{-1}(b)B_K(1)' \equiv U_K(b), \qquad (9)$$

where

$$\Xi_K(b) = \frac{2}{b}\int_0^1 \bar{B}_K(r)\bar{B}_K(r)'dr - \frac{1}{b}\int_0^{1-b}\left[\bar{B}_K(r+b)\bar{B}_K(r)' + \bar{B}_K(r)\bar{B}_K(r+b)'\right]dr. \qquad (10)$$

Here, $\bar{B}(r)$ is a K-vector of Brownian bridges defined as before. The modified Lobato test has a limiting null distribution that depends on b (but is otherwise nuisance free) and its critical values can be derived using the asymptotic critical value function given in Kiefer and Vogelsang (2003, Table I).

3 Size and Power in Finite Samples

In this section we report the results of Monte Carlo experiments designed to investigate the size and power of the modified Lobato test ($\hat{Q}_K(b)$) for K=1. Simulations are performed in GAUSS. Throughout this section, the simulation results are calculated using 10,000 iterations at sample sizes 100 and 500 and the empirical rejection probabilities are reported at 5% significance level for the modified test with b=0.1, 0.2, …, 0.9, 1.0. Note that when b=1.0, the modified test and the Lobato test are exactly equivalent.

Finite sample performance in size is studied first. Following Lobato (2001), we first consider three uncorrelated processes. The first two are iid sequences with innovations drawn from $N(0,1)$ and t distribution with six degrees of freedom (T(6)). The third process is a uncorrelated none martingale difference sequence (non-MDS): $y_t = z_{t-2}z_{t-1}(z_{t-2} + z_t + 1)$, where z_t is a sequence of iid $N(0,1)$ random variables. We report the simulation results in Table 1. Table 1 shows that the modified Lobato test has correct size regardless the choice of b in the cases of $N(0,1)$ and T(6). For the case of non-MDS, the modified test is under-sized, particularly when b and T are both small.

Still following Lobato (2001), two empirically relevant models are considered: a GARCH(1,1) and a bilinear model. Let z_t be sequence of iid $N(0,1)$, the GARCH(1,1) model is given by $y_t = z_t s_t$, where and $s_t^2 = 0.001 + 0.02y_{t-1}^2 + 0.8s_{t-1}^2$, and the bilinear model is given by $y_t = z_t + 0.5\, z_{t-1}y_{t-2}$. It is well-known that the GARCH(1,1) is uncorrelated but not independent over time. So is the bilinear model, see Granger and Teräsvirta (1993). We report the simulation result in Table 2. Table 2 shows that rejection rate of the modified test is invariant to the choice of b. In both models, the test is very accurate at T=500 but slightly over-sized at T=100.

We now turn our attention to finite sample power. We generate the data by an AR(1) process with N(0,1), T(6), non_MDS, GARCH and bilinear innovations. The AR coefficients (f's) considered are 0.10 and 0.20. We report power and size-adjusted power (in the parentheses) – power using simulated finite sample critical values – for each experiment in Tables 3-4. The key facts observed in Tables 3-4 can be summarized as follows. It is generally that the power of each test becomes larger as the AR coefficient increases and as the sample size increases. Also, tests with smaller b tend to be more powerful than those with larger b. Among all the tests considered, the test with b=0.1 seems to enjoy the largest (size-adjusted) power in all but one case (non-MDs, T=100). As a matter of fact, in the case of non-MDs, the power of the Lobato test and the modified test (regardless the choice of b) is relatively low comparing to other cases. For all other cases, power improvement of the L-KV test is substantial, particularly when b=0.3 or smaller is chosen. For example, when $f = 0.10$ and T=500 is considered, the size-adjusted power of the Lobato test (i.e. b=1) is 0.420 (N(0,1)), 0.398 (T(6)), 0.393 (GARCH), 0.303 (Bilinear) while the power of the modified test with b=0.1 is 0.532 (N(0,1)), 0.519 (T(6)), 0.509 (GARCH), 0.384 (Bilinear). In other words, at T=500, the modified test with b=0.1 enjoys more than 25% increase in power comparing to the Lobato test.

4 Conclusions

In this paper, we suggest modifying the Lobato test by using the bandwidth parameter (M) of the covariance estimator as a fixed proportion of the sample size (T): M=bT, where $b \in (0,1]$ is a constant. It is shown by means of simulations that the modified test has good control over size regardless the choice of b and a higher testing power can be achieved when b is small.

