E C O N O M I C S B U L L E T I N

Increasing optimism and demand uncertainty

Julien Prat
Department of Economics, Vienna University

Abstract

By allowing the initial prior over market size to be a mixture of distributions, this paper extends the model of irreversible investment under uncertainty proposed by Rob (1991). We find that capacity expansion fuels investors' optimism. It is shown in the paper that the crash is always preceded by a boom when the initial prior is a mixture of exponential distributions.

I am most thankful to Boyan Jovanovic for his encouragement. I am also very grateful to Giuseppe Bertola for his guidance, and to Carlos Alos−Ferrer, Christopher Flinn, Mohamad L. Hammour, Omar Licandro, Ludovic Renou and Emily Sinnot for their very helpful comments.
Citation: Prat, Julien, (2005) "Increasing optimism and demand uncertainty." *Economics Bulletin*, Vol. 12, No. 10 pp. 1−8
Submitted: April 13, 2005. **Accepted:** June 15, 2005.
URL: http://www.economicsbulletin.com/2005/volume12/EB−05L10014A.pdf

1 Introduction

It is well known that anticipated changes in market fundamentals may produce asset price paths that would appear similar to speculative bubbles. A classical example of such changes is the "Peso problem" considered by Krasker (1980), where fundamentals are subject to regime switches. Rob (1991) investigates the implications of the "Peso problem" when investment is irreversible.

More recently, Barbarino and Jovanovic (2004) have elaborated on Rob's (1991) model in order to explain the crash of the NASDAQ index in March 2001. They find that, for the model to fit the data, investors must become more and more optimistic about future prospects as output grows. Such a requirement hardly pleads in favor of the model because intuition suggests that the likelihood of saturating the market increases with installed capacity. Moreover, most of the statistical distributions do not have a decreasing hazard rate (Fudenberg and Tirole (1991), p.267). This is probably why increasing optimism is often assimilated to irrational exuberance.

This paper questions the common wisdom by showing that increasing optimism arises endogenously in Rob's (1991) set-up when parameter uncertainty is introduced. Instead of assuming that anticipations are based on a specific distribution, we consider the more general hypothesis according to which the initial prior is a mixture of distributions. We show that when this is the case, market expansion fuels investors optimism. Consequently, we can relax the requirements needed to generate booms. We find that when the initial prior is a mixture of exponential distributions, the crash is always preceded by a run-up in capacity building.

Section 2 lays out the set up of the model. Optimal accumulation paths are characterized in Section 3. Section 4 concludes.

2 Set up of the model

2.1 The structure of the industry

(i) Consumers' demand: We assume that consumers' demand is given by a *stationary* inverse demand function. As in Barbarino and Jovanovic (2004), we restrict our attention to the rectangular-demand case in order to alleviate the algebra. Therefore

$$D(k_t; z) = \begin{cases} p; \text{ if } k_t \leq z \\ 0; \text{ if } k_t > z \end{cases}$$

where p is the price paid for one unit of output and k_t is the total output of the industry at date t. The variable z is the maximum number of new consumers demanding one unit of the good per unit of time. Therefore $z \in \mathbb{R}_+$. z does not change over time but it is initially unknown. In the remainder of the paper, we will say that the market crashes when k_t exceeds z for the first time. Since the willingness to pay of each consumer p is known, the only uncertain variable is z.

(ii) Production: Capital does not depreciate and investment is irreversible. A unit of capital produces one unit of output and its salvage value is negligible. The free-entry condition holds and each entrant incurs a fixed cost c. Machines take one period to be activated and are operated without costs.

2.2 The Investors' problem

We study a new industry so that installed capacity at the initial date is equal to zero. Investors think of z as a random variable drawn at time 0 from a prior distribution $I(z)$. The prior distribution is common knowledge among the potential entrants. Given the lack of historical data, investors' beliefs are quite vague. To capture their uncertainty, we consider that $I(z)$ is a mixture of distributions. As explained by De Groot (1970, p.190-191), this assumption is more accurate when uncertainty is pervasive.

