E C O N O M I C S   B U L L E T I N

# The optimal provision of public inputs in a second best scenario

Diego Martinez−Lopez
*centrA and University Pablo de Olavide*

## *Abstract*

This paper provides a simple general equilibrium model with productive public spending and distorting taxes. The optimal conditions for the provision of public inputs are obtained under different tax systems. Also we discuss which factors affect the marginal cost of public funds.

I am grateful to J.M. Gonzalez−Paramo, J. Rodero, A. J. Sanchez, and seminar participants at the University of Jaén and at the University Complutense of Madrid for their comments on an earlier version. This paper has been discussed at VI and VII Spanish Meetings of Young Economists in Economic Analysis, at X Spanish Meetings on Public Economics, and at XXVII Symposium on Economic Analysis. The usual disclaimer applies.
**Citation:** Martinez−Lopez, Diego, (2004) "The optimal provision of public inputs in a second best scenario." *Economics Bulletin*, Vol. 8, No. 2 pp. 1−9
**Submitted:** April 14, 2004. **Accepted:** May 19, 2004.
**URL:** http://www.economicsbulletin.com/2004/volume8/EB−04H40001A.pdf

# 1 Introduction

The optimal provision of public goods calls the existence of distorting taxation into question. When governments use lump-sum taxes to finance public spending, the conventional rule claims that the efficient provision of public goods must be carried up to the point where the sum of the marginal rates of substitution equals the marginal cost of providing the public goods (Samuelson, 1954). However, things are different when distorting taxes are used. Pigou (1947) stressed the deadweight loss caused by non lump-sum taxes, and argued that the conventional rule may generate an over-supply of public goods. Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974) enlarge Pigou's view and find another effect: a tax revenue effect that public spending may generate when the consumption of taxed goods is encouraged.

Others assumptions and scenarios have been considered. Wildasin (1979, 1984) uses arbitrary distorting taxation to discuss the optimal provision of public goods. King (1986), Wilson (1991), Konishi (1993), and Gaube (2000) introduce heterogeneous agents. Aronsson and Sjogren (2001) study the effect of a non-competitive labour market on the provision of public goods.

Regarding public input provision, the number of contributions is smaller. Feehan and Matsumoto (2000) study the use of benefit taxation to provide public inputs. Also Feehan and Matsumoto (2002) show the differences between the first best and the second best rules in the provision of public inputs.

In this paper we are concerned with the optimal provision of a public good that yields both productive services for firms and utility gains for consumers. Many examples of this can be quoted: roads, airports, ports, railways, etc. Whereas in the case of a consumption public good the reduction in the provision cost requires fulfilling some conditions, our model offers outcomes where these requirements are relaxed. We present results in which the fall in the marginal cost of public funds does not exclusively depend on the complementary relationship between the public input and the taxed goods. Moreover, we highlight the relevance of deciding which is the untaxed good.

The structure of the paper is as follows. Section 2 presents the theoretical framework. In the section 3 the conditions for the optimal provision of productive public spending with optimal taxes are obtained. Next section achieves a similar result but when taxes are designed arbitrarily. Section 5 presents some concluding remarks.

# 2 The model

Our model consists of $H$ identical consumers, $n+1$ firms and a government. The utility function for the consumer is

$$U(\mathbf{x}, g), \tag{1}$$

where $\mathbf{x}$ is a vector of $n+1$ private goods and $g$ is a pure public good[1]. Consumer offers a fixed amount of labour and capital ($l$ and $k$) at prices $w$ and $r$. This agent faces consumer prices $\mathbf{q}$, and considers $g$ and his income $m$ (expressed in terms of good 0 taken as the numeraire) as given. Notice that $m$ comes from the sum of $wl$ and $rk$. The next two expressions are obtained from the consumer's optimization problem:

$$x_i = x_i(\mathbf{q},\ g, m) \tag{2}$$

$$V(\mathbf{q},\ g, m) = \underset{\{\mathbf{x}\}}{Max}\ U(\mathbf{x}, g), \quad s.t.\ \mathbf{q}\ \mathbf{x}' = m. \tag{3}$$