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Table 1: Size (I) at the 5% level

	N(0,1)		T(6)		Non-MDS	
test	T=100	T=500	T=100	T=500	T=100	T=500
Lobato	0.049	0.049	0.048	0.051	0.029	0.042
Modified L(0.1)	0.050	0.048	0.046	0.049	0.024	0.038
Modified L(0.2)	0.049	0.048	0.045	0.048	0.025	0.037
Modified L(0.3)	0.049	0.048	0.045	0.048	0.026	0.038
Modified L(0.4)	0.049	0.049	0.048	0.050	0.028	0.040
Modified L(0.5)	0.050	0.050	0.049	0.050	0.031	0.040
Modified L(0.6)	0.051	0.048	0.050	0.049	0.030	0.041
Modified L(0.7)	0.049	0.048	0.049	0.049	0.029	0.041
Modified L(0.8)	0.049	0.048	0.048	0.049	0.029	0.040
Modified L(0.9)	0.048	0.047	0.048	0.049	0.029	0.040

Table 2: Size (II) at the 5% level

	GARCH		Bilinear	
test	T=100	T=100	T=100	T=500
Lobato	0.052	0.050	0.053	0.049
Modified L(0.1)	0.052	0.050	0.054	0.048
Modified L (0.2)	0.050	0.050	0.052	0.049
Modified L (0.3)	0.052	0.052	0.053	0.049
Modified L (0.4)	0.052	0.050	0.054	0.048
Modified L (0.5)	0.054	0.050	0.054	0.049
Modified L (0.6)	0.052	0.049	0.055	0.049
Modified L (0.7)	0.052	0.048	0.053	0.049
Modified L (0.8)	0.052	0.048	0.052	0.048
Modified L (0.9)	0.051	0.048	0.052	0.048

Table 3: Power (I) at the 5% level

(A) ϕ=0.1, adjusted power in the parentheses

	Normal		T(6)		Mon_MDs	
test	T=100	T=500	T=100	T=500	T=100	T=500
Lobato	0.108	0.415	0.106	0.404	0.029	0.086
	(0.110)	(0.420)	(0.111)	(0.398)	(0.048)	(0.097)
Modified L(0.1)	0.120	0.521	0.123	0.515	0.026	0.086
	(0.120)	(0.532)	(0.132)	(0.519)	(0.051)	(0.111)
Modified L(0.2)	0.113	0.478	0.112	0.471	0.023	0.080
	(0.115)	(0.485)	(0.122)	(0.484)	(0.053)	(0.109)
Modified L(0.3)	0.111	0.452	0.108	0.440	0.025	0.077
	(0.110)	(0.463)	(0.116)	(0.446)	(0.053)	(0.100)
Modified L(0.4)	0.111	0.434	0.106	0.421	0.028	0.080
	(0.112)	(0.438)	(0.110)	(0.424)	(0.051)	(0.098)
Modified L(0.5)	0.111	0.424	0.105	0.410	0.029	0.081
	(0.111)	(0.426)	(0.107)	(0.410)	(0.050)	(0.096)
Modified L(0.6)	0.110	0.414	0.105	0.400	0.029	0.082
	(0.110)	(0.421)	(0.105)	(0.403)	(0.049)	(0.099)
Modified L(0.7)	0.109	0.411	0.106	0.398	0.029	0.081
	(0.110)	(0.419)	(0.107)	(0.401)	(0.049)	(0.095)
Modified L(0.8)	0.108	0.409	0.104	0.394	0.030	0.082
	(0.111)	(0.418)	(0.107)	(0.399)	(0.048)	(0.095)
Modified L(0.9)	0.107	0.410	0.105	0.397	0.029	0.083
	(0.109)	(0.420)	(0.111)	(0.400)	(0.048)	(0.097)

