E C O N O M I C S   B U L L E T I N

Capital accumulation, unemployment, and the putty−clay

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Abstract

This note studies the dynamics of labor markets in a putty−clay framework. It analyzes the evolution of job creation and job destruction in an economy without market frictions. Unemployment and labor market flows emerge under putty−clay technologies because low productive jobs become unused factors. As capital accumulates, firms destruct low productive jobs by obsolescence. Simultaneously, the use of capital intensive technologies creates new jobs by the low substitution between capital and labor.

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1 Introduction

This note studies unemployment in a putty-clay framework or in economies with limited factor substitution. In the putty-clay model, heterogenous technologies compite for scarce labor. The most profitable jobs are filled first and although low productive jobs have positive marginal products, their productivity is unable to cover the cost of labor in alternative technologies. Those jobs are unused. Thus, as wages increase the profitability of jobs declines and job destruction increases. Conversely, higher wages lead firms to invest in more capital intensive technologies and create new jobs. Therefore, putty-clay technologies generate simultaneous job destruction and creation due to employment obsolescence and investment reallocation.

The putty-clay model of Johansen [12] has been widely employed. Solow [17] studies long run economic growth, Bischoff [5] relates putty-clay technologies, investment and adjustment costs, and Abel [1] studies the response of capital utilization to price uncertainty. Akerlof [3] studies static putty-clay economies to analyze unemployment. Recently, putty-clay economies have been used to analyze short run fluctuations over the business cycles for investment and capacity utilization by Gilchrist and Williams [11], the response of energy utilization to oil price variations by Atkeson and Kehoe [4] and the response of the stock market to changes in energy price shocks by Wei [19].

The model complements well-known formulations of job creation and destruction based on search and matching frictions, see for example Mortensen and Pissarides [14], and Mortensen [13].[1] Pissarides [16] provide a complete summary of equilibrium unemployment models with frictions. Caballero and Hammour [6], [7] also study job flows when productivity differences reflect the vintage of the installed capital. Our model considers a version of Cass and Stiglitz [9], and Calvo [8] where capital varies over a continuum for a given distribution of employment in each technology. This source of heterogeneity simplifies some of the computations of the vintage models of Caballero and Hammour [6], although it generates a less interesting steady state behavior.

2 A putty-clay economy

Consider an economy that produces a final good by labor and different capital goods k. Each capital type $k \in \mathbf{K} \subseteq \mathbf{R}_+$ (capital-labor ratio) specifies a stationary output-labor ratio $\phi(k)$.

Assumption 1 The output-labor ratio $\phi : \mathbf{K} \to \mathbf{R}_+$ is twice continuously differentiable, strictly concave, and bounded from above, with

$$\phi(0) = 0,\ \phi_k(0) = \infty,\ \phi_k(\infty) = 0, \text{ and } \phi\left(\bar{k}\right) < \delta\bar{k} \text{ for some } \bar{k} \in \mathbf{K}.$$

Let $\theta(k)$ represent the frequency of jobs on a type k machine and assume

Assumption 2 The density of jobs $\theta : \mathbf{K} \to \mathbf{R}_+$ is continuous with $\theta(k) \geq 0$ for all $k \in \mathbf{K}$ and

$$0 \leq \Phi(\mathbf{K}) \equiv \int_{\mathbf{K}} \theta(k) dk \leq 1. \tag{1}$$

[1] As Akerlof [3] suggests, the particular view of the labor market embedded in the paper is capable to explain: i) the inability of wage flexibility to restore full employment, ii) the harmful effects of minimum wages for unskilled workers, iii) the ineffectiveness of labor subsidies at low skill levels, iv) the higher unemployment rates for unskilled workers (compared to skilled workers), and v) the negative correlation between unemployment and output growth.

The total amount of capital available in technology k is $k\theta(k)$, i.e., the product of the capital-labor ratio and the number of jobs in that technology. Total population is normalized to one.

2.1 The labor market

A representative firm allocates labor into the different jobs in order to maximize total output. Output maximization corresponds to the following assignment program

$$\max \int_{\mathbf{K}} \phi(k)\theta(k)dk,$$

subject to the constraint on labor use (1). From the nature of the problem, the optimal policy corresponds to an extensive margin condition.

