E C O N O M I C S B U L L E T I N

Small departures from rationality magnify fluctuations

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Abstract

This note shows that introducing into the economy a few number of agents who do not support sunspots theories magnifies endogenous business cycles fluctuations when the current state of the economic system is sensitive enough to traders' forecasts, or equivalently, in presence of indeterminacy.

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1. Introduction

The sunspot literature usually assumes that all the agents have the same rational belief. Some have argued that introducing agents who do not support sunspot theories should stabilize such endogenous competitive business cycles. For instance, in a misperceived cycle, prices evolve according to a deterministic cycle, though some agents expect the price to cycle and others wrongly expect the price to be drawn randomly in each period; see Azariadis and Guesnerie (1982), Evans, Honkapohja and Sargent (1993) or Woordford (1990). The presence of non-sunspots believers is stabilizing in the sense that misperceived cycles do not exist when the proportion of these agents is large enough. As this note highlights, however, in the remaining polar case where this proportion is sufficiently low, price fluctuations will be magnified as soon as the economy is sensitive to traders' forecasts. In other words, a small departure from rationality may magnify fluctuations.

2. General Framework

I shall use a simple version of the overlapping generations model that involves one non storable normal consumption good and fiat money, whose stock $\bar{m} \equiv 1$ is fixed. At each date t ($t \geq 0$), there is a size 1 continuum of newborns who live for two consecutive periods. In the first, they work during n_t units of time, which allows them to produce $y_t = n_t$ units of goods. Each unit is then sold at price p_t and the collected fiat money is used to transfer wealth from one period to the next, $t+1$, where they enjoy consuming c_{t+1} units of goods. Preferences of a newborn at date t are represented by a separable utility function $E_t\left[u\left(c_{t+1}\right)\right] - v\left(n_t\right)$, where $u(\cdot)$ is increasing and concave, and $v(\cdot)$ is increasing and convex.

The operator $E_t(\cdot)$ depends on traders' beliefs. Let us distinguish two types of agents.

First there are α ($0 \leq \alpha \leq 1$) *informed* agents. Young agents of this type observe a sunspot signal which alternates between two states l and h, i.e. period $(t+1)$ state is $s_{t+1} = l$ (resp. h) when period t state is $s_t = h$ (resp. l). All these agents *a priori* believe that price should be perfectly correlated with sunspot signals: they expect the period $(t+1)$ price to be p_h^e (resp. p_l^e) if $s_t = l$ (resp. h). If $s_t = l$, they consequently face the intertemporal budget constraint $p_h^e c_{t+1} \leq p_t n_t$, and their optimal current labor supply is some function $n^i\left(p_t/p_h^e\right)$. By the same way, if $s_t = h$, their labor supply is $n^i\left(p_t/p_l^e\right)$.

The $(1-\alpha)$ remaining agents are *uninformed* of the true law of the signal. They actually use the sample moments of the signal in order to form their forecasts, i.e. they expect the period $(t+1)$ price to be p_l^e with probability 1/2 and p_h^e with probability 1/2, whatever s_t is. Let $n^u\left(p_t/p_l^e, p_t/p_h^e\right)$ be their optimal labor supply in the current period.

Since the aggregate demand for consumption goods is equal to real holdings of old agents at date t, $\bar{m}/p_t \equiv 1/p_t$, the current market clearing price, if $s_t = s$ $(s = l, h)$, satisfies

$$\frac{1}{p_t} - \alpha n^i\left(\frac{p_t}{p_{s'}^e}\right) - (1-\alpha)\, n^u\left(\frac{p_t}{p_l^e}, \frac{p_t}{p_h^e}\right) = 0 \tag{1}$$

for $s' = l, h$ and $s' \neq s$. This allows us to define a *misperceived two-period cycle* as a triplet $((p_l, p_h), \alpha)$ such that $p_t = p_l$ (resp. p_h) in (1) if $s_t = l$ (resp. $s_t = h$), $p_l^e = p_l$ and $p_h^e = p_h$. Namely,

$$\frac{1}{p_l} - \alpha n^i\left(\frac{p_l}{p_h}\right) - (1-\alpha)\, n^u\left(1, \frac{p_l}{p_h}\right) = 0, \tag{2}$$

$$\frac{1}{p_h} - \alpha n^i\left(\frac{p_h}{p_l}\right) - (1-\alpha)\, n^u\left(\frac{p_h}{p_l}, 1\right) = 0. \tag{3}$$

In a misperceived equilibrium, the informed agents' belief fits the actual law of the economy, whereas the one of uninformed agents is merely statistically correct, except in the special case where $p_l = p_h$. If $\alpha = 1$ in (2) and (3), then a misperceived two-period cycle coincides the usual concept of two-period cycle, which is known to exist (in the so-called Samuelson case) whenever the elasticity $\varepsilon^i(\cdot)$ of labor supply $n^i(\cdot)$ with respect to expected real wage is less than $(-1/2)$ at point $p_l = p_h$ (see, e.g. Azariadis and Guesnerie 1986). Under this condition, and provided that α is close enough to 1, a misperceived equilibrium also exists.

3. Destabilizing Effects of Belief Heterogeneity

The purpose of this section is to describe how equilibrium prices (p_l, p_h) react to a raise in the proportion of uninformed traders, when most of the agents are informed in the economy, i.e. when α is close enough to 1.