Equation (2) is the demand function for good $i$, and the equation (3) is the indirect utility function. The firms produce exclusively the good $i$ according to the following production function:

$$X_i = F_i(L_i,\ K_i,\ g), \tag{4}$$

where $L_i$ and $K_i$ are the quantities of labour and capital used. Public good $g$ enters the production function as a public input. The marginal productivity of each factor is positive and decreasing. Moreover, additional increases in $g$ raise the productivity of private factors. It is assumed that there are constant returns to scale in the private factors. On the other hand, the markets for goods and services are competitive, with perfect international mobility for capital and goods, and no obstacles for labour and capital mobility among sectors. Hence, firms' optimization problem allows us to obtain the optimal factors demands:

[1] It is assumed that $U(.)$ is a well-behaved function

$$
\begin{aligned}
p_i \, \frac{\partial \, X_i}{\partial \, L_i} &= w \\
p_i \, \frac{\partial \, X_i}{\partial \, K_i} &= r,
\end{aligned}
$$

where $p_i$ is the international price for good $X_i$. Since labour supply is fixed in our economy, when the public input $g$ increases, the labour price $w$ rises.

The government is supposed to maximize a Bentham-type social welfare function subject to a budget constraint that includes two main elements. The first is the production cost of public spending $c(g)$[2]. The second are the resources needed for financing productive public spending. We establish *ad valorem* taxes so that $q_i = p_i(1 + t_i)$, where $t_i$ is the tax rate. Thus, the behaviour of the government is given by the following optimization problems:

$$
\begin{aligned}
&\underset{\{g,\ t_i\}}{Max} \quad H \ V(\mathbf{q}(\mathbf{t}), \ g, \ m(g)), \\
s.\ t. \quad & c(g) = H \ \textstyle\sum_{i=1}^{n} t_i \ p_i \ x_i(\mathbf{q}(\mathbf{t}), \ g, \ m(g))
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
&\underset{\{g,\ t_i\}}{Max} \quad H \ V(\mathbf{q}(\mathbf{t}), \ g, \ m(g)), \\
s.\ t. \quad & c(g) = H \ \sum_{i=1}^{n} t_i \ p_i \ x_i(\mathbf{q}(\mathbf{t}), \ g, \ m(g)) \\
& t_i \ p_i \ x_i(\mathbf{q}(\mathbf{t}), \ g, \ m(g)) = \frac{c(g)}{N \ H}, \quad \forall i
\end{aligned}
\tag{6}
$$

where $\mathbf{t}$ is the vector of tax rates. Note that the scheme for government is considerably different in (5) and (6). Whereas the public sector can determine which the optimal taxes are in problem (5), there is a non efficient requirement in problem (6) for choosing tax rates: each consumption tax must collect the same resources. This allows us to include government's restrictions by deciding the tax policy[3].

[2] $C(g)$ is assumed to be linearly increasing in $g$.

[3] Wildasin (1979, 1984) discusses a similar case, where taxes are arbitrarily determined.

# 3 Optimal provision of public spending with efficient taxes.

This section deals with the optimal productive public spending and the taxes that minimize the tax burden. This implies solving the optimization problem (5). The first order conditions for $g$ and $t_k$ are given by:

$$H\ \left(\frac{\partial\ V}{\partial\ g}+\frac{\partial\ V}{\partial\ m}\frac{\partial\ m}{\partial\ g}\right)=\lambda\left(c^{'}\left(g\right)-H\ \sum_{i=1}^{n}t_i\ p_i\left(\frac{\partial\ x_i}{\partial\ g}+\frac{\partial\ x_i}{\partial\ m}\frac{\partial\ m}{\partial\ g}\right)\right) \tag{7}$$

$$H\ \frac{\partial\ V}{\partial\ t_k}=\lambda\left(-H\ \sum_{i=1}^{n}\frac{\partial\ x_i}{\partial\ t_k}t_i\ p_i-H\ x_k\ p_k\right),\quad \forall\ k,\ \ k=1,2,...,n, \tag{8}$$

