E C O N O M I C S   B U L L E T I N

Monetary Policy and Equilibrium Indeterminacy in a Cash−in−Advance Economy with Investment

Chong Kee Yip
Department of Economics, The Chinese University of Hong Kong

Ka Fai Li
Department of Economics, The Chinese University of Hong Kong

Abstract

In a cash−in−advance economy where cash is required in advance of purchasing both consumption and investment goods, we find that active interest rate rules generate equilibrium uniqueness, but passive rules can lead to real indeterminacy. Simulation shows that even in the presence of investment, passive rules are very likely to render indeterminacy.

Citation: Yip, Chong Kee and Ka Fai Li, (2004) "Monetary Policy and Equilibrium Indeterminacy in a Cash−in−Advance Economy with Investment." *Economics Bulletin,* Vol. 5, No. 2 pp. 1−7
Submitted: December 5, 2003. **Accepted:** January 15, 2004.
URL: http://www.economicsbulletin.com/2004/volume5/EB−03E50006A.pdf

1 Introduction

The conventional conclusion in the literature on interest rate feedback rules is that an active monetary policy that fights inflation by raising the nominal interest rate by more than the increase in inflation stabilizes the economy by ensuring the uniqueness of equilibrium. On the other hand, a passive monetary policy that increases inflation by raising the nominal interest rate by less than the observed increase in inflation yields indeterminacy by giving rise to expectations-driven fluctuations. [e.g., Leeper (1991), Clarida et al. (2000) and Benhabib et al. (2001a)]

There are recent challenges to these conventional results. Benhabib et al. (2001a) show that depending on the way in which money is assumed to enter preferences and technology, an active monetary policy does not necessarily bring about the determinacy of equilibrium, and that passive monetary policy may. By appending endogenous investment to a benchmark continuous-time flexible-price model, Meng and Yip (2004) find that equilibrium uniqueness is ensured regardless of the interest rate policy rules. However, if endogenous labor supply is considered, then Meng and Yip show that indeterminacy of equilibrium can be obtained for both types of monetary rules. Nevertheless, the condition for indeterminacy carries an intuition that the labor demand and supply curves may cross with wrong slopes.[1] Likewise, in an endogenous growth model with a Clower CIA constraint and endogenous labor supply, Itaya and Mino (2002) also find that in the presence of external increasing returns, indeterminacy can occur regardless the interest rate rule is active or passive. But as in Meng and Yip (2004), the indeterminacy conditions offer the same interpretation that the labor demand and supply curves cross with wrong slopes. In addition, the magnitude of the external increasing returns required for indeterminacy is not

[1] Similar findings are present in the work by Benhabib and Farmer (1994) and Farmer (1997).

assessed.

The present note studies monetary policy and multiple equilibria in a cash-in-advance (CIA) economy where cash is required in advance of purchasing both consumption and investment goods. We find that active interest rate rules generate equilibrium uniqueness, but passive rules can lead to real indeterminacy. This provides an alternative, less controversial environment that indeterminacy of equilibrium can occur in the presence of capital accumulation. Moreover, we assess quantitatively the likelihood of the indeterminacy condition and our simulation shows that even in the presence of investment, passive rules are very likely to render indeterminacy. Finally, corroborating the findings of Meng (2002), whether indeterminacy can occur under passive rules depend on, but not too sensitive to, the magnitude of the steady-state inflation rate.

2 A Flexible-Price, Cash in Advance Economy

The household's lifetime utility function is given by

$$Max \int_0^\infty U(c) \; e^{-\rho t} dt \tag{1}$$

where $\rho > 0$ represents the rate of time preference and c is consumption. The instantaneous utility function satisfies $U_c > 0 > U_{cc}$.[2] The household may hold wealth in terms of either capital, money or bonds. The law of motion of capital is:[3]

$$\dot{k} = i \tag{2}$$

where i denotes physical investment. The real value of non-capital wealth a (i.e. money and bonds) evolves according to

$$\dot{a} = (R - \pi) a + f(k) - Rm - i - \tau - c \tag{3}$$

[2] U_c and U_{cc} denote the first- and second-order derivative of U respectively.
[3] For simplicity, we abstract from depreciation of capital stock.

