E C O N O M I C S   B U L L E T I N

Combining Pricing Bands with Price Cap Regulation

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Abstract

Within the context of natural monopoly regulation, price cap schemes are often combined with price bands in which a maximum percentage by which the price in a market can be raised or lowered is stipulated. In this note, we examine the feasibility of combining average−revenue−lagged and Laspeyres price cap regulation with price band regulation. Specifically, we study the impact of pricing bands on the known efficiency properties of these two price cap schemes.

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1. Introduction

Historically, natural monopolies have been regulated via cost-based rate of return regulation (ROR). Over the last 20 years, however, price cap regulation (PC) has been developed and has gradually replaced ROR in response to a number of known inefficiencies associated with ROR (Averch and Johnson, 1962; Littlechild, 1983; Brennan, 1989; Crew and Kleindorfer, 1996; and Vogelsang, 2002).

PC schemes are often combined with price band regulation (PB) in which a maximum percentage by which the price in a market can be raised or lowered is specified, e.g., the Federal Communications Commission's regulatory plan for AT&T. With PB regulation, consumers are protected from large price changes. In addition, PB regulation provides assurances that customers in one market are not favored by the firm at the expense of customers in other markets. In general, PB provides stability to any PC regime by slowing the rate at which prices are permitted to change (Sappington, 2002).

Price caps for some regulated sectors in the U.K. (e.g., gas, airports) have utilized average revenue constraints. One form of this, average-revenue-lagged regulation (ARL) is attractive in that steady-state prices are known to be efficient, i.e., to satisfy the Ramsey Rule (Cowan, 1997; Baumol and Bradford, 1970).

Other PC schemes have employed a price cap based on a Laspeyres index of the firm's prices (e.g., regulation of British Telecom and AT&T). As is the case with ARL, under Laspeyres based (L) regulation, steady-state prices are known to be efficient (Brennan, 1989).

In this note, we examine the feasibility of combining ARL and L price cap regulation with pricing band regulation. Specifically, we study the impact of PB on the known efficiency properties of these two price cap schemes.

2. The Model

We define consumer surplus as $V(p)$, where $p = (p_1, \ldots, p_N)$ denotes the vector of prices of a single product in each of N-markets. V is assumed to be C^2 and strictly convex. By Roy's Identity $\nabla V(p) = -q(p)$ where ∇V denotes the gradient of V and $q(p)$ denotes the N-vector of consumer demands. The aggregate output is $Q = \sum_{i=1}^{N} q_i$.

The regulated firm's profit function is $\pi(p) = p \bullet q(p) - C[q(p)]$ where $C(\cdot)$ denotes the cost function. We assume that π is C^2, strictly concave, and attains a unique global maximum at p^*. In addition, welfare is $W = V + \pi$, the unweighted sum of consumer surplus and profit.

The firm's average revenue is

$$AR = \frac{p \bullet q}{\sum_{i=1}^{N} q_i}$$

Now, let $p^0 = \left(\bar{p}^0, \ldots, \bar{p}^0\right)$. Under ARL regulation, the firm's period t prices are required to satisfy

$$\frac{p^t \bullet q\left(p^{t-1}\right)}{\sum_{i=1}^{N} q_i\left(p^{t-1}\right)} \le \bar{p}^0. \tag{1}$$

ARL regulation allows prices in any time period t to be set equal to p^0, where $\bar{p}^0$ is the level of the cap. Following Cowan (1997), we shall refer to the left-hand side of inequality (1) as *calculated average revenue* for period t, ARC^t. ARC^t is a measure of average revenue that uses the previous period's shares of total output to weight the current period's prices. Letting $q^{t-1} \equiv q(p^{t-1})$, (1) may be expressed as $p \bullet q^{t-1} \le \bar{p}^0 Q^{t-1}$. We define $A^t = \{p \mid p \bullet q^{t-1} \le \bar{p}^0 Q^{t-1}\}$.

In the absence of additional pricing constraints, in time period t, the firm selects p^t to maximize profit subject to $p^t \in A^t$. As the ARL process is iterated, a sequence of price vectors is generated. Cowan (1997) has demonstrated that steady-state prices are "efficient" in the sense that welfare is maximized, given the profit level of the firm, i.e., steady-state prices are Ramsey prices.

Under L regulation, given an initial price vector p^0, the firm's period t prices must satisfy $p^t \bullet q^{t-1} \le p^{t-1} \bullet q^{t-1}$. Letting $L^t = \{p \mid p \bullet q^{t-1} \le p^{t-1} \bullet q^{t-1}\}$, in the absence of additional constraints, the firm selects p^t to maximize profit subject to $p^t \in L^t$. As is the case with ARL, the Laspeyres process generates a sequence of price vectors for which steady-state prices are efficient Ramsey prices (Brennan, 1989).

Finally, under price band regulation, given an initial price vector p^0, the firm's period t prices must satisfy

$$(1-k)\, p_i^{t-1} \le p_i^t \le (1+k)\, p_i^{t-1} \qquad i = 1, \ldots, N \qquad (2)$$

$0 < k < 1$. Thus, period t prices are not permitted to deviate from period $t-1$ prices by more than $100k$ percent in either direction (i.e., a band of $\pm 100k$ percent). We shall define

$$B^t = \left[(1-k)p_1^{t-1}, (1+k)p_1^{t-1}\right] x \ldots x \left[(1-k)p_N^{t-1}, (1+k)p_N^{t-1}\right]$$

where x denotes the Cartesian product.

In the following section, we present some implications of combining pricing bands with ARL and L regulation.