where $\lambda$ is the Lagrange multiplier. Since indirect utility function can be expressed in terms of the direct utility function, and the marginal utility of income is $\alpha=\sum_{i=0}^{n}\frac{\partial\ U}{\partial\ x}\frac{\partial\ x}{\partial\ m}$, equation (7) becomes:

$$H\ \left(RMS_g^m+\frac{\partial\ m}{\partial\ g}+\frac{\sum_{i=0}^{n}\frac{\partial\ U}{\partial\ x_i}\frac{\partial\ x_i}{\partial\ g}}{\alpha}\right)=$$
$$\frac{\lambda}{\alpha}\left(c^{'}\left(g\right)-H\ \sum_{i=1}^{n}t_i\ p_i\left(\frac{\partial\ x_i}{\partial\ g}+\frac{\partial\ x_i}{\partial\ m}\frac{\partial\ m}{\partial\ g}\right)\right) \tag{9}$$

The LHS of equation (9) is the sum of marginal benefits of $g$ in terms of income over all individuals. These benefits consist of the marginal rates of substitution, the increment in the consumer's income, and how $g$ affects the consumption of other goods, and hence the utility level as well. The RHS of equation (9) comprises the social cost involved in providing $g$, i. e. the marginal cost of public funds (MCPF). At this point, the marginal production cost $c^{'}\left(g\right)$ must be reduced by the revenue effect that one unit of public input causes over tax collection. On the other hand, the social cost of providing $g$ must take into account the deadweight loss by using distorting taxes (ratio $\lambda/\alpha$)[4].

[4] As is well known, the envelope theorem allows us to interpret the Lagrange multiplier $\lambda$ as the marginal cost of using taxes different to lump-sum.

Regarding the revenue effect, our model goes beyond the previous literature. When a consumption public good is considered, the only way to lower the MCPF is assuming complementarity between the public good and the taxed good. This paper extends this framework and allows the possibility that the MCPF decreases even when the taxed goods and $g$ are substitutes. Only a non restricted assumption is required: at least a taxed good must be normal. In that case, $\frac{\partial\, x_i}{\partial\, g}$ may be negative but the positive sign of $\frac{\partial\, x_i}{\partial\, m}\frac{\partial\, m}{\partial\, g}$ is able to overcome the sign of $\frac{\partial\, x_i}{\partial\, g}$, and a reduction in $c^{'}(g)$ takes places.

Another interesting point is the effect of distorting taxes on welfare. The next proposition sheds light about that.

**Proposition 1** *If taxed goods are normal, the social marginal cost of raising tax revenues is bigger than the marginal utility of income, so that* $\alpha/\lambda < 1$.

**Proof.** Following Dasgupta and Stiglitz (1971) and Atkinson and Stern (1974), we will use the optimal condition for the tax choice in equation (8):

$$H\,\frac{\partial\, V}{\partial\, t_k} = \lambda \left( -H\, \sum_{i=1}^{n} \frac{\partial\, x_i}{\partial\, t_k} t_i\; p_i - H\; x_k\; p_k \right), \quad \forall\; k. \tag{10}$$

Using $\frac{\partial\, V}{\partial\, t_k} = \frac{\partial\, V}{\partial\, q_k}\frac{\partial\, q_k}{\partial\, t_k}$, Roy's identity and rearranging terms, equation (10) can be written as

$$\frac{\alpha}{\lambda} = \frac{\sum_{i=1}^{n} H\; t_i\; p_i\; \frac{\partial\, x_i}{\partial\, t_k} + H\; x_k\; p_k}{H\; x_k\; p_k}, \quad \forall\; k. \tag{11}$$

If we apply the Slutsky's decomposition in the RHS of (11), the new expression achieved allows us to know which factors are after the ratio $\alpha/\lambda$:

$$\frac{\alpha}{\lambda} = 1 + \frac{\sum_{i=1}^{n} S_{i\; k}\; t_i\; p_i}{H\; x_k} - \sum_{i=1}^{n} \frac{\partial\, x_i}{\partial\, m}\; t_i\; p_i, \quad \forall\; k, \tag{12}$$

where $S_{ik}$ is the Slutsky's term. Because of the concavity of the expenditure function, the second term in the RHS of (12) is non positive so that $\alpha$ will be necessarily smaller than $\lambda$ if $\sum_{i=1}^{n} \frac{\partial\, x_i}{\partial\, m} > 0$, i. e. if taxed goods are normal. ■

