E C O N O M I C S B U L L E T I N

Endogenous Monetary Growth Rules and Determinacy in Cash−in−Advance Models

Mark Weder
Humboldt University Berlin

Abstract

It is well known that a high degree of relative risk aversion induces equilibrium indeterminacy in cash−in−advance economies. I find that by endogenously adjusting the nominal money supply to output fluctuations, these equilibria can be effectively eliminated.

This paper was written while I was a DFG Heisenberg Fellow. I would like to thank the Federal Reserve Bank of San Francisco for its hospitality.
Citation: Weder, Mark, (2004) "Endogenous Monetary Growth Rules and Determinacy in Cash−in−Advance Models." *Economics Bulletin*, Vol. 5, No. 11 pp. 1−7
Submitted: May 15, 2004. **Accepted:** May 20, 2004.
URL: http://www.economicsbulletin.com/2004/volume5/EB−04E50003A.pdf

1 Introduction

Over the course of the recent years, it has become fashionable in macroeconomics to formulate monetary policy as Taylor rules, i.e. it is assumed that the central bank targets the short-run nominal interest rate in response to economic variables such as inflation or output. In fact, many central banks' policies are exemplary of this rule – at least over certain periods.

In the current paper, monetary policy is alternatively formulated so that it is nominal money supply growth that reacts to output movements. Phrased alternatively, the monetary authority regulates money injections but leaves interest rates to be determined by market forces. The motivation for using money supply rules is as follows. First, in a sense, there exists an correspondence between the two formulations of policy: every interest rate rule endogenously determines money supply and *vice versa*. Nevertheless, the policy-effects of using growth rates instead of interest rates as an instrument may differ and are therefore analyzed here individually. Second, versions of money supply rules have been adapted by several central banks over time. For example, it was the *Bundesbank*'s proclaimed policy to keep money growth within announced growth-target corridors. This policy had obviously been formulated in response to economic conditions. In 1979 the *Federal Reserve* and the *Bank of England* dropped interest rate targets in favor of money supply rules. These policy switches were dramatically less successful than the *Bundesbank*'s record. Faced with these results, these "monetarist experiments" were abandoned a few years later.

The current paper employs monetary policy to stabilize the aggregate economy. In particular, it asks if a well-defined endogenous money-growth policy is able to eliminate cycles that arise in response to (self-fulfilling) expectations.[1] The specific monetary economy which is considered here builds on the standard cash-in-advance superstructure. It is well established that the cash-in-advance model displays sunspot equilibria for large values of the relative risk aversion parameter (see for example Farmer, 1999). The reason for real indeterminacy is as follows. Suppose that people increase current consumption without any cause other than believing that it is best to do so: they follow some extrinsic sunspot signal. The consumption surge implies an increase in the expected inflation rate which decreases future consumption. Unless the intertemporal elasticity of substitution is high – i.e. risk aversion

[1] In a companion paper, Weder (2003) examines the related effects of interest rate rules.

is low – the fall in future consumption does not generate a strong enough effect on today's consumption and the fall is thereby not strong enough to defeat the initial beliefs. The present note shows that the central bank can preempt such self-fulfilling equilibria quite easily by simply obeying endogenous monetary growth rules.

2 Model

The model is a dynamic general equilibrium model. The economy is populated by atomistic firm-households of measure one. Money is introduced by imposing a cash-in-advance constraint on consumption purchases. All markets are perfectly competitive and prices are perfectly flexible.

2.1 Preferences and technology

Lifetime utility for each representative agent is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - l_t)$$

$$u(.) \equiv \frac{c_t^{1-\sigma}}{1-\sigma} - Al_t \qquad 0 < \beta < 1,\ \sigma > 0,\ A > 0$$

where β denotes the discount factor, c_t consumption, l_t labor and σ is the coefficient of relative risk-aversion.[2] E_0 is the usual rational expectations operator. The cash-in-advance constraint

$$M_t + N_t \geq P_t c_t$$

states that period t nominal consumption purchases must be covered by the agents' cash holdings, M_t at the beginning of period t and the current period's lump-sum transfers from the central bank, N_t. P_t stands for the price level. The production technology is given by

$$y_t = l_t.$$

[2] The assumption of indivisible labor was made to buy convenience of the analytical evaluation. All results carry over for less elastic labor supplies. See also footnote 4.

2.2 Policy

There is no government consumption. The central bank follows a money supply growth rule. Nominal money supply, M_t^s, grows at the gross rate, G_t:

$$M_{t+1}^s = G_t M_t^s.$$

Phrased alternatively, monetary injections into the household are given by

$$N_t = M_{t+1}^s - M_t^s.$$

I assume that monetary policy targets output movements. That is, policy is described by the time-invariant function

$$G_t = G(y_{t+\tau}/y) \qquad G_t > 0 \nabla t. \tag{1}$$

Here, $y > 0$ denotes the economy's steady state. Policies that imply $\tau = 0$ are coined current-looking. If $\tau = 1$ ($\tau = -1$), policy is forward-looking (backward-looking). Policies in which the central bank accommodates output movements are given by $G' > 0$. In some sense, this response could be interpreted as taming inflationary pressures arising from overheating economic activities. Leaning-against-the-wind policies imply that $G' < 0$.

