E C O N O M I C S B U L L E T I N

A new approach to causality in the frequency domain

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Abstract

This study refers to the earlier work of analysis in the frequency domain. A different definition of causality is made, and its implications to the general idea of causality are discussed. The causality relationship between two monetary aggregates, simple sum and Divisia indices, and their relation with the personal income is analyzed using wavelet time−scale decomposition.

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1 Introduction

Many economists are aware that there is actually more time scales in between the short run and the long run; however, economic theory followed the convenience of representing the time in just two scales. The advance in computing technology, together with the fast advance in the field of harmonic analysis now make it possible to investigate empirical economic phenomena under many different time scales. This study investigates different time scales in the frequency domain. Lower frequencies in the data will be taken as related to longer run phenomena, whereas higher frequencies for short run events. This will bring the analysis the convenience to investigate different "runs" of the underlying economic system from the frequency domain point of view. The business cycles theory is full of models generating cycles (earlier works include that of Slutzky 1927, and Yule 1927) be them generated by a disturbance exogenous to the system, or endogenously generated (Kaldor 1940, Hicks 1950, Goodwin 1951).

Causality analysis is largely stands on the seminal work by Granger (1986) and his followers. His definition will be considered as the canonical definition of causality throughout this paper. Ramsey (2002), while investigating the money-income relationship, makes use of the stylized fact that causality relationship is a phase difference relationship in the frequency domain. Following this statement by Ramsey, this paper derives the phase difference (as defined in Koopmans 1947, p.147 and also in Brillinger 1975) between two time series and investigates the lead-lag relationship from the theoretical and empirical points of view. Section 2 shows that the Granger causality test may yield different results for different frequency bands of the data. For some cases, the result may even be spurious. Section 3 is an introduction to wavelet decomposition, section 4 briefs the Divisia index, and section 5 reports and discusses the empirical results. Section 6 concludes.

2 Derivation of the phase difference

Koopmans (1974) defines the bivariate spectral density of two stochastic processes Y, X as a function of the frequency in the complex domain as

$$f_{Y,X}(\lambda) = EZ_Y(d\lambda)\overline{Z_X(d\lambda)} = c_{Y,X}(\lambda) - i\ q_{Y,X}(\lambda) \tag{1}$$

where $Z_Y(d\lambda)$ and $Z_X(d\lambda)$ are the spectral measures of the two processes, E is the expectations operator, $c_{Y,X}(\lambda)$ and $q_{Y,X}(\lambda)$ are the cospectrum and quadrature spectrum of the series respectively. $\overline{Z}$ defines complex conjugate of Z, and $i^2 = -1$. The average phase lead of Y over X, $\phi_{Y,X}(\lambda)$ is defined accordingly:

$$\phi_{Y,X}(\lambda) = -\arctan(q_{Y,X}(\lambda)/c_{Y,X}(\lambda)) \tag{2}$$

Using the transfer function $a(\lambda) = A(e^{-i\lambda})$ for any lag polynomial $A(L)$, one can derive the phase difference for two economic aggregates where there is an underlying causality relationship in the Granger sense:

$$\begin{aligned} Y_t &= A_1(L)Y_t + A_2(L)X_t + U_t \\ X_t &= B_1(L)X_t + B_2(L)Y_t + V_t \end{aligned} \tag{3}$$

where A_1, A_2 and B_1, B_2 are lag polynomials of order p, q, r, s respectively, and U and V are i.i.d. disturbance terms. Then,

$$\begin{aligned} f_{Y,X}(\lambda) &= EZ_Y(d\lambda)\overline{Z_X(d\lambda)} \\ &= K \times \{[1 - A_1(e^{i\lambda})]A_2(e^{-i\lambda})\sigma_V^2 + [1 - B_1(e^{-i\lambda})]B_2(e^{i\lambda})\sigma_U^2\} \end{aligned} \tag{4}$$

where $K \in \mathbb{R}$, and σ_V^2 and σ_U^2 are variances of stochastic processes U and V respectively. Defining

$$\begin{aligned} A_1(L) &= \sum_{j=1}^{p} \alpha_j L^j \\ A_2(L) &= \sum_{k=1}^{q} \beta_k L^k \\ B_1(L) &= \sum_{h=1}^{r} \gamma_h L^h \\ B_2(L) &= \sum_{l=1}^{s} \theta_l L^l \end{aligned} \tag{5}$$

one can define the phase difference

$$\phi_{Y,X}(\lambda) = -\arctan[Imf_{Y,X}(\lambda)/Ref_{Y,X}(\lambda)] \tag{6}$$