# 4 Optimal provision of public spending with arbitrary taxes.

We are now concerned with the efficiency conditions when taxes are not chosen optimally, in order to show the implications derived from restrictions in the design of taxes. As is well-known, the governments have not very scope to define tax rates minimizing the tax burden. Although we use a non very realistic scheme, its simplicity allows us to obtain some interesting results. From (6), the first order condition for the optimal provision of $g$ is given by:

$$H\left(\frac{\partial V}{\partial g}+\frac{\partial V}{\partial m}\frac{\partial m}{\partial g}+\sum_{i=1}^{n}\frac{\partial V}{\partial q_i}\frac{d t_i}{d g}\right)=$$
$$\lambda\left(\begin{array}{c} c'(g)-H\sum_{i=1}^{n}t_i\,p_i\frac{\partial x_i}{\partial g}-H\sum_{i=1}^{n}t_i\,p_i\frac{\partial x_i}{\partial m}\frac{\partial m}{\partial g}- \\ -H\sum_{i=j=1}^{n}t_i\,p_i\frac{\partial x_i}{\partial q_j}\frac{d t_j}{d g}-H\sum_{i=1}^{n}x_i\,p_i\,\frac{d t_i}{d g}\end{array}\right), \tag{13}$$

where the second constraint of problem (6) has been used to express $t$ as a function of $g$. Using Roy's identity and dividing both terms by $\frac{\partial V}{\partial m}=\alpha$, the above expression can be rewritten as:

$$H\left(\frac{\partial V}{\partial g}/\frac{\partial V}{\partial m}+\frac{\partial m}{\partial g}-\sum_{i=1}^{n}x_i\frac{d t_i}{d g}\right)=$$
$$\frac{\lambda}{\alpha}\left(\begin{array}{c} c'(g)-H\sum_{i=1}^{n}t_i\,p_i\frac{\partial x_i}{\partial g}-H\sum_{i=1}^{n}t_i\,p_i\frac{\partial x_i}{\partial m^h}\frac{\partial m}{\partial g}- \\ -H\sum_{i=j=1}^{n}t_i\,p_i\frac{\partial x_i}{\partial q_j}\frac{d t_j}{d g}-H\sum_{i=1}^{n}x_i\,p_i\,\frac{d t_i}{d g}\end{array}\right) \tag{14}$$

In essence, the economic interpretation is similar to equation (9). LHS of equation (14) are the net marginal benefits derived from the provision of $g$. A third term arises here: it displays the negative consequences that an increase of $g$ has on the consumption of taxed goods by means of non-optimal taxes. RHS of equation (14) is the MCPF of providing $g$. Also, welfare effects of distorting taxes and a tax revenue effect have to be considered. An additional step can be sketched in the proposition 2.

**Proposition 2** *The more Hicksian complementarity between a numeraire untaxed good and each taxed good, the smaller MCPF of providing g (Sufficient Condition).*

**Proof.** The MCPF is given by the RHS of equation (14). Among others, its magnitude depends on the terms situated after $c^{'}(g)$. All these terms are always positive except $H \sum_{i=j=1}^{n} t_i \; p_i \frac{\partial \; x_i}{\partial \; q_j} \frac{d \; t_j}{d \; g}$, which is indeterminate. In order to know more about its sign, the Slutsky's decomposition is used so that we have: $H \left( \sum_{i=j=1}^{n} t_i \; p_i \; s_{ij} - \sum_{i=j=1}^{n} t_i \; p_i \; \frac{\partial \; x_i}{\partial \; m} \right) \frac{d \; t_j}{d \; g}$. The sign of $\sum_{i=j=1}^{n} t_i \; p_i \; s_{ij}$ must be studied here. Considering all goods in our economy (the inclusion of the untaxed good 0 has no consequences), the last term can be rewritten as follows: $\sum_{i=j=0}^{n} t_i \; p_i \; s_{ij}$. When we multiply each summand by $\frac{q_j}{q_j}$, we obtain: $\sum_{i=j=0}^{n} \frac{t_i}{1+t_i} \; q_i \; s_{ij}$. Since demand functions are homogeneous in degree 0, and $s_{ii} < 0$, the more Hicksian complementarity between a numeraire untaxed good and each taxed good, the more intense the substituibility relationship between taxed goods (bigger $s_{ij} > 0$). Thus, it is more likely that the sign of $\sum_{i=j=1}^{n} t_i \; p_i \; s_{ij}$ is closer to become positive, and a smaller MCPF will be found. ■