Proposition 1. *Consider a two-period misperceived cycle $((p_l, p_h), \alpha)$, with $p_l < p_h$ by definition. Assume that $\varepsilon^i(p_l/p_h) < 0$. Then, there exists a threshold proportion $\bar{\alpha} < 1$ such that, for any $\alpha \geq \bar{\alpha}$, both $dp_h/d\alpha < 0$ and $dp_l/d\alpha > 0$ if $1 + \varepsilon^i(p_l/p_h) + \varepsilon^i(p_h/p_l) < 0$, i.e. the usual two-period cycle $((p_l, p_h), 1)$ is locally indeterminate. If, on the contrary, $1 + \varepsilon^i(p_l/p_h) + \varepsilon^i(p_h/p_l) > 0$, i.e. the usual two-period cycle $((p_l, p_h), 1)$ is locally determinate, then both $dp_h/d\alpha > 0$ and $dp_l/d\alpha < 0$ for any $\alpha \geq \bar{\alpha}$.*

Proof. Let us differentiate (2) and (3) with respect to p_l, p_h, and α. At point $\alpha = 1$, one obtains, after straightforward computations,

$$\begin{pmatrix} \dfrac{dp_l}{d\alpha} \\ \\ \dfrac{dp_h}{d\alpha} \end{pmatrix} = \frac{1}{\Delta}\mathbf{DF}(p_l, p_h) \begin{pmatrix} n^u\left(1, \dfrac{p_l}{p_h}\right) - n^i\left(\dfrac{p_l}{p_h}\right) \\ \\ n^u\left(\dfrac{p_h}{p_l}, 1\right) - n^i\left(\dfrac{p_h}{p_l}\right) \end{pmatrix},$$

with

$$\mathbf{DF}(p_l, p_h) = \begin{pmatrix} \dfrac{1 + \varepsilon^i\left(p_h/p_l\right)}{p_h/p_l} & \varepsilon^i\left(\dfrac{p_h}{p_l}\right) \\ \varepsilon^i\left(\dfrac{p_l}{p_h}\right) & \dfrac{1 + \varepsilon^i\left(p_l/p_h\right)}{p_l/p_h} \end{pmatrix}$$

and $\Delta = (1 + \varepsilon^i(p_l/p_h) + \varepsilon^i(p_h/p_l))/(p_h p_l)$ (Δ is assumed to differ from 0). Observe now that both convexity properties of individual preferences and the fact that the first-order condition of the problem solved by an uninformed agent is a weighted average of the first-order conditions of the problems solved by informed agents in each state of the signal, imply that

$$\inf\left\{n^i\left(\frac{p_t}{p_l}\right), n^i\left(\frac{p_t}{p_h}\right)\right\} < n^u\left(\frac{p_t}{p_l}, \frac{p_t}{p_h}\right) < \sup\left\{n^i\left(\frac{p_t}{p_l}\right), n^i\left(\frac{p_t}{p_h}\right)\right\} \quad (4)$$

whatever p_t is. Let respectively $p_t = p_l$ and $p_t = p_h$ in the preceding relation. Then we have both $n^u\left(1, p_l/p_h\right) < n^i\left(p_l/p_h\right)$ and $n^i\left(p_h/p_l\right) < n^u\left(p_h/p_l, 1\right)$. Finally $\varepsilon^i(\cdot) < 0$ by hypothesis, and $\varepsilon^i(\cdot) > -1$ when leisure is normal. Thus $\Delta dp_l/d\alpha < 0$ and $\Delta dp_h/d\alpha > 0$. But $\Delta < 0$ if and only if the two-period cycle is locally indeterminate, which shows the result. □

The property of indeterminacy plays a crucial role: in presence of indeterminacy, introducing a small number of uninformed agents raises the highest price of the cycle and reduces the lowest one.

Actually, their presence affects excess demand through two distinct channels, to be called the *size* and the *expectations* effects. The size effect corresponds to the change in the current price p_t in (1) that results from a lower α, with p_l^e and p_h^e fixed to their initial level. In order to restore equilibrium, where price forecasts are correct, adjustments in expected prices are consequently needed. These changes in forecasts induce a new change in the current price, and thus triggers a feedback process from current to expected prices that continues until price beliefs are self-fulfilling; this is the expectations effect.

The size effect is actually always stabilizing. In the indeterminate configuration, the expectations effect opposes to the size effect, and is responsible for magnifying price fluctuations.

The reason why the size effect is stabilizing stems from the inertia in the behavior of uninformed agents; their behavior less volatile than the one of informed agents since it can not rely on sunspots. In fact, when the current price is low, so that the aggregate supply is large, the quantity produced by an uninformed agent is less than the one produced by an informed agent, as (4) highlights. Therefore, at a low current price, reducing the proportion of informed agents involves an increase in the excess demand, and so, in order to clear the market, there must be a raise in the current price. On the contrary, a decrease in the current price would be required for high current prices. To summarize: the price, through the size effect, tends to raise when it is low, and to fall when it is high.

The assumption $\varepsilon^i(p_l/p_h) < 0$ now implies that, for α close enough to 1 in (1), if the signal is l (resp. h), the current price decreases (resp. increases) whenever the highest expected price p_h^e increases and the smallest one p_l^e decreases.[1] Thus, for price forecasts to be correct when agents expect fluctuations to be magnified, and given that the size effect has always a stabilizing effect, there must be a destabilizing change in the current price due to changes in traders' forecasts. This effect overcomes the stabilizing size effect in the indeterminate configuration, where changes in current price are amplified by changes in expected prices (that is, $|dp_t| > |dp_l^e|$ in state l, and $|dp_t| > |dp_h^e|$ in state h).

[1]To see this point most clearly, assume for instance that the current state of the signal is l. An increase in the expected price p_h^e leads to a decrease in the expected real wage, and so, under assumption (H), to an increase in informed agents' output. For the market for consumption goods to clear, a decrease in the current temporary equilibrium price is required, which is destabilizing in state l.

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