For the sake of illustration, lets consider the simplified case $p = r = 2$ and $q = s = 1$. Now the system of equations under consideration is

$$Y_t = \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \beta X_{t-1} + U_t \quad (7)$$
$$X_t = \gamma_1 X_{t-1} + \gamma_2 X_{t-2} + \theta Y_{t-1} + V_t$$

For the system of equations (7), substituting equation (4) in equation (6) and solving for β, we get

$$\beta = \theta \frac{\sigma_U^2}{\sigma_V^2} \times \frac{\gamma_1 \sin\lambda + \gamma_2 \sin 2\lambda + (1 - \gamma_1 \cos\lambda + \gamma_2 \cos 2\lambda)\tan\phi_{Y,X}(\lambda)}{\alpha_1 \sin\lambda + \alpha_2 \sin 2\lambda + (1 - \alpha_1 \cos\lambda + \alpha_2 \cos 2\lambda)\tan\phi_{Y,X}(\lambda)} \quad (8)$$

Assuming the last factor nonzero, for the simplified case above notice that $\beta = 0 \Leftrightarrow \theta = 0$ and $\beta \neq 0 \Leftrightarrow \theta \neq 0$, except for particular frequencies λ given $\phi_{Y,X}(\lambda) \neq \frac{\pi}{2}$. So, a Granger causality test will generally yield either feedback or inconclusive result for most frequency bands in the data. The result could be generalized for other values of p, q, r, s. The result of the Granger causality test depends on the frequency band under consideration.

For this simplified case, it is shown that the null hypotheses of feedback and inconclusiveness depends on many factors: the own lag structure of the series, the variances of the disturbance terms, the phase difference between the series, and most of all, the frequency λ . The above formula may yield zero or nonzero values for β for different values of the frequency. If the data is decomposed into different frequency bands, it is possible to observe causality relationships in opposing directions, or even spurious relation between the series under consideration.

3 Wavelet decomposition

This section is devoted to introduce the wavelet transform to the reader. An attempt is made to keep the text least technical as possible, but detailed enough to be able to discuss various aspects of the application. The interested reader may refer to many references present such as, but not limited to Chui (1992), Gençay *et al.* (2002), Percival and Walden (2000), and Daubechies (1994). This study also refers to an excellent and still compact introduction by Ramsey (2002).

The Fourier transform, which preceded wavelets in harmonic analysis, was a tool to convert the data series in time domain to the frequency domain. However, Fourier transform does not provide the analyst the valuable information in time domain, like possible changes in time in harmonic structure. This requirement has been partially fulfilled by the windowed Fourier transform where a window of fixed length is slid in the time domain to yield time dependent frequency spectra for analysis. Wavelet bases do a similar task of scanning the time series for frequencies of a certain range.

One basic superiority of the wavelet transform over the windowed Fourier transform is that wavelet transform is a band-pass filter at the same time. That is, it gives the strength of frequencies in a given interval, and by following that over time, the analyst may observe which frequencies actually dominate the series at a certain time.

Another superiority of the wavelet transform is that it is coarser for lower frequencies, and gives a more detailed graphical illustration for higher frequencies. For the lowest frequencies that could be captured, the wavelet transform gets closer to a Fourier transform with a wider "window", except for the fact that the former captures a limited number of frequencies. As the spectrum of frequencies increase, the "window" of the wavelet transform gets narrower, so changes in the harmonic structure for these frequencies could be followed in more detail in time scale.