Proposition 1 *Let $\phi(k)$ and $\theta(k)$ satisfy Assumptions 1 and 2, and let w be the real wage. Then, there exists a unique capital type $\lambda(w) \in \mathbf{K}$ such that $\theta(k) = 0$ for $k \leq \lambda(w)$ and $\theta(k) > 0$ otherwise. Moreover, $\lambda(w)$ is increasing in w.*

Proof. Consider the Lagrangian

$$y = \max \int_{\mathbf{K}} [\phi(k) - w]\, \theta(k)dk + wl \equiv \Pi(\theta, w) + w, \tag{2}$$

where w represents the Lagrange multiplier on (1) and the competitive wage. For all $k \in \mathbf{K}$, $\phi(k) - w \geq 0$ represents the first order condition. Thus, the equilibrium in the competitive market settles the output-labor ratio of the marginal capital type utilized to the current wage rate by $\phi(\lambda) = w$. By Assumption 1, λ is an increasing function of w. ■

As the Proposition suggests, the most profitable jobs are filled first until the economy exhaust all profitable opportunities (profits are represented by $\Pi(\theta, w)$). Although technologies with low capital-output ratios have a positive marginal labor productivity, their contribution to output is zero. Since the real wage or the opportunity cost of the labor required exceeds the value of its output, those available jobs should not be filled, see Akerlof [3].

From (2) it follows that for a given θ, $\Pi(\theta, w)$ is weakly decreasing in w because higher wages reduce the number of technologies in operation.

As in Akerlof [3], define the *unemployment rate* by the fraction of unused jobs, i.e.,

$$u(\theta, w) = 1 - \int_{k \geq \lambda(w)} \theta(k)dk. \tag{3}$$

For a given θ, $u(\theta, w)$ is weakly increasing in w; as the wage increases, the number of unused jobs increases because a higher fraction of low skilled jobs becomes obsolete. In other words, the unemployment rate is an increasing function of the real wage.

The next Corollary follows directly from previous assumptions.

Corollary 2 *Let $\phi(k)$ and $\theta(k)$ satisfy Assumptions 1 and 2, and let $\hat{\theta}(k)$ be an alternative distribution of jobs that satisfies Assumption 2. Assume that θ and $\hat{\theta}$ have the same mean and single-cross at $\lambda(w)$. Then, $u(\hat{\theta}, w) \geq u(\theta, w)$.*

Since the distribution $\hat{\theta}$ puts more weight on low capital intensive jobs, it generates more unused jobs and lower employment opportunities in high capital intensive technologies. These fewer employment opportunities reduce final output and produce higher unemployment.

2.2 Capital accumulation

We next consider an accumulation program in the putty-clay economy. A representative consumer maximizes

$$\sum_{t=0}^{\infty} \beta^t \{U(c_t)\}, \tag{4}$$

where c_t represents the consumption of the homogeneous good and the discount factor satisfies $0 < \beta < 1$.

Assumption 3 The utility function $U : \mathbf{R}_+ \to \mathbf{R}$ is twice continuously differentiable, strictly increasing, strictly concave, and bounded from above, with $U_c(0) = \infty$, $U(0) = 0$, and $U_c(\infty) = 0$.

Consumer maximization is subject to the evolution of capital, and the budget constraint. The dynamics of capital in technology k are given by $\left[k\theta'(k)\right] = (1-\delta)\left[k\theta(k)\right] + i(k)$, or in terms of labor utilization

$$\theta'(k) = (1-\delta)\,\theta(k) + \frac{i(k)}{k}, \tag{5}$$

with $\delta > 0$ as the constant (among capital types) depreciation rate and $i(k)$ as the investment in technology k. That is, the distribution of employment evolves according to the number of people technically required to operate each capital type (see Calvo [8] and Cass and Stiglitz [9]).

The period budget constraint is

$$\int_{\mathbf{K}} i(k)dk + c = y.$$

Define $V : \mathbf{R}_+ \to \mathbf{R}$ as the value of the maximized objective function (4) for a given θ. Then, the problem of the representative consumer must satisfy the following Bellman's optimality equation (see Stokey, Lucas and Prescott [18])

$$V(\theta) = \max_{i(k)} \left\{ U\left(y - \int_{\mathbf{K}} i(k)\, dk \right) + \beta V(\theta') \right\}, \tag{6}$$

in which θ' is given by (5) for all capital types $k \in \mathbf{K}$.

The previous problem is not particularly difficult and its solution exhibits well established properties.

Proposition 3 *Let Assumptions 1-3 hold. Then, for each θ there exists a unique function V increasing, once differentiable and concave satisfying (6).*

Proof. The proof is standard. Since U is bounded, continuous and concave, the previous properties follow because the set of all bounded continuous and concave functions (under the sup metric) is complete. It is not difficult to show that $V(\theta)$ is generated by a contraction mapping and therefore is Cauchy-convergent. Banach's fixed point theorem ensures uniqueness of $V(\theta)$, and differentiability in U (under interior solutions) ensure differentiability in V. See for example Stokey, Lucas and Prescott [18]. ■

The first order condition for optimal investment is

$$\left[U_c(c) - \beta \frac{V_\theta\left(\theta'(k), w'\right)}{k} \right] i(k) = 0, \text{ for all } k \in \mathbf{K}.$$