In order to keep the analysis tractable, we restrict our attention to the case where the prior is formed over a conjugate family of distributions $F(z|\lambda)$ that can be parametrized using the scalar parameter λ. The initial prior might be a mixture of exponential distributions or, for example, a mixture of log-normal distributions ranked according to their location parameter. The probability measure $P_0(\lambda)$ assigns to each distribution a relative weight, so the initial prior reads

$$I(x) \equiv \Pr(z \leq x|\, k_0 = 0) = \int_{\underline{\lambda}}^{\overline{\lambda}} F(x|\,\lambda) dP_0(\lambda) \tag{1}$$

where it is assumed that λ belongs to $\left[\underline{\lambda}, \overline{\lambda}\right] \subset \mathbb{R}_+$. The specification used in Rob (1991) is a particular case of (1), with $P_0(\lambda)$ equal to the Dirac's delta function $\delta(\lambda - \lambda^*)$ for a given λ^*.

One may think of each λ as indexing a specific evaluation of the market. In emerging industries, investors usually face different estimates of their business prospects. When making their investment decision, they have to weight these different predictions: a process that is captured by the probability measure $P_0(\lambda)$. As opposed to z which refers to an objective constraint, λ is a subjective parameter that allows to analyze the process through which agents revise their expectations.

As time goes by until z is reached, new firms enter the market and aggregate output k_t increases. All investors perfectly observe industry's output. In light of this experience, potential entrants revise their belief about z. Parameter uncertainty introduces a new dimension into the learning process: agents do not solely update each distribution $F(x|\,\lambda)$, but they also revise $P_t(\lambda)$ by reallocating the relative weights attached to each λ. Since the only new information acquired during the periods preceding the crash is that $z \geq k_t \geq k_{t-1}$, Bayes' rule yields

$$P_t(\lambda|\, k_t) = \frac{\int_{\underline{\lambda}}^{\lambda} (1 - F(k_t|\,\lambda)) dP_0(\lambda)}{\int_{\underline{\lambda}}^{\overline{\lambda}} (1 - F(k_t|\,\lambda)) dP_0(\lambda)} \tag{2}$$

Relation (2) follows from Bayes' rule because $(1 - F(k_t|\,\lambda))$ is the probability that the economy will reach capacity k_t without experiencing a crash. Intuitively, pessimistic predictions about the size of the market become less and less credible as the industry expands. This new dimension of the learning process generates increasing optimism. To formalize the intuition, we have to order the distributions $F(x|\,\lambda)$ according to their "optimism". We propose to consider the case where they can be ranked using the Monotone Hazard Rate Condition (henceforth MHRC), so that

$$\frac{f(x|\,\lambda_1)}{1 - F(x|\,\lambda_1)} < \frac{f(x|\,\lambda_2)}{1 - F(x|\,\lambda_2)}, \text{ for all } x > 0 \text{ and } \lambda_1 < \lambda_2 \tag{3}$$

where $f(x|\,\lambda)$ is the density associated to $F(x|\,\lambda)$. It is easy to verify that families of exponential or Pareto distributions satisfy condition (3). According to the MHRC, the lower is λ, the smaller is the hazard rate of the crash for any given output. Using this criterion, Proposition 1 proves that posteriors become more and more skewed towards low values of λ as k_t increases.

Proposition 1 *Assume that the distributions $F(x|\,\lambda)$ can be ranked according to the MHRC defined in* (3). *Then the prior $P_t(\lambda|\,k_t)$ First Order Stochastically Dominates the posterior $P_{t+1}(\lambda|\,k_{t+1})$.*
Proof. *Let $p_t(\lambda|\,k_t)$ denote the conditional density of λ. From* (2), *it follows that*