These features of wavelets have some virtues for economic analysis: first of all, as a band-pass filter, it enables us to make a distinction on the behavior of different frequencies through time. For instance, the wavelet transformation of a monetary aggregate could be analyzed separately in a succession of time scales. Lower frequencies will represent a periodic structure to cover years or decades, higher frequencies seasons, and the highest frequency band represents months in our case. As we go from lower frequencies to higher, the transformed series will get finer and finer, enabling us a month by month observation of a periodic structure that could be defined in terms of months, and year by year observation of a periodic structure that could be defined in years. In this study, the causality relationship between money and income will be analyzed in different time scales, thanks to this appealing property of wavelets. We would be able to see the causal relationship between these two aggregates in terms of their month-by-month, quarter-by quarter, and year-by-year behavior.

4 Monetary aggregation

The monetary aggregates used most widely by the Central Banks of the world have been criticized for being simple sum aggregates. Indeed, the simple sum aggregate assigns equal weights of one for each component, which may not be reliable for monetary aggregates since perfect substitutability of all monetary assets are assumed. An alternative index derived by Barnett (1980) is the Divisia index which utilizes the user cost of different forms of money for weighting rate of change of components to get the rate of change of the index, namely

$$\frac{d\log M_t}{dt} = \sum_i s_{it} \frac{d\log m_{it}}{dt} \tag{9}$$

where M is the monetary aggregate, m_{it}'s are the components of the aggregate, and s_{it}'s are "the ith asset's share in expenditure on the total portfolio's service flow." (Barnett, 1987)

The Divisia index is a member of "superlative indexes" class as defined by Diewert (1976). The changes in the values of components of the index that have lower user costs (higher return monetary assets are in this group) will affect the Divisia index more. So, distinction is made between the components in Divisia index unlike the simple sum.

5 The method and results

This study has a number of aims. First, to verify the direction of causality between monetary aggregates and output. Second, to compare the results obtained by using two different monetary aggregates. Third, to observe the change in causality relationships in different intervals of time, with an emphasis on the "monetary experiment" of the late seventies-early eighties in the United States.

5.1 The method

The data covers monthly personal income (PI) and monetary aggregates for a thirty year period between 1962 and 1992. The monetary aggregates are simple sum M1 and M2 (SSM1 and SSM2), and Divisia M1 and Divisia M2 (DM1 and DM2). Personal income is used as a proxy for the GDP series since the latter does not come as a monthly series. Monthly series are required for this study to observe the behavior of the data for higher frequencies of the frequency spectrum. For a discussion of using personal income as a proxy for GDP, see Ramsey and Lampart (1998). The Granger causality test is the Wald variant of the Granger test as explained in Granger and Newbold (1986, p.260). Nelson and Schwert (1982) report a serious loss of power of the test when it is based on heavily parameterized models. For that reason, the Schwartz Bayesian Criteria, which penalizes number of parameters more heavily compared to the Akaike Information Criteria, is selected to determine the lag lengths. Also, Geweke, Meese, and Dent (1983) indicate a preference for the Wald variant of the Granger test which has been used in this study. The procedure is thus estimating the following models separately using ordinary least squares.

$$Y_t = \sum_{j=1}^{K} \delta_j Y_{t-j} + \sum_{j=1}^{K} \rho_j M_{t-j} + \epsilon_t \tag{10}$$

$$Y_t = \sum_{j=1}^{K} \eta_j Y_{t-j} + \xi_t \tag{11}$$

where K is the lag length determined using the Schwartz Bayesian criteria, Y is income, M is the monetary aggregate, and $\{\epsilon_t\}$ and $\{\xi_t\}$ are the residual series. Let $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ denote the residual variance estimates for the models above, respectively. Then the test statistic for sample size T is

$$\tau = T \frac{\hat{\sigma}_2^2 - \hat{\sigma}_1^2}{\hat{\sigma}_1^2} \tag{12}$$

which has an asymptotic χ^2 distribution with K degrees of freedom under the null hypothesis that M does not cause Y. For testing the causality relationship other way around, Y becomes the independent variable, and M becomes the dependent variable in equation (10), and equation (11) is estimated for M.