If next period's consumption c' is positive, the optimal allocation satisfies the following envelope conditions

$$V_\theta\left(\theta'\right) = U_c\left(c'\right)\Pi_\theta\left(\theta', w'\right) = U_c\left(c'\right)\left[\phi\left(k\right) - w'\right],$$

for all $k \in \mathbf{K}$. Combining the previous expressions gives

$$U_c\left(c\right) \geq \beta U_c\left(c'\right)\left\{\frac{\phi\left(k\right) - w'}{k}\right\}, \text{ for all } k \in \mathbf{K},$$

with strict equality if $i\left(k\right) > 0$. Since a policy with zero investment at all dates leads to zero income and employment, it must be the case that $i\left(k\right) > 0$ for at least some capital types and periods. In the putty-clay economy, optimal investment policies are lumpy and concentrate in only one technology at each time.[2]

Proposition 4 *Let Assumptions 1-3 hold. Then, there exists a unique capital type $\kappa(w') \in \mathbf{K}$ with $i(k) > 0$ for $k = \kappa(w')$ and $i(k) = 0$ otherwise.*

Proof. Assume to the contrary that there is positive investment in two types of capital, $\check{k}$ and $\hat{k}$ with $\check{k} \neq \hat{k}$. Thus, investment in technology $\hat{k}$ is carried out until

$$U_c\left(c\right) - \beta\frac{V_\theta(\theta'(\hat{k}))}{\hat{k}} = 0,$$

and investment in $\check{k}$ until

$$U_c\left(c\right) - \beta\frac{V_\theta(\theta'(\check{k}))}{\check{k}} = 0.$$

Since $V_\theta(\theta'(\check{k})) = V_\theta(\theta'(\hat{k}))$, both expressions imply that $\check{k} = \hat{k}$ contradicting the initial statement. ■

Maximization of returns require that investment is allocated into the capita type that produces the highest marginal gain per unit of capital. If firms are only concerned with instantaneous profits, the solution is straightforward

Corollary 5 *Let Assumptions 1-3 hold, and assume that firms are only concerned with instantaneous profits. Then, the Euler equation (for the technology with positive investment) is*

$$U_c(c) = \beta U_c(c')\left[\max_{k\in\mathbf{K}}\left\{\frac{\phi(k) - w}{k}\right\}\right],$$

with $\kappa(w')$ strictly increasing in w.

Proof. Note that $\kappa(w')$ satisfies

$$\phi\left(\kappa\left(w'\right)\right) - \kappa\left(w'\right)\phi_k\left(\kappa\left(w'\right)\right) = w',$$

with

$$\kappa_w\left(w'\right) = \frac{-1}{\kappa\left(w'\right)\phi_{kk}(\kappa\left(w'\right))} > 0.$$

■

[2] Non-concavity in ϕ is capable of producing investment in multiple technologies because it produces.multiple maximands in the previous expressions (Cass and Stiglitz [9]).

Since $\kappa(w')$ is increasing in w', as the wage increases, firms switch from high labor costs to more capital intensive technologies in order to maximize profits. This reallocation of investment induces job creation in technology $\kappa(w')$.

2.3 Job creation and destruction

The presence of unused jobs or unemployment in the putty-clay economy allows us to study the creation and destruction of jobs due to investment change and job obsolescence. Given θ and w, note

$$u(\theta', w') - u(\theta, w) = \delta(1 - u(\theta, w)) + (1-\delta)\int_{\min\{\lambda(w'),\lambda(w)\}}^{\max\{\lambda(w'),\lambda(w)\}} \theta(k)\, dk - \frac{i(\kappa(w'))}{\kappa(w')}. \qquad (7)$$

The previous equation represents the dynamics of the labor market. The transition rates depend on the depreciation rate of capital and the amount of investment in the economy. In particular, job destruction has an exogenous component since capital depreciation induces a constant entry rate into unemployment δ for all employed workers, $1 - u(\theta, w)$.

Job destruction also has an endogenous component related to labor obsolescence. Wage movements determine labor obsolescence since the marginal technologies employed are functions of w and w'. For instance, assume that wages increase over time. Then, there exists an endogenous destruction of jobs due to the obsolescence of skills in jobs that were profitable at a wage w but that result unprofitable at wages w'. Finally, the positive investment in technology $\kappa(w')$ creates new jobs.

Equation (7) accounts for a simultaneous process of job creation and job destruction with the particular feature that the jobs destructed correspond to the lower tail of the productivity distribution, while the jobs created correspond to more productive technologies. In the model, unemployment is related to job obsolesence. If wages decline, the profitability of the position may rise again; however, due to a positive depreciation of physical capital, the number of positions will not exceed the available jobs before the beginning of the downturn. In the model, as well as in the analysis of Davis, Haltiwanger and Schuh ([10], chap. 6), higher unemployment rates during recessions result largely from increases in job destruction.