Let is be clear that the present paper does not necessarily claim that any central bank actually follows this class of rules. Even the *Bundesbank*'s growth targets were most likely less rigidly formulated than in (1).[3] What the current paper does claim, however, is that (i) assumed growth-rules have prime effects on aggregate dynamics and (ii) they can be exploited to stabilize the economy. These suppositions will be analyzed in the next Section.

3 Equilibrium dynamics

In symmetric equilibrium, $M_t^s = M_t$ and $y_t = l_t = c_t$ must hold for ∇t. Let us begin the discussion of dynamics by considering backward-looking policies.

[3] Moreover, the Bundesbank's targets may be better described as being determined by inflation.

3.1 Backward-looking policies

Under the backward-looking policy $G_t = G(y_{t-1}/y)$, the model reduces to the nonlinear difference equation

$$A = E_t \frac{\beta}{G(y_t/y)} \frac{y_{t+1}^{1-\sigma}}{y_t}.$$

Local dynamics can be discussed from the linearized model version

$$E_t \widehat{y}_{t+1} = \frac{1+\eta}{1-\sigma} \widehat{y}_t. \tag{2}$$

In equation (2), the elasticity

$$\eta \equiv \frac{G'(1)}{G(1)}$$

is evaluated at the unique economy's steady state. Figure 1 plots the determinacy (D) and indeterminacy (I) regions. The following results emerge. When the central bank reacts to economic fluctuations by a sufficient degree, then sunspot equilibria can be pre-emptied. More concretely, unlike the case in which the central bank follows an exogenous money-growth policy, the current cash-in-advance model is determinate even when the relative risk aversion parameter is greater than 2. For example, when setting $\sigma = 3$, which is entirely reasonable from an empirical standpoint, then either $\eta < -3$ or $\eta > 1$ delivers unique dynamics. No general monetary policy appears to allow the central bank to rule out endogenous fluctuations independent of the degree of risk-aversion. That is, the central bank must have knowledge of the risk parameter before fixing the policy rule – however it suffices to possesses a general idea of the parameter's empirical magnitude.[4] Most economists would agree that the parameter falls somewhere in the range of 1 to 5. Sufficient accommodation of output movements – i.e. $\eta = 3$ or larger – always preempt sunspot equilibria.

[4] If I had asumed the more general utility function $u(.) = (1-\sigma)^{-1} c^{1-\sigma} - v(l_t)$, then the local dynamics were given by

$$E_t \widehat{y}_{t+1} = \frac{1+\eta+\varepsilon}{1-\sigma} \widehat{y}_t$$

where $\varepsilon \geq 0$ stands for the inverse of the Frischian labor supply. Consequentely, the determinacy conditions are simply shifted.

Intuitively, the mechanism that keeps in check the sunspot forces can be thought of like this. Changes in consumption imply changes in output. Once the central bank targets output, the supply of money adjusts to the incipient sunspot belief. Policy causes the transformation of the dynamics. In the model with constant money growth, the initial sunspot-caused surge in consumption is met by a drop in future consumption. Once $\sigma > 2$, the numerator on the right-hand-side of

$$\frac{AG}{\beta} = E_t \frac{y_{t+1}^{1-\sigma}}{y_t}$$

keeps the sunspot tilt of y_t in check. On the other side, in the model with variable money growth, the initial sunspot-caused movement in consumption is supported by a reaction of the growth rate. That is, depending on the size of the change in money growth, the process

$$\frac{A}{\beta} G(\overbrace{y_t}^{\uparrow})\overbrace{y_t}^{\uparrow} = E_t \overbrace{y_{t+1}^{1-\sigma}}^{\downarrow\uparrow}$$

can be stationary (i.e. indeterminacy) or unstable (i.e. determinacy). In sum, simple policy rules can eliminate sunspot equilibria by destabilizing the economy. Curiously, the analysis does not pin down the direction of monetary policy: all that matters for uniqueness is that the central bank's reaction to changes in the economy is considerably forceful. However, a caveat to this finding does exist: the policy's success crucially depends on the central bank's ability to control money aggregates. Otherwise the bank may run into choppy water similar to that experienced by the *Federal Reserve* in the early 1980s. Even the otherwise celebrated record of the *Bundesbank* shows 11 misses out of the 24 targets for monetary growth the bank announced.