3. Combining Price Bands with ARL and L Regulation

We begin by considering the implications of employing price band regulation in conjunction with ARL. In this scenario, the firm solves

$$\underset{p^t}{\text{Maximize}}\ \pi(p^t) \text{ subject to } p^t \in A^t \cap B^t$$

where $p^0 = (\bar{p}^0, \ldots, \bar{p}^0)$ and $\bar{p}^0$ denotes the level of the cap.

Proposition 1. *Suppose that ARL with cap $\bar{p}^0$ is combined with price band regulation where the band width is ±100k percent. Then, if for some t, $\frac{\bar{p}^0}{1-k} < AR^{t-1}$, the ARL process breaks down in the sense that the firm's period t feasible set is empty.*

Proof. Recall that ARL requires that $\text{ARC}^t \leq \;\bar{p}^0$ where

$$\text{ARC}^t = \frac{p^t \bullet q^{t-1}}{Q^{t-1}}.$$

Now if $\frac{\bar{p}^0}{1-k} < \text{AR}^{t-1}$, then $\bar{p}^0 < \frac{(1-k)p^{t-1} \bullet q^{t-1}}{Q^{t-1}}$. In addition, using (2), we have $p_i^t \geq (1-k)p_i^{t-1}$ for $i = 1, \ldots, N$. Hence

$$\bar{p}^0 < \frac{(1-k)p^{t-1} \bullet q^{t-1}}{Q^{t-1}} \leq \frac{p^t \bullet q^{t-1}}{Q^{t-1}} = \text{ARC}^t,$$

contradicting the fact that ARL requires $\text{ARC}^t \leq \;\bar{p}^0$. Thus $A^t \cap B^t = \phi$.

This result establishes that if ARL is applied with price band regulation that is sufficiently tight (i.e., k is sufficiently small), the firm will not be able to find any price vector for which all pricing constraints are satisfied.

We now consider application of L in conjunction with price band regulation. Within this context, the firm solves

$$\underset{p^t}{\text{Maximize}}\ \pi\left(p^t\right) \text{ subject to } p^t \in L^t \cap B^t \tag{3}$$

where, it is assumed, $\pi\left(p^0\right) \geq 0$.

PROPOSITION 2. *Suppose that L is combined with price band regulation where the band width is ±100k percent. Then* $L^t \cap B^t \neq \phi$ *for any value of k. Moreover, the process generates a sequence of price vectors for which any steady state is an efficient (Ramsey) price vector.*

Proof. Let $S = \{p \,|\, \pi(p) \geq 0\}$. Since $p^0 \in S$ and $p^{t-1} \in L^t \cap B^t$ for any $t \geq 1$, $\pi\left(p^t\right) \geq 0$ for all t. Thus, the firm's maximization problem (3) defines a continuous mapping F: $S \rightarrow S$. Moreover, since $L^t \cap B^t \subset L^t$, we have $V\left(p^t\right) > V\left(p^{t-1}\right)$ for all t. Now, let $\{p^t\}$ denote the sequence of price vectors generated by (3). Since $\{p^t\} \subset S$ and S is compact, there exists a convergent subsequence $p^\upsilon \rightarrow \bar{p},\ \upsilon \in \Omega$. Since V is C^2, $\lim_{\upsilon \in \Omega} V(p^\upsilon) = V(\bar{p})$. Since V is monotonically increasing, $\lim_{t \rightarrow \infty} V\left(p^t\right) = V(\bar{p})$.

Now, consider the subsequence $\left\{p^{\upsilon+1}\right\},\ \upsilon \in \Omega$. Since S is compact, there exists $\Omega' \subset \Omega$ such that $p^{\upsilon+1} \rightarrow \bar{\bar{p}},\ \upsilon \in \Omega'$. Again, continuity of V and the fact that V increases monotonically implies that $\lim_{\upsilon \in \Omega'} V\left(p^{\upsilon+1}\right) = V(\bar{\bar{p}})$ and $\lim_{\upsilon \rightarrow \infty} V\left(p^{\upsilon+1}\right) = V(\bar{\bar{p}})$. Thus, $V(\bar{p}) = V(\bar{\bar{p}})$.

Now, suppose that $\bar{p}$ is *not* an efficient price vector. Since $p^{\upsilon+1} = F(p^\upsilon), p^\upsilon \rightarrow \bar{p}$ and $p^{\upsilon+1} \rightarrow \bar{\bar{p}}$ for $\upsilon \in \Omega'$, continuity of F implies that $\bar{\bar{p}} = F(\bar{p})$. Since $\bar{p}$ is assumed not to be an efficient price vector, $V(\bar{\bar{p}}) > V(\bar{p})$, which contradicts our above conclusion that $V(\bar{p}) = V(\bar{\bar{p}})$. Hence, $\bar{p}$ is an efficient price vector.

Proposition 2 establishes that L regulation is compatible with price bands of any size. Moreover, the desirable convergence properties of the Laspeyres-based process are retained in the presence of pricing bands.

4. CONCLUSIONS

The purpose of this note was to examine the compatibility of two forms of price cap regulation that possess desirable efficiency properties (ARL and L) with price band regulation. Price bands are frequently employed to add stability to a price cap regime. We have shown that application of price bands in conjunction with ARL regulation may be problematical. Specifically, if the band is sufficiently tight, there may be no price vectors that simultaneously satisfy the price cap and the price band. Thus the desirable properties of ARL may be forfeited. In addition, it is straightforward to show that this result continues to hold even if only a subset of the regulated firms' markets are subject to price band regulation.

The L process is, however, compatible with a price band of any size. Moreover, though the price band may be expected to reduce the speed of convergence, efficiency of steady-state prices in the presence of price bands is guaranteed with L regulation.

5. REFERENCES

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