$$\begin{aligned}\left(\frac{p_t(\lambda_1|\,k_t)}{p_t(\lambda_2|\,k_t)}\right) / \left(\frac{p_{t+1}(\lambda_1|\,k_{t+1})}{p_{t+1}(\lambda_2|\,k_{t+1})}\right) &= \left(\frac{1 - F(k_t|\,\lambda_1)}{1 - F(k_{t+1}|\,\lambda_1)}\right) / \left(\frac{1 - F(k_t|\,\lambda_2)}{1 - F(k_{t+1}|\,\lambda_2)}\right) \\ &= e^{-\int_{k_t}^{k_{t+1}}\left[\frac{f(x|\lambda_2)}{1-F(x|\lambda_2)} - \frac{f(x|\lambda_1)}{1-F(x|\lambda_1)}\right]dx} < 1\end{aligned}$$

where $\lambda_1 > \lambda_2$. Hence $P_t(\lambda|\,k_t) \succ_{MLR} P_{t+1}(\lambda|\,k_{t+1})$, where $\succ_{MLR}$ orders distributions according to the Monotone Likelihood Ratio Condition (MLRC henceforth). Since the MLRC implies First Order Stochastic Dominance, the claim follows. ■

Now that we have characterized the learning process and shown that parameter uncertainty delivers increasing optimism, we proceed by deriving the optimal accumulation paths.

3 The optimal accumulation path

After the crash, investment collapses to zero and output remains constant. So we restrict our discussion to the only periods of interest, namely those during which $k_t \leq z$. At any point in time before the crash, the value of an installed unit is made of current profits plus the expected value of the unit next period times the discount factor β

$$V(k_t|\,z \geq k_t) = p + \beta E\left[V(k_{t+1}|\,z \geq k_t)\right] \tag{4}$$

Given that machines take one period to be activated and that c is the cost of entering the industry, the free-entry condition is satisfied if and only if

$$c = \beta E\left[V(k_{t+1}|\, z \geq k_t)\right] \tag{5}$$

Substituting from (5) into (4) and iterating one period ahead, yields

$$V(k_t|\, z \geq k_t) = V(k_{t+1}|\, z \geq k_{t+1}) = p + c \tag{6}$$

Hence, the value of a unit of capital remains constant through time prior to the crash. Using this result, we can decompose the expected value. Equation (6) and Bayes rule yield

$$\begin{aligned} E\left[V(k_{t+1}|\, z \geq k_t)\right] &= V(k_{t+1}|\, z \geq k_{t+1}) \Pr\left\{z \geq k_{t+1}|\, z \geq k_t\right\} \\ &= (p+c)\left(\frac{\int_{\underline{\lambda}}^{\overline{\lambda}}(1 - F(k_{t+1}|\, \lambda))dP_t(\lambda|\, k_t)}{\int_{\underline{\lambda}}^{\overline{\lambda}}(1 - F(k_t|\, \lambda))dP_t(\lambda|\, k_t)}\right) \end{aligned} \tag{7}$$

The first equality holds because the salvage value of capital is negligible. The final expression is derived using Bayes rule to compute the likelihood of the crash given current output. Finally, substituting (6) and (7) into (4), we obtain

$$\frac{c}{\beta\,(p+c)} = \frac{\int_{\underline{\lambda}}^{\overline{\lambda}}(1 - F(k_{t+1}|\, \lambda))dP_t(\lambda|\, k_t)}{\int_{\underline{\lambda}}^{\overline{\lambda}}(1 - F(k_t|\, \lambda))dP_t(\lambda|\, k_t)} \tag{8}$$

Equation (8) states that the likelihood of the crash occurring next period remains constant. Since the right hand side of (8) is a probability, the industry is viable if and only if $\beta > \left(1 + \frac{p}{c}\right)^{-1}$. So investors have to be patient enough.

Barbarino and Jovanovic (2004) analyze in details the accumulation paths under the specific assumption according to which $P_t\,(\lambda|\, k_t)$ is given by the Dirac's delta function for a given λ^*.[1] They show that the hazard rate of the prior distribution $F(x|\, \lambda^*)$ is the key statistic. For investment to be increasing from one period to the other, the hazard rate of the prior must be a decreasing function of output. Conversely, when the hazard rate is increasing, optimal investment decreases. Obviously, when the hazard rate of $F(x|\, \lambda^*)$ is constant, investment remains even through time.