Following Ramsey and Lampart (1998), the data is taken "as is" for wavelet decomposition together with its log difference transformation. Gençay, Selçuk, and Whitcher (2002, p.152, Table 4.4), display their results on the unit root tests for monthly U.S. M1 and production indexes for the period 1959-1999. Similarly, Serletis (2001, p.144, Table 11.1) report the unit root test results for different quarterly monetary indexes for the period 1960-1999. Both report that M1 is integrated of degree 1. Serletis also reports that all monetary indexes used in this study are of $I(1)$. Gençay, Selçuk, and Whitcher (2002, p.153) is using the logged data, and a cointegration-robust version of the Granger causality test to get around the problem of integration. We follow Ramsey and Lampart's (1998) approach to include log difference series in our study. The wavelet decomposition series have been obtained from the raw data without any transformation. Wavelet decomposition is applied to the raw data using Daubechies 8 (or D(8)) orthogonal wavelet.

The Daubechies discrete wavelet filters may be interpreted as generalized differences of adjacent averages (Gençay *et al.*, 2002) and they are related with difference operators. The wavelet filter used in this study, Daubechies 8 has four vanishing moments. Thus, it has the ability to generate stationary series from a series integrated up to and including level 4. One of the reasons why this group of wavelets has been chosen is to overcome the problem of nonstationarity in the income and monetary aggregate series. Another reason for this choice is the lower leakage of the filter (as a band-pass filter) at all frequency levels, and smoothness of the wavelet function for the purposes of the data series to be examined. The Daubechies discrete wavelet filters are widely used, and available in most mathematical software. Tests with another group of filters, quadratic spline wavelets, showed the results are basically robust to the choice of the wavelet base.

5.2 Results

In an earlier study, Ramsey and Lampart (1998) find interesting causality relationships between personal income (PI) and M1. For the finest scale, the relationship is "PI causes M1," and for more rough scales, the direction of causality turns the opposite way. At the lowest frequencies, feedback is reported to be the relation. They also report that, the causality relationship depends on the time period chosen. For instance, they report a change in the direction of the causality relationship at the finest scale for the periods 1970-1979 and 1980-1994. In a similar study, Gençay, Selçuk, and Whitcher (2002) report similar results for the US industrial production index and the M1. They use a cointegration-robust version of the causality test for the period 1959-1999 with logarithmic data. Their test is inconclusive for the main series, and similar with Ramsey and Lampart for the decomposed series. For the rest of the analysis, the reader may refer to Tables I-IV of the appendix II. In order to be able to observe the changes in the causality relation, three periods of 128 months each is taken together with a combination of the later two periods (1972:9 to 1992:12). The result for the 3rd level decomposition of DM2 is particularly interesting. For periodic movements of periods 5 to 8 months, the causality relation changes direction for all three sub-series. This is an expected result for the second and third periods because of the so-called "monetary experiment" of the early eighties. The substantial change in the monetary policy should be showing up in the causality relations. On the contrary, there is no change in causality for the 4th level decomposition (9-16 months) of DM1 for these periods. Also, no change could be observed for SSM2 first level decomposed series. As for the change of direction between the first and second period series, we have 3rd level decomposed series of both DM1 and DM2, which is not easy to explain from the economic theory viewpoint. Here, the Divisia series generate a number of interesting results above even under a relatively short period of 128 months. Also, Divisia index generates less feedback and inconclusive results compared with simple sum index for both M1 and M2. Therefore, one may conclude that the Divisia indices reflect the causality relation with a higher degree of success compared to the simple sum indices, even with shorter sets of data. As for the consistency of the prior studies, none of the series tested here verified a causality of "income causes money" for the finest scale decomposition, and "money causes income" for the two consecutive levels. The conclusion so far is that the causality relation between money and income depends on the period of interest, the method of aggregation (simple sum vs. Divisia), and the level of aggregation (M1 vs. M2).