where m is real money balances, R is nominal interest rate and π is inflation rate. Household has perfect foresight and takes price and lump-sum transfer τ as given. The neoclassical production function f satisfies $f^{'}(k) > 0 > f^{''}(k)$.[4]

In addition, the household is subject to the following liquidity constraint

$$c + \phi \dot{k} \leq m \tag{4}$$

where $\phi \in [0, 1]$. According to (4), the CIA constraint applies to all consumption goods and a fraction of investment goods as proposed by Wang and Yip (1992).[5] The household then maximizes (1) subject to(2) - (4) and the standard no-Ponzi-game condition. The optimality conditions are

$$U_c = \lambda + \psi \tag{5}$$

$$\psi = R\lambda \tag{6}$$

$$\lambda + \phi\psi = \lambda_k \tag{7}$$

$$\dot{\lambda} = \lambda (\rho + \pi - R) \tag{8}$$

$$\dot{\lambda}_k = \rho\lambda_k - \lambda f^{'}(k) \tag{9}$$

as well as the transversality conditions associated with a and k, where λ and λ_k are the costate variables for a and k respectively and ψ is the Lagrangian multiplier for (4). Equilibrium in goods market yields

$$\dot{k} = f(k) - c \tag{10}$$

[4] f' and f'' denote the first- and second-order derivative of f respectively.

[5] In the analysis below, we would not study the special case where $\phi = 0$ (the Clower CIA constraint). This is because the equilibrium dynamics of this special case is identical to the money-in-the-utility-function models. To be specific, in the Clower CIA model, there exists a unique perfect foresight equilibrium under both active and passive monetary policies which coincides with the main finding of Meng and Yip (2004). The intuition is that these two classes of models are functionally and qualitatively equivalent as shown by Feenstra (1986) and Wang and Yip (1992) respectively.

Following Leeper (1991) and Benhabib et al. (2001a), the monetary authority sets the nominal interest rate as a function of the inflation rate, i.e.

$$R = R(\pi) \tag{11}$$

where $R(\cdot)$ is continuous, nondecreasing, and strictly positive and there exists at least one $\pi^* > -\rho$ such that $R(\pi^*) = \rho + \pi^*$.[6] We refer the monetary policy as active if $R'(\pi^*) > 1$ and passive if $R'(\pi^*) < 1$.[7] We further assume $R'(\pi^*) \neq 1$.[8]

3 Equilibrium Dynamics

From (5) to (7), we can solve

$$c = c(\lambda, \lambda_k) \quad \text{and} \quad \pi = \pi(\lambda, \lambda_k) \tag{12}$$

where $\partial c/\partial\lambda = (\phi - 1)/\phi U_{cc} > 0$, $\partial c/\partial\lambda_k = 1/\phi U_{cc} < 0$, $\partial\pi/\partial\lambda = -(1 + \phi R)/\phi\lambda R' < 0$ and $\partial\pi/\partial\lambda_k = 1/\lambda R' > 0$. Substituting (11) and (12) into the dynamic equations (8) - (10), we have

$$\dot{\lambda} = \lambda\left[\rho + \pi(\lambda, \lambda_k) - R(\pi(\lambda, \lambda_k))\right] \tag{13}$$

$$\dot{\lambda}_k = \rho\lambda_k - \lambda f'(k) \tag{14}$$

$$\dot{k} = f(k) - c(\lambda, \lambda_k) \tag{15}$$

which characterize the dynamics of the system. Linearizing (13) - (15) around the steady state $(\lambda^*, \lambda_k^*, k^*)$, we have

$$\begin{pmatrix} \dot{\lambda} \\ \dot{\lambda}_k \\ \dot{k} \end{pmatrix} = A \begin{pmatrix} \lambda - \lambda^* \\ \lambda_k - \lambda_k^* \\ k - k^* \end{pmatrix} \tag{16}$$

[6] The asterisk * denotes the steady-state value of a variable.

[7] R' denotes the derivative of R with respect to π.