Once parameter uncertainty is introduced, this simple result does not hold anymore. As discussed in section 2.2., when output increases, posteriors attach more and more weight to optimistic opinions about z. This new mechanism drives a positive and increasing wedge between the optimal rate of investment in our set-up and the one obtained in the absence of parameter uncertainty. To illustrate this fact, Proposition 2 analyses the threshold case where the prior is a mixture of distributions with a constant hazard rate. As opposed to the single prior case, Proposition 2 shows that the accumulation path is always convex.

[1] However, they also consider a more general set-up where c is not anymore constant but is an increasing function of the aggregate rate of investment.

Proposition 2 *The optimal rate of investment is increasing through time if:*
1. The distributions $F(k|\,\lambda)$ have a constant hazard rate.
2. The prior distribution $P(\lambda)$ is non-degenerate and absolutely continuous.
Proof. *Since the hazard rate of $F(x|\,\lambda)$ is constant, we can assume without loss of generality that $F(x|\,\lambda) = 1 - e^{-\lambda x}$. Inserting this expression in (2) and (8) yields the following optimality condition*

$$\int_{\underline{\lambda}}^{\overline{\lambda}} \left(e^{-\lambda k_{t+1}} - \left(\frac{c}{\beta\,(p+c)} \right) e^{-\lambda k_t} \right) e^{-\lambda k_t} p_0(\lambda) d\lambda = 0 \tag{9}$$

where $p_0(\lambda)$ denotes the density associated to $P_0(\lambda)$. The proof proceeds by contradiction: (i) Let $\Delta = \Delta k_t = k_{t+1} - k_t$ denote aggregate investment in period t. Assume that investment remains constant in the next period, so that $\Delta k_{t+1} = \Delta$. By iteration, we obtain

$$g(s|\,k_t) = \int_{\underline{\lambda}}^{\overline{\lambda}} e^{-2\lambda(k_t+\Delta(s-t))} \left(e^{-\lambda\Delta} - \left(\frac{c}{\beta\,(p+c)} \right) \right) p_0(\lambda) d\lambda \tag{10}$$

$$= \int_{\underline{\lambda}}^{\overline{\lambda}} h\left(\lambda|\, s, k_t\right) d\lambda = 0; \text{ for } s = \{t, t+1\}. \tag{11}$$

where $h\left(\lambda|\, s, k_t\right)$ is given by

$$h\left(\lambda|\, s, k_t\right) = e^{-2\lambda(k_t+\Delta(s-t))} \left(e^{-\lambda\Delta} - \left(\frac{c}{\beta\,(p+c)} \right) \right) p_0(\lambda)$$

According to condition (10), $g'(t|\,k_t)$ must be equal to zero. However

$$\begin{aligned} \frac{dg(s|\,k_t)}{ds} &= -2\Delta \int_{\underline{\lambda}}^{\overline{\lambda}} \lambda h\left(\lambda|\, s, k_t\right) d\lambda \\ &= -2\Delta \left[\overline{\lambda} H\left(\overline{\lambda}\middle|\, s, k_t\right) - \underline{\lambda} H\left(\underline{\lambda}|\, s, k_t\right) - \int_{\underline{\lambda}}^{\overline{\lambda}} H\left(\lambda|\, s, k_t\right) d\lambda \right] \end{aligned}$$

where $H\left(\lambda|\, s, k_t\right)$ is the primitive of $h\left(\lambda|\, s, k_t\right)$. For the optimality condition (10) to be satisfied, the function $h\left(\lambda|\, s, k_t\right)\big|_{s=t}$ must change sign in $\left[\underline{\lambda}, \overline{\lambda}\right]$. But $h\left(\lambda|\, s, k_t\right)$ is decreasing in λ. Hence the primitive $H\left(\lambda|\, s, k_t\right)\big|_{s=t}$ reaches its minimum at $\overline{\lambda}$, so that

$$\int_{\underline{\lambda}}^{\overline{\lambda}} H\left(\lambda|\, t, k_t\right) d\lambda > (\overline{\lambda} - \underline{\lambda}) H\left(\overline{\lambda}\middle|\, t, k_t\right)$$