6 Conclusion

This study sheds some light on the causality relationship between money and output using personal income as a proxy. The wavelet transform makes it possible to see this relationship in different levels of resolution. Wavelet decomposition makes it possible to see in which timescale is the change: for the majority of the cases, the direction of causality is found to be "money $\rightarrow$ income" for all scales of resolution. This result illustrates the effectiveness of monetary policies in the U.S. economy. There are serious differences both in the direction of causality, and the conclusiveness of the test for periods 1962-1971, 1971-1982 and 1982-1992 in all scales. This indicates a change in the monetary policy between these periods. Monetary policy seems to become more effective in the 80s and early 90s compared to the 70s. The change in the causality relationship in the lower frequencies (wider time scope) may be due to change in the general monetary policy, or, as Ramsey and Lampart argue, may be a result of the phase difference between the series. Under the findings of section 2, our conclusion is that the causality relationship depends on the scale under which the series is observed. Wavelet decomposition is a valuable tool in that sense. The second conclusion is that since the null could hold spuriously for certain values of lag coefficients, phase difference, and frequency, the empirical results should be taken skeptically.

References

Barnett, W.A. (1980) "Economic Monetary Aggregates: An Application of Aggregation and Index Number Theory" *Journal of Econometrics* **14**, 11-48

Barnett, W.A. (1987) "The Microeconomic Theory of Monetary Aggregation" in *New Approaches to Monetary Economics,* Proceedings of the 2nd international symposium in economic theory and econometrics, W.A. Barnett and K.J. Singleton, eds., Cambridge University Press.

Brillinger, D.R. (1975) *Time Series: Data Analysis and Theory* Holt, Rinehart and Winston, Inc.

Chui, C.K. (1992) *An Introduction to Wavelets, Wavelet Analysis and its Applications* Volume 1, Academic Press

Daubechies, I., (1992) *Ten Lectures on Wavelets* CBMS-NSF Regional Conference Series on Applied Mathematics **61**, Society for Industrial and Applied Mathematics: Philadelphia

Diewert, E. (1976) "Exact and Superlative Index Numbers" *Journal of Econometrics* **4**, 115-145

Gençay, R., Selçuk, F., and Whitcher, B. (2002) *An Introduction to Wavelets and Other Filtering Methods in Finance and Economics* Academic Press

Geweke, J., Meese, R., and Dent, W., (1983) "Comparing alternative tests of causality in temporal systems," *Journal of Econometrics,* **21**, 161-194,

Goodwin, R.M. (1947) "Dynamical coupling with espacial reference to markets having production lags" *Econometrica* **15**, 181-204,

Granger, C.W.J., and Newbold, P. (1986) *Forecasting Economic Time Series* 2nd ed., Academic Press: New York

Hicks, J.R., (1950) *A contribution to the theory of the trade cycle*, Oxford University Press: Oxford, 2nd edition, (1965), Claredon Press: Oxford

Kaldor, N. (1940) "A model of the trade cycle" *Economic Journal* **50**, 78-92

Koopmans, L.H. (1974) *The Spectral Analysis of Time Series* Monograph and Textbook Series in Probability and Mathematical Statistics, Academic Press: London

Nelson, C.R., and Schwert, G.W. (1982) "Tests for Predictive Relationships between Time Series Variables: A Monte Carlo Investigation" *Journal of American Statistical Association* **77**, 11-18

Percival, D.B., and Walden, A.T. (2000) *Wavelet Methods for Time Series Analysis* Cambridge University Press

Ramsey, J.B. (2002) *Wavelets in Economics and Finance: Past and Future* Studies in Nonlinear Dynamics and Economics, **6** iss.3

Ramsey, J.B., and Lampart, C. (1998) "Decomposition of Economic Relationships by Timescale

Using Wavelets: Money and Income" *Macroeconomic Dynamics* **2**, 49-71

Serletis, A. (2001) *The Demand for Money* Kluwer Academic Publishers

Slutzky, E. (1937) "The summation of random causes as the source of cyclic processes" *Econometrica* **5**, 105-146

Yule G.U.(1927) "On a Method of Investigating Periodicities in Disturbed Series, with Special Reference to Wolfer's Sunspot Numbers" in *The foundations of econometric analysis* by D.F. Hendry and M.S. Morgan, Eds., Cambridge University Press: Cambridge, New York and Melbourne 1995, 159-65. Previously published: 1927.