2.4 The dynamical system

We can represent the dynamics of the previous economy by the evolution of consumption and employment $\{c, \theta(k) : k \in \mathbf{K}\}$. However, since the number of state variables for employment is given by the dimensionality of $\mathbf{K}$, it is more practical to reduce the state to $\{c, y\}$.

The putty-clay economy can be represented by the Euler equation in consumption and by a difference equation in y:

$$U_c(c) = \beta U_c(c')\, \phi_k(\kappa(w')),$$

$$y' = [1 - \delta + \phi(\kappa(w'))]\, y + \frac{\phi(\kappa(w'))}{\kappa(w')} c + \int_{\min\{\lambda'(w'),\lambda(w)\}}^{\max\{\lambda(w'),\lambda(w)\}} \phi(k)\, \theta(k)\, dk.$$

As the previous expression shows, output grows because investment creates more efficient technologies or because firms discard fewer technologies. Note that although the Euler equation in consumption is similar to its counterpart in neoclassical accumulation programs, the return to capital $\phi_k(\kappa(w'))$ does not depend on the state of the economy (unless an exogenous process for wages is specified).

Next consider the steady state. In the steady state, the competitive wage represents the marginal product of labor of the unique technology in use, see Akerlof [2], $w^* = \phi(\kappa^*) - \kappa^*\phi_k(\kappa^*)$, and zero growth in output, consumption, investment and wages leads to full specialization in production $y^* = \phi(\kappa^*)\theta(\kappa^*)$. The unemployment rate is trivially $u^* = 1 - \theta(\kappa^*)$ and $c^* = [\phi(\kappa^*) - \delta\kappa^*]\,\theta(\kappa^*)$. As in capital accumulation problems, $\phi_k(\kappa^*)\beta = 1$ or $\kappa^* = \phi_k^{-1}(1/\beta)$. The putty-clay model specifies a steady-state value for wages w but no dynamics.

The analysis of the stability of the previous economy is relatively simple. Although the transitional dynamics are not particularly interesting, there exists monotone convergence in output and a constant consumption along the transition. The conditions for stability are very restrictive in the putty-clay case because there are no corrective forces against excessive consumption.

Proposition 6 *Let Assumptions 1-4 hold. Assume in addition that $\theta(\kappa^*) > 0$. Then, given any initial distribution of jobs $\{\theta_0(k) : k \in \mathbf{K}\}$, $\{c_0, y_0\} \to \{c^*, y^*\} \in \mathbf{R}^2_{++}$ if and only if $c_0 = c^*$.*

Proof. Rewrite the dynamics of consumption and output as

$$U_c(c') - U_c(c^*) = \frac{1}{\beta\phi_k(\kappa(w'))}(U_c(c) - U_c(c^*)) + \left[\frac{1}{\beta\phi_k(\kappa(w'))} - 1\right]U_c(c^*),$$

$$(y' - y^*) = \left[1 - \delta + \frac{\phi(\kappa(w'))}{\kappa(w')}\right](y - y^*) - \frac{\phi(\kappa(w'))}{\kappa(w')}(c - c^*) + \int_{\min\{\lambda(w'),\lambda(w)\}}^{\max\{\lambda(w'),\lambda(w)\}} \phi(k)\,\theta(k)\,dk.$$

Consider an approximation around the steady state. It follows that consumption must be such that $U_c(c_t) = U_c(c^*)$ for all $t \geq 0$ because otherwise either output will go to zero (as consumption increases) or consumption will go to zero. For that reason, the solution is given by $c_t - c^* = 0$, and $y_t - y^* = (y_0 - y^*)\left[1 - \delta + \phi(\kappa^*)/\kappa^*\right]^t$. By the bound in Assumption 1, $\phi(k^*)/k^* < \delta^* < 1$ so $y_t \to y^*$ as $t \to \infty$ for all y_0. ■

3 Conclusion

This paper studies a competitive labor market under putty-clay technologies. Since putty-clay production limits capacity utilization, this technology generates unused jobs (or unemployment) even when those jobs have a positive marginal productivity. The main reason for job destruction in the model is job obsolescence. As the wage increases, the opportunity cost to use low productive technologies rises. Low productive jobs are not able to generate positive profits to remain in use. Wage change also forces firms to invest in more capital intensive technologies. The new investments produce new capital and consequently new jobs. This job creation reduces the impact of job destruction on unemployment. At the end, as in modern business cycles, shocks that increase the cost of a job induce higher job destruction and reduce job creation leading to higher unemployment and lower aggregate output.

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