Equation (10) also implies that $H\left(\overline{\lambda}\middle|\, t, k_t\right) = H\left(\underline{\lambda}|\, t, k_t\right)$. Therefore

$$\left. \frac{dg(s|\,k_t)}{ds} \right|_{s=t} = -2\Delta \left[(\overline{\lambda} - \underline{\lambda}) H\left(\overline{\lambda}\middle|\, t, k_t\right) - \int_{\underline{\lambda}}^{\overline{\lambda}} H\left(\lambda|\, t, k_t\right) d\lambda \right] > 0$$

which contradicts the optimality condition at date $t+1$.
(ii) Let's assume that $\Delta k_t > \Delta k_{t+1}$. *At date* t, *condition* (9) *still holds. Iterating one period ahead, the optimality condition at date* $t+1$ *reads*

$$\int_{\underline{\lambda}}^{\overline{\lambda}} e^{-2\lambda(k_t+\Delta k_t)} \left(e^{-\lambda \Delta k_{t+1}} - \left(\frac{c}{\beta\,(p+c)}\right)\right) p_0(\lambda) d\lambda > g(t+1|\, k_t) > 0$$

The only case which does not lead to a contradiction is when $\Delta k_t < \Delta k_{t+1}$. ■

To conclude, we illustrate Propositions 1 and 2 through an example. We assume that the initial prior over λ is also exponential, so that $P_0(\lambda) = 1 - e^{-\gamma\lambda}$, $F(x|\,\lambda) = 1 - e^{-\lambda x}$ and $\lambda \in [0, +\infty]$. The initial expectation of λ is equal to γ^{-1}. Since λ is the crash rate of the underlying distributions, γ measures investors' initial optimism. This stylized example has the advantage of delivering closed-form solutions. Inserting the expression of $p_0(\lambda)$ in (2) yields

$$P_t(\lambda|\, k_t) = 1 - e^{-\lambda(k_t+\gamma)} \tag{12}$$

As claimed in Proposition 1, the investors' beliefs become more skewed towards low values of λ as output gets bigger. Figure 1 depicts the updating process of $P_t(\lambda|\, k_t)$ for given values of the exogenous parameters. The legend in the lower-right corner of Figure 1 reports the time periods attached to each curve. Hence, the investors' initial beliefs are depicted by the lowest curve of the graph with a time index equal to 0. The curves with positive time indexes depict investors' priors over λ when investment went on for t periods without experiencing a crash. Figure 1 clearly illustrates the regular shift of beliefs towards low crash rate distributions.

We can also solve explicitly for the optimal rate of investment. Inserting $p_0(\lambda) = \gamma e^{-\gamma\lambda}$ in (9) and solving for the integrals yields

$$k_{t+1} - k_t = (2k_t + \gamma)\left(\frac{\beta\,(p+c) - c}{c}\right) \tag{13}$$

Investment is an increasing function of output and therefore of time, as predicted by Proposition 2. Figure 2 compares the accumulation paths with and without parameter uncertainty. The horizontal line represents the limit size of the market z that we have arbitrarily set to 0.2 for the simulation. The dashed curve depicts the accumulation path for the set of parameters reported in Figure 1. The plain curve is the accumulation path that one would obtain in the absence of parameter uncertainty. More precisely, we have set $P_0(\lambda|\, k_t)$ equal to the Dirac's delta function for a given λ^*. To make the two curves comparable, we have picked λ^* equal to the initial expectation of λ, namely γ^{-1}. Then the investment rates in the first period are almost identical in both cases.

After industry capacity has overshot z, firms stop investing and capacity remains constant because of the irreversibility constraint. As one might expect, parameter uncertainty fuels investment as time elapses. More surprising is the rate at which the two curves diverge. Obviously, the example is stylized and the impact of the learning process is particularly strong because the lower bound of the crash rate has been set to zero.