[8] We assume, as in Meng (2002), that fiscal policy is Ricardian so that the present discounted value of total government liabilities converges to zero both in and off equilibrium. For details, see Benhabib et al. (2001a) and the reference cited therein.

where the Jacobian matrix A is given by[9]

$$A=\begin{bmatrix} -\left(1-R'\right)\frac{1+\phi R(\pi^*)}{\phi R'} & \left(1-R'\right)\frac{1}{\phi R'} & 0 \\ -\rho\left(1+\phi R\left(\pi^*\right)\right) & \rho & -\lambda^* f''\left(k^*\right) \\ -\frac{(\phi-1)}{\phi U_{cc}} & -\frac{1}{\phi U_{cc}} & \rho\left(1+\phi R\left(\pi^*\right)\right) \end{bmatrix}.$$

The trace and the determinant of the Jacobian matrix is given by:

$$tr\left(A\right)=-\frac{\left(1+\phi R\left(\pi^*\right)\right)\left(1-R'\right)}{\phi R'}+\rho+\rho\left(1+\phi R\left(\pi^*\right)\right) \tag{17}$$

$$\det\left(A\right)=\frac{\left(1+R\left(\pi^*\right)\right)\left(1-R'\right)}{\phi R'}\frac{\lambda^* f''\left(k^*\right)}{U_{cc}}. \tag{18}$$

According to (18), $\det\left(A\right)>(<)0$ if we have passive (active) rules. From (17), $tr\left(A\right)>0$ under active rules but is ambiguous under passive rules. Thus, we can conclude:

Proposition 1 *For active policy rule where $R'>1$, we must have two characteristic roots with positive real parts and one characteristic root with negative real part. The steady-state equilibrium is a saddle and there is no real indeterminacy.*

Proposition 2 *For passive policy rule where $R'<1$, we either have three characteristic roots with positive real parts or one characteristic root with positive real part and two with negative real parts. In the latter case, real indeterminacy is possible.*

The intuition behind these results is as follows. Suppose consumption is reduced below its steady-state level, it follows that the nominal interest rate has to rise above its steady-state level as well. This in turn pushes up the shadow price of capital so that consumption falls further. Under active interest rate rules, the real interest rate also rises which then leads to a decline in capital stock. This lowers production and reduces consumption further from the steady

[9] All derivatives are evaluated at their steady-state values.

state so that such a trajectory is not consistent with equilibrium. On the other hand, if the interest rate rule is passive, then the real interest rate falls instead. As a result, capital stock must rise so that consumption will increase and return to its steady state level. If this income-type effect dominates the effect of the shadow price of capital on consumption, then the trajectory under consideration is consistent with the steady state equilibrium.[10]

In order to investigate further on real indeterminacy under passive rules, we note that a negative trace is a sufficient condition for real indeterminacy in this case. This sufficient condition is equivalent to the following restriction:

$$R' < \frac{1 + \phi R(\pi^*)}{1 + \phi \left[\rho + R(\pi^*) + \rho \left(1 + \phi R(\pi^*)\right)\right]} \equiv B. \tag{19}$$

Notice that the steady-state inflation rate affects the nominal interest rate and hence the indeterminacy result. We then have

Corollary 3 *For passive rule, if $R' < B$, we have real indeterminacy.*

Numerical examples can be constructed as follows. We take $\rho = 0.0045$. For the CIA parameter ϕ, we consider several values ranging from 0.25 to 1. For the steady-state inflation rate π^*, we start from 0 and all the way up to 1000%. The values of B is summarized in Table 1.

(Table 1 about here)

As B ranges from 0.9911 to 0.9978, we believe that it is very likely to have indeterminacy under passive rules. We also provide the following example for illustrative purposes. The utility function is the constant intertemporal elasticity of substitution type, i.e. $c^{1-\sigma}/1-\sigma$, where σ is the inverse of intertemporal

[10] In the case where the CIA constraint does not apply to investment goods (i.e., $\phi = 0$), then only the shadow price effect on consumption is at work so that equilibrium determinacy occurs regardless whether monetary policy is active or passive. This is consistent with the main finding of Meng and Yip (2004) although their analysis is conducted in a money-in-the-utility-function model.

elasticity of substitution. In addition, we assumed a simple Cobb Douglas production function, i.e.$y = AK^{\alpha}$, where A is a constant scaling factor measuring the productivity of the general capital and α is the capital share.