Appendix I: Wavelet decomposition

This appendix refers to the works by Ramsey (2002), Gençay *e*t al. (2002), Daubechies (1992), and Percival and Walden (2000).
A wavelet could be defined as a function that satisfies the wavelet admissibility condition

$$\int_0^\infty \frac{|\Psi(\lambda)|}{\lambda} d\lambda < \infty \tag{13}$$

where $\Psi(\lambda) = \int_{-\infty}^{\infty} \psi(t) e^{-i\lambda t} dt$ is the Fourier transform of $\psi(t)$. An equivalent condition is $\Psi(0) = 0$, or

$$\int_{-\infty}^{\infty} \psi(t) dt = 0. \tag{14}$$

A secondary condition for $\psi(t)$ is unit energy,

$$\int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1. \tag{15}$$

An example is the wavelet related to the first derivative of the density of the normal distribution, which satisfy all the conditions above:

$$\psi(t) = \frac{\sqrt{2}t}{\sigma^{3/2}\pi^{1/4}} e^{-t^2/2\sigma^2}. \tag{16}$$

For a suitable choice of function $\Phi(\cdot)$ satisfying $\int_{-\infty}^{\infty} \Phi(t) dt = 1$ the corresponding "scaling function" is

$$\Phi_{J,k} = 2^{-\frac{J}{2}} \Phi(\frac{t - 2^J k}{2^J}) \tag{17}$$

and the corresponding wavelets are

$$\psi_{j,k} = 2^{-\frac{j}{2}} \psi(\frac{t - 2^j k}{2^j}), \quad j = 1, \ldots, J \tag{18}$$

with the properties of $\psi(\cdot)$ defined above.
Define

$$\begin{aligned} s_{J,k} &= \int f(t)\Phi_{J,k}(t)dt & (19) \\ w_{j,k} &= \int f(t)\psi_{j,k}(t)dt, \qquad j = 1,\ldots,J. & (20) \end{aligned}$$

Now the function $f(\cdot)$ could be expressed by

$$f(t) = S_J + W_J + W_{J-1} + \ldots + W_j + \ldots + W_1 \tag{21}$$

where

$$\begin{aligned} S_J &= \sum_k s_{J,k}\Phi_{J,k}(t) & (22) \\ W_j &= \sum_k w_{j,k}\psi_{j,k}(t), \qquad j = 1,\ldots,J. & (23) \end{aligned}$$

If we think $f(\cdot)$ as a binary color coded picture, S_J gives us a smooth background, and with the addition of each W_j, $j = J,\ldots,1$ we start to see more and more details, and in the end, the entire picture, $f(t)$, becomes apparent. Wavelets make it possible to examine continuous functions or time series in different levels of detail, namely,

$$\begin{aligned} S_j &= S_J + W_J + \ldots + W_{j+1} & (24) \\ f &\equiv S_0. \end{aligned}$$

In order to represent discrete time series in vector form $\mathbf{y} = (y_1, y_2, \ldots, y_T)'$, the discrete wavelet transform could be defined as

$$\mathbf{w} = \mathcal{W}\mathbf{y} \tag{25}$$

where the orthonormal $T \times T$ matrix $\mathcal{W}$ is to be determined by a procedure called the "pyramid algorithm."(see Gençay *et al.*) The vector $\mathbf{w}$ has T (assumed to be a power of 2) elements where the first $T/2$ entries are $w_{1,k}$'s, the following $T/2^2$ entries are $w_{2,k}$'s and so on. The last $T/2^J$ entries are $s_{J,k}$'s.
The wavelet used in this study, Daubechies extremal phase wavelet (denoted by $D(L)$) gets

closer to an ideal filter with increasing L. The wavelet $D(2)$ (also known as the "Haar wavelet," the first wavelet in history) is simply differences of adjacent moving averages of a series, with increasing number of data points averaged as the scale becomes coarser. Specifically, the choice wavelet employed in this study, $D(8)$, does not have a closed form. The transform in this case is entirely numerical. Further technical details of wavelet transform requires far more space than a shorter appendix. Interested reader may refer to the references cited at the beginning of this section.