Our model defines effects on the MCPF which are beyond the immediate complementarity between $g$ and taxed goods. Unlike Wildasin (1984) or Chang (2000), we link reductions in the MCPF to the choice of the untaxed goods.

# 5 Concluding remarks

The aim of this paper has been the discussion of the optimal provision of productive public spending when distorting taxation exists. We have built a general equilibrium model with public expenditure as argument in the utility and production functions. Two tax schemes have been considered: tax rates chosen optimally and an arbitrary tax design.

Our results show how the provision of public inputs should take at least two circumstances into account. Firstly, distorting taxation that causes deadweight loss. Secondly, a tax revenue effect that reduces the MCPF. We have proved that the MCPF may decrease even when the taxed goods and the public inputs are substitutes. On the other hand, we have also shown that the choice of the untaxed good is relevant for the magnitude of the MCPF.

This paper has pointed out that the projects of public spending must consider the costs in terms of social welfare derived from a second best taxation. However, if public goods exert productive services in the economy, governments should take the tax revenue effect into consideration. So public investment seems to be a good policy instrument for improving not only the productivity of the economy but also the social welfare.

# References


[1] Aronson, T. and T. Sjögren (2001) "Income taxation, commodity taxation and the provision of public goods under labor market distortions" *FinanzArchiv* **59**, 347-370.

[2] Atkinson, A. B. and N. H. Stern (1974) "Pigou, taxation and public goods" *Review of Economic Studies* **41**, 119-128.

[3] Chang, M. (2000) "Rules and levels in the provision of public goods: the role of complementarities between public good and taxed commodities" *International Tax and Public Finance* **7**, 83-91.

[4] Dasgupta, P. and J. E. Stiglitz (1971) "Differential taxation, public goods and economic efficiency" *Review of Economic Studies* **38** (114), 151-174.

[5] Feehan, J. P. and M. Matsumoto (2000) "Productivity-enhancing public investment and benefit taxation: the case of factor-augmenting public inputs" *Canadian Journal of Economics* **33** (1), 114-121.

[6] Feehan, J. P. and M. Matsumoto (2002) "Distortionary taxation and optimal public spending on productive activities" *Economic Inquiry* **40** (1), 60-68.

[7] Gaube, T. (2000) "When do distortionary taxes reduce the optimal supply of public goods?" *Journal of Public Economics* **76**, 151-180.

[8] King, M. A. (1986) "A pigouvian rule for the optimum provision of public goods" *Journal of Public Economics* **30**, 273-291.

[9] Konishi, H. (1993) "A note on public good provision and commodity taxes" *The Economic Studies Quarterly* **44** (2), 178-184.

[10] Pigou, A. C. (1947) *A study in Public Finance*, Third edition, Macmillan: London.

[11] Samuelson, P. A. (1954) "The pure theory of public expenditure" *The Review of Economics and Statistics* **36**, 387-389.

[12] Wildasin, D. E. (1979) "Public good provision with optimal and non-optimal commodity taxation. The single-consumer case" *Economic Letters* **4**, 59-64.

[13] Wildasin, D. E. (1984) "On public good provision with distortionary taxation" *Economic Inquiry* **22**, 227-243.

[14] Wilson, J. D. (1991) "Optimal public good provision with limited lump-sum taxation" *American Economic Review* **81**, 1, 153-166.