(B) ϕ=0.2, adjusted power in the parentheses

	Normal		T(6)		Mon_MDs	
test	T=100	T=500	T=100	T=500	T=100	T=500
Lobato	0.288	0.879	0.289	0.867	0.068	0.270
	(0.291)	(0.883)	(0.298)	(0.863)	(0.108)	(0.297)
Modified L(0.1)	0.378	0.981	0.374	0.980	0.069	0.361
	(0.378)	(0.983)	(0.391)	(0.981)	(0.129)	(0.417)
Modified L(0.2)	0.335	0.961	0.332	0.959	0.061	0.308
	(0.340)	(0.963)	(0.351)	(0.964)	(0.126)	(0.372)
Modified L(0.3)	0.313	0.935	0.312	0.932	0.064	0.285
	(0.319)	(0.939)	(0.326)	(0.936)	(0.118)	(0.334)
Modified L(0.4)	0.304	0.911	0.300	0.907	0.068	0.270
	(0.309)	(0.913)	(0.310)	(0.909)	(0.111)	(0.313)
Modified L(0.5)	0.296	0.895	0.291	0.886	0.070	0.263
	(0.297)	(0.897)	(0.293)	(0.886)	(0.108)	(0.294)
Modified L(0.6)	0.291	0.882	0.288	0.873	0.070	0.263
	(0.288)	(0.886)	(0.289)	(0.876)	(0.106)	(0.300)
Modified L(0.7)	0.289	0.876	0.287	0.866	0.067	0.261
	(0.290)	(0.881)	(0.291)	(0.867)	(0.108)	(0.295)
Modified L(0.8)	0.286	0.874	0.286	0.863	0.067	0.262
	(0.291)	(0.881)	(0.294)	(0.866)	(0.107)	(0.293)
Modified L(0.9)	0.286	0.872	0.286	0.862	0.068	0.262
	(0.291)	(0.882)	(0.298)	(0.865)	(0.108)	(0.297)

Table 4: Power (II) at the 5% level

(A) ϕ=0.1, adjusted power in the parentheses

test	GARCH		Bilinear	
	T=100	T=500	T=100	T=500
Lobato	0.104	0.395	0.082	0.299
	(0.100)	(0.393)	(0.077)	(0.303)
Modified L(0.1)	0.118	0.509	0.099	0.375
	(0.115)	(0.509)	(0.092)	(0.384)
Modified L(0.2)	0.112	0.460	0.088	0.342
	(0.112)	(0.460)	(0.087)	(0.348)
Modified L(0.3)	0.108	0.429	0.086	0.322
	(0.106)	(0.424)	(0.082)	(0.327)
Modified L(0.4)	0.109	0.411	0.085	0.309
	(0.106)	(0.412)	(0.079)	(0.314)
Modified L(0.5)	0.108	0.398	0.082	0.304
	(0.102)	(0.398)	(0.077)	(0.307)
Modified L(0.6)	0.106	0.391	0.083	0.298
	(0.101)	(0.394)	(0.075)	(0.300)
Modified L(0.7)	0.104	0.388	0.081	0.294
	(0.100)	(0.396)	(0.076)	(0.297)
Modified L(0.8)	0.103	0.387	0.082	0.291
	(0.100)	(0.394)	(0.079)	(0.299)
Modified L(0.9)	0.103	0.388	0.081	0.292
	(0.100)	(0.396)	(0.077)	(0.302)

(B) ϕ=0.2, adjusted power in the parentheses

test	GARCH		Bilinear	
	T=100	T=100	T=100	T=500
Lobato	0.277	0.227	0.227	0.761
	(0.271)	(0.218)	(0.218)	(0.763)
Modified L(0.1)	0.367	0.295	0.295	0.920
	(0.357)	(0.283)	(0.283)	(0.923)
Modified L(0.2)	0.322	0.262	0.262	0.879
	(0.322)	(0.259)	(0.259)	(0.883)
Modified L(0.3)	0.303	0.243	0.243	0.838
	(0.299)	(0.236)	(0.236)	(0.843)
Modified L(0.4)	0.292	0.235	0.235	0.802
	(0.286)	(0.223)	(0.223)	(0.807)
Modified L(0.5)	0.285	0.232	0.232	0.777
	(0.268)	(0.220)	(0.220)	(0.781)
Modified L(0.6)	0.277	0.226	0.226	0.762
	(0.266)	(0.214)	(0.214)	(0.763)
Modified L(0.7)	0.276	0.224	0.224	0.755
	(0.268)	(0.216)	(0.216)	(0.759)
Modified L(0.8)	0.275	0.225	0.225	0.753
	(0.268)	(0.220)	(0.220)	(0.760)
Modified L(0.9)	0.275	0.226	0.226	0.754
	(0.270)	(0.218)	(0.218)	(0.763)