Example 1. Consider the following parameterization: $\rho = 0.0045$, $R^{'} = 0.5$, $\pi^{*} = 0\%$, $A = 2$, $\alpha = 0.33$ and $\sigma = 1.5$. In this case, the eigenvalues are -1, 0.0045207 and -4.53548×10^{-7}, implying that local indeterminacy occurs.[11]

4 Conclusion

In a neoclassical growth model where both consumption and a fraction of investment are subject to CIA constraint, we find that passive rules are very likely to generate real indeterminacy while active rules render equilibrium uniqueness. However, we must remind the readers that local equilibrium uniqueness does not imply global uniqueness.[12] Nevertheless, the present paper focuses on the nature of the CIA constraint and investiagtes the local dynamic properties of the steady state equilibrium. The global analysis with endogenous investment awaits for future research.

[11] It is likely that when $R^{'}$ changes across B, the system may experience a Hopf bifurcation. Such analysis of periodic equilibra of closed orbits deserves further study.

[12] See Benhabib et al (2001b) for a global analysis on the dynamics of interest rate feedback rules.

Table 1

ϕ	π^* (%)	B
0.25	0	0.9978
0.5	0	0.9956
0.75	0	0.9933
1	0	0.9911
1	5	0.9913
1	10	0.9915
1	50	0.9926
1	100	0.9933
1	250	0.9942
1	500	0.9948
1	1000	0.9951

The column under B (where B is defined below) shows the critical values for generating local indeterminacy under different combinations of steady state inflation rates (π^*) and the CIA fraction parameter (ϕ). When the policy rules' parameter is less than the critical value, local indeterminacy occurs. (i.e. when $R^{'} < B$, local indeterminacy occurs)

Note: $B \equiv \dfrac{1+\phi R(\pi^*)}{1+\phi\left[\rho + R(\pi^*) + \rho\left(1+R(\pi^*)\right)\right]}$

where ρ is the rate of time preference and is set to 0.0045 and $R(\pi^*) = \rho + \pi^*$.

References

[1] Benhabib, J. and R. E. A. Farmer, "Indeterminacy and Increasing Returns," *Journal of Economic Theory*, **63** (1994), 19-41.

[2] Benhabib, J., S. Schmitt-Grohé and M. Uribe, "Monetary Policy and Multiple Equilibria," *American Economic Review*, **91** (2001a), 167-186.

[3] Benhabib, J., S. Schmitt-Grohé and M. Uribe, "The Perils of Taylor Rules," *Journal of Economic Theory*, **96** (2001b), 40-69.

[4] Clarida, R., J. Gali and M. Gertler, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics*, **115** (2000), 147-180.

[5] Farmer, R. E. A., "Money in a Real Business cycle Model," *Journal of Money, Credit, and Banking*, **29** (1997), 568-611.

[6] Feenstra, R. C., "Functional Equivalence between Liquidity Costs and the Utility of Money," *Journal of Monetary Economics*, **17** (1986), 271-291.

[7] Leeper, E., "Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies," *Journal of Monetary Economics*, **27** (1991), 129-147.

[8] Meng, Q., "Monetary Policy and Multiple Equilibria in a Cash-in-Advace Economy," *Economics Letters*, **74** (2002), 165-170.

[9] Meng, Q. and C. K. Yip, "Investment, Interest Rate Rules, and Equilibrium Determinacy," *Economic Theory*, **23** (2004), 863-878.

[10] Itaya, J. and K. Mino, "Interest Rate Rule and multiple Equilibria with Endogenous Growth," Working Paper (2002), Kobe University, Japan.

[11] Wang, P. and C. K. Yip, "Alternative Approaches to Money and Growth," *Journal of Money, Credit, and Banking*, **24** (1992), 553-562.