3.2 Current-looking policies

When policy is current-looking, the local dynamics are given by

$$E_t \widehat{y}_{t+1} = \frac{1}{1-\sigma-\eta} \widehat{y}_t.$$

Now, determinacy requires that policy falls into a narrow band that describes an inverse relationship between σ and η (see Figure 2). For example, when

the relative risk aversion parameter is equal to 2, the model is determinate if and only if $\eta \in (-2, 0)$. The complementarity of the effects of intertemporal substitution and the money rule elasticity is easily understood. The mild response is required since the money supply now grows hand in hand with $t+1$ output: large values of η are equivalent to assuming high risk aversion. Moreover, as in the scenario of backward-looking rules, no policy exists that effectively rules out endogenous fluctuations independently of the degree of risk aversion.

3.3 Forward-looking policies

When monetary policy is forward-looking, the model becomes

$$A = E_t \frac{\beta}{G(y_{t+2}/y)} \frac{y_{t+1}^{1-\sigma}}{y_t}$$

and the local dynamics are given by

$$\eta E_t \widehat{y}_{t+2} - (1-\sigma) E_t \widehat{y}_{t+1} + \widehat{y}_t = 0. \tag{3}$$

Determinacy requires that both roots of (3) must lay outside the unit circle. Figure 3 summarizes the dynamical structure. In a nutshell, policy must not show too strong responses to expected output fluctuations. Is it possible to sift out a general strategy that can eliminate sunspot equilibria for all reasonable values of σ? The answer again is: no. Quite on the contrary, the space of desirable forward-looking rules is small in the following sense. Money-growth rules are only able to eliminate sunspot equilibria for $\sigma < 3$. On this account, forward-looking policies are not able to counter the indeterminacy effects of high degrees of relative risk aversion.

4 Summary

In recent years, it has become fashionable to formulate monetary policy in the form of Taylor rules. According to this, it is assumed that the central bank sets the short-run nominal interest rate in response to macroeconomic variables. The current paper presents policy in an alternative framework: it is nominal money growth that endogenously reacts to macroeconomic activity. The main insight that emerges from this paper is that monetary policy

can be used to eliminate sunspot equilibria. This is shown within a monetary model in which indeterminacy solely arises from preferences: it is well known that high degrees of relative risk aversion induce multiple equilibria in cash-in-advance economies. I find that by endogenously adjusting nominal money supply growth to output fluctuations, these equilibria can be effectively eliminated. Policies that accommodate past output fluctuations constitute this kind of a preemptive strategy.

References

[1] Farmer, R. (1999): *The Macroeconomics of Self-fulfilling Prophecies* (2nd edition), MIT Press, Cambridge.

[2] Weder, M. (2003): "Taylor Rules in Practice: How The Central Bank Can Intercept Sunspot Expectations", CEPR Discussion Paper #3899.

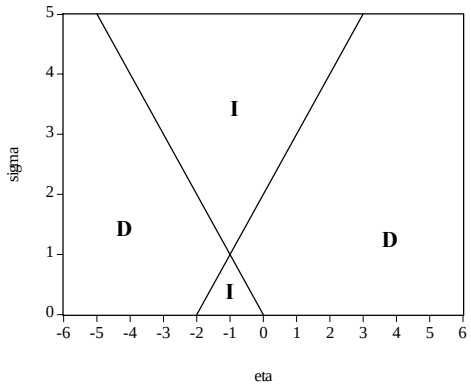

Figure 1: Backward-looking rules: dynamics

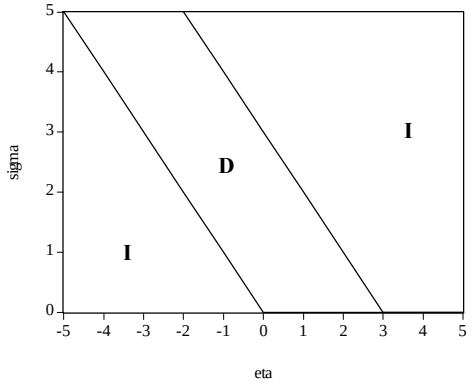

Figure 2: Current-looking rules: dynamics

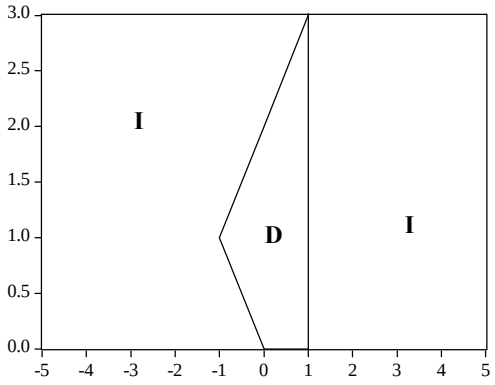

Figure 3: Forward-looking rules: dynamics