In this study, the wavelet coefficient vector $\mathbf{w}$ is derived using the TimeStat^{TM} software. For the highest frequency component, the second half of the vector (i.e., the part which has no information concerning the highest frequency component) has been set to zero, and the vector is transformed back to the time domain using the inverse wavelet transform. Likewise, for isolating a certain frequency level, the coefficients other than those representing that frequency level have been set to zero, and the series have been converted back to the time domain. Then, Granger causality tests (that are defined in the time domain) have been applied to those time series where a certain band of frequencies are isolated. That way, it was possible to observe causality relationships within a certain frequency band.

Appendix II: Tables

Table I Granger causality test results between personal income (PI) and simple sum M1 index (SSM1).
Significance level is 1% for cells marked by a †. Otherwise it is 5%.
(F: feedback, I: inconclusive, →: income causes money, ← : money causes income.)

		period in months	Time Periods Included			
			61:1-71:8	71:9-82:4	82:5-92:12	71:9-92:12
PI	m1	*not filtered*	I	I	←	←
$\triangle \log PI$	$\triangle \log m1$	*not filtered*	I	→	I	I
PI[6]	m1[6]	33-128	F	F	I	←
PI[5]	m1[5]	17-32	I	F	F	F
PI[4]	m1[4]	9-16	F	←	I	I
PI[3]	m1[3]	5-8	F	F	F	I
PI[2]	m1[2]	3-4	F	F	F	← †
PI[1]	m1[1]	2	F	←	I	F

Table II Granger causality test results between personal income (PI) and simple sum M2 index (SSM2).
Significance level is 1% for cells marked by a †. Otherwise it is 5%.
(F: feedback, I: inconclusive, →: income causes money, ← : money causes income.)

		period in months	Time Periods Included			
			61:1-71:8	71:9-82:4	82:5-92:12	71:9-92:12
PI	m1	*not filtered*	I	F	I	I
$\triangle \log PI$	$\triangle \log m1$	*not filtered*	I	I	I	I
PI[6]	m1[6]	33-128	F	← †	F	F
PI[5]	m1[5]	17-32	F	F	F	←†
PI[4]	m1[4]	9-16	F	F	F	→
PI[3]	m1[3]	5-8	F	F	→†	→
PI[2]	m1[2]	3-4	F	F	F	←
PI[1]	m1[1]	2	F	←	←	F

Table III Granger causality test results between personal income (PI) and Divisia M1 index (DM1).

Significance level is 1% for cells marked by a †. Otherwise it is 5%.

(F: feedback, I: inconclusive, →: income causes money, ←: money causes income.)

		period in months	Time Periods Included			
			61:1-71:8	71:9-82:4	82:5-92:12	71:9-92:12
PI	m1	*not filtered*	I	I	←	←
$\triangle \log PI$	$\triangle \log m1$	*not filtered*	I	I	I	I
PI[6]	m1[6]	33-128	F	F	F	I
PI[5]	m1[5]	17-32	I	F	F	F
PI[4]	m1[4]	9-16	F	←	←†	←
PI[3]	m1[3]	5-8	→	←	F	F
PI[2]	m1[2]	3-4	F	F	F	←
PI[1]	m1[1]	2	F	F	I	F

Table IV Granger causality test results between personal income (PI) and Divisia M2 index (DM2).

Significance level is 1% for cells marked by a †. Otherwise it is 5%.

(F: feedback, I: inconclusive, ←: income causes money, →: money causes income.)

		period in months	Time Periods Included			
			61:1-71:8	71:9-82:4	82:5-92:12	71:9-92:12
PI	m1	*not filtered*	I	←	I	I
$\triangle \log PI$	$\triangle \log m1$	*not filtered*	I	I	I	I
PI[6]	m1[6]	33-128	F	I	←	F
PI[5]	m1[5]	17-32	I	F	F	←
PI[4]	m1[4]	9-16	F	F	F	I
PI[3]	m1[3]	5-8	→	←	→	→
PI[2]	m1[2]	3-4	F	F	←†	←
PI[1]	m1[1]	2	F	←†	I	F