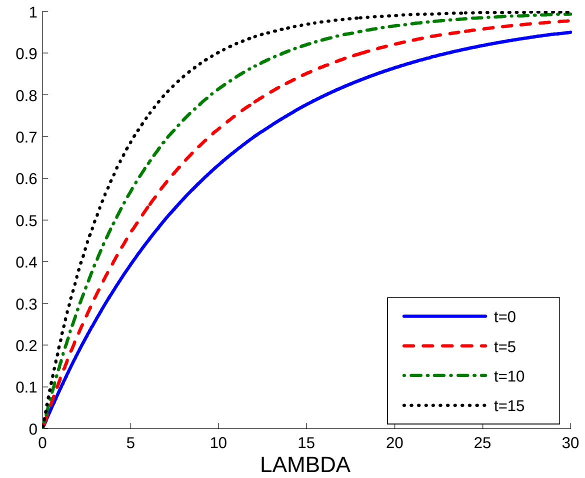

Figure 1: $P_t(\lambda |\, k_t)$ at different point in time.
Parameters: $\gamma = 0.1, \beta = 0.95, c = 1, p = 0.1$.

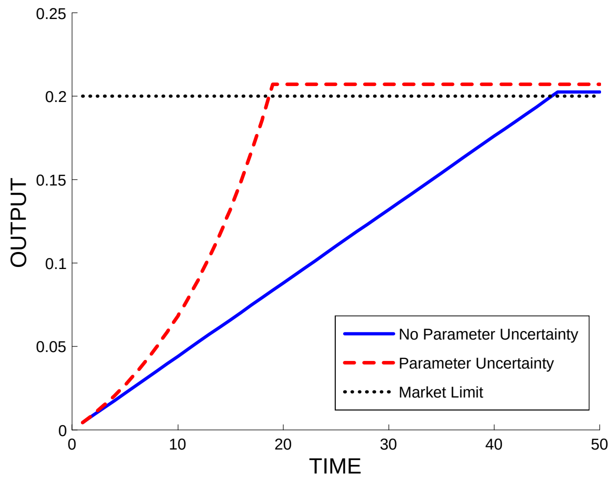

Figure 2: Accumulation paths with and without parameter uncertainty.
Parameters: as reported in Figure 1 except for $\lambda^* = \gamma^{-1} = 10$.

Nevertheless, Figure 2 underlines that parameter updating is far from having solely a second-order effect.

Another issue of interest is the impact of parameter uncertainty on social welfare. As explained in Rob (1991), competitive firms ignore the positive informational externality generated by their investment. Hence, they are too cautious and capacity accumulation occurs at a slower path than the social optimum. Obviously, parameter uncertainty increases the informational externality. However, the risk of exceeding the market limit is also higher, as can be seen from Figure 2. Given that the planner determines the optimal trade-off between these two effects, an interesting avenue of research would be to ascertain whether or not parameter uncertainty worsens the inefficiency of the competitive equilibrium.

4 Conclusion

The prospects of emerging industries are often quite uncertain. So the prior of investors over market size is likely to be better described by a mixture than a single distribution. Parameter uncertainty introduces a new dimension into the learning process. In Rob's (1991) model, expectations are that capacity will eventually saturate the market. As the market grows without reaching this threshold, investors put less and less weight on pessimistic predictions. In other words, they become more and more optimistic. According to this mechanism, when demand is more sustained than what was initially expected, periods of euphoria similar to the end of the 1990's naturally arise.

At a technical level, the declining hazard rate assumed in Barbarino and Jovanovic (2004) can be interpreted as a reduced-form property of the full-fledged investors' problem. We have restricted our attention to mixtures of exponential distributions because only them allow to obtain a conclusion as generic as the one stated in Proposition 2. Nevertheless, the basic mechanism described in this paper carries over to more elaborated set-ups. Further empirical research would enable us to decide whether the impact of parameter uncertainty is significant, as suggested by the example in section 3.

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