E C O N O M I C S B U L L E T I N

Schooling, Working Experiences, and Human Capital Formation

Yunfang Hu
Research Institute for Economics Business Administration, Kobe University

Kazuo Mino
Faculty of Economics, Osaka University

Abstract

This paper assumes that human capital is a composite of two types of knowledge and skills: one is accumulated by formal education in schools and the other is accumulated through working experiences in production activities. Introducing such a concept of human capital into the standard Lucas−Uzawa model of endogenous growth, we show that a higher rate of long−run growth is not necessarily associated with a higher level of education attainment.

We are grateful to an anonymous referee for his or her penetrating and detailed comments. We also thank Ping Wang for his useful comments on an earlier version of this note.
Citation: Hu, Yunfang and Kazuo Mino, (2005) "Schooling, Working Experiences, and Human Capital Formation." *Economics Bulletin*, Vol. 15, No. 3 pp. 1−8
Submitted: October 21, 2004. **Accepted:** January 12, 2005.
URL: http://www.economicsbulletin.com/2005/volume15/EB−04O40006A.pdf

1 Introduction

In the standard Lucas-Uzawa model of endogenous growth (Lucas 1988 and Uzawa 1965), human capital is enhanced by learning in the process of education. As a result, the Lucas-Uzawa model concludes that the long-term growth rate of the economy solely depends on the allocation of resources to education activities. In particular, in the Lucas-Uzawa setting the education sector is assumed to use human capital alone, so that a higher devotion of human capital to education will promote long-term economic growth. Such a specification of human capital formation facilitates the analytical investigation of equilibrium dynamics, which provides us with clear results concerning the transitional as well as balanced-growth impacts of various policy experiments.[1] At the same time, the Lucas-Uzawa model fails to capture some relevant aspects of the relationship between long-term growth and human capital formation. In particular, the model concludes that a higher long-term growth rate is associated with a higher fraction of human capital devoted to education. This result does not fit well the reality at least in advanced countries. Continuing increase in participation rate in higher education and expansion of the average schooling years have not accelerated long-term economic growth in many advanced countries.[2] This fact suggests that the mechanics of human capital formation assumed in the Lucas-Uzawa model needs some modification, if we intend to explain the observed relationship between education and growth.[3]

The purpose of this note is to demonstrate that a slight modification of the Lucas-Uzawa model may reduce the discrepancy between the theoretical outcome and the empirical finding. We assume that human capital is not a homogenous stock created by schooling alone but a composite of two types of stock variables: one is accumulated by formal education in schools and the other is accumulated through working experiences in production activities. This assumption means that the relation between education and human capital formation is not so straightforward as that in the Lucas-Uzawa model. Since time available to each agent is limited, a longer schooling that promotes investment for human capital reduces labor participation rate in production activities, which depresses accumulation of human capital due to learning by working. Because of the presence of such a trade-off, a higher rate of human capital allocation to education may not accelerate long-term growth. In fact, in our modified version of the Lucas-Uzawa

[1]See Faig (1995) and Chapter 5 in Barro and Sala-i-Martin (2004) for through analytical discussions of the Lucas model.

[2]Pritchett (2001) showes that there is no association between increases in labor-efficinecy-promoting human capital accumulation with the growth rate of the economy. In examining the relation between schooling and growth, Bils and Klenow (2000) find that the impact of schooling on growth explains less than one-third of the empirical cross-country relationship.

[3]Noting that human capital used in applied work has been poorly proxied, Barro and Lee (2001), Temple (2001) and Woessmann (2000) construct new measures for human capital stock. They suggest that human capital accumulated by education exhibit diminishing returns.

model, we find that a lower rate of long-run growth would be associated with a higher level of resource allocation to education activities.

2 The Model

The structure of our model basically follows Lucas (1988). There is a homogenous final good and its production technology is specified as

$$y = Ak^{\alpha} (uh)^{1-\alpha}, \quad 0 < \alpha < 1, \tag{1}$$

where y is output, k is physical capital, h is human capital and u denotes the ratio of human capital devoted to the final goods production. Unlike Lucas (1988), we assume that human capital is a composite of two types of stock variables. One of them represents knowledge and skills obtained in the process of schooling (formal education), while the other expresses knowledge and skills obtained on the job. Namely, we assume that human capital is enhanced by schooling as well as by learning-by-doing in production activities.[4] More specifically, we assume that

$$h = E^{\theta} L^{1-\theta}, \quad 0 < \theta < 1, \tag{2}$$

where E is a cumulative stock of knowledge obtained in the process of education, while L is a stock of knowledge accumulated through working experiences. Therefore, we define

$$\begin{aligned} E(t) &= \delta \int_{-\infty}^{t} [1 - u(\tau)] h(\tau) d\tau, \quad \delta > 0, \\ L(t) &= \eta \int_{-\infty}^{t} u(\tau) h(\tau) d\tau, \quad \eta > 0. \end{aligned}$$

These expressions mean that E and L are represented by cumulative sums of human capital services used for education and production, respectively. The above definitions give

$$\dot{E} = \delta (1 - u) h, \tag{3}$$

$$\dot{L} = \eta u h. \tag{4}$$

Here, parameters δ and η respectively denote the speeds of leaning in schooling and working.

There is no population growth and the number of households is normalized to one. In what follows, we assume that each household purposefully selects its

[4] In the labor economics literature, it has been a popular idea that human capital formation depends on the job training as well as on the formal education: see, for example, Ben-Porath (1967). In order to construct an analytically tractable growth model, we use a much simpler formulation of human capital investment than those used by the microeconomic studies on education and human capital.

education level by controlling u, but the learning by doing effect in production is external to an individual: accumulation of human capital through working experiences is a by-product of production and the household takes it as an external effect. The flow budget constraint for the representative household is

$$\dot{k} = rk + wuh + s\left(1-u\right)wh - c - T, \tag{5}$$

where c is consumption, r the rate of return to physical capital, w the real wage rate, s the rate of education subsidy ($0 \leq s < 1$), and T denotes a lump-sum tax. Following Lucas (1988), we assume that education service is not a market good so that there is no education industry. As a result, the household's income equals $rk + wuh$. In addition, the household is assumed to receive an education subsidy from the government that is proportional to the opportunity cost of education, $(1-u)\,wh$. The government finances its expenditure for subsidy by lump-sum taxation, so that the government budget constraint is $s\left(1-u\right)wh = T$. We assume that the government adjusts T to satisfy this equation under a fixed level of s.

The representative household maximizes a discounted sum of utility

$$U = \int_0^\infty \frac{c^{1-\sigma}-1}{1-\sigma}e^{-\rho t}dt, \quad \rho > 0, \quad \sigma > 0$$

subject to (5), $h = E^{\theta}\bar{L}^{1-\theta}$, and the dynamic equation of the stock of knowledge created by education:

$$\dot{E} = \delta\left(1-u\right)E^{\theta}\bar{L}^{1-\theta}, \tag{6}$$

In the above, $\bar{L}$ indicates that the level of L is external to the household: when solving its optimization problem, the household takes a sequence of $\left\{\bar{L}\right\}_{t=0}^{\infty}$ as given. For notational simplicity, we assume that neither physical capital nor knowledge capital depreciates.

To derive the optimization conditions for the household, we set up the Hamiltonian function such that

$$\begin{aligned} H &= \frac{c^{1-\sigma}-1}{1-\sigma} + p\left[rk + wuE^{\theta}\bar{L}^{1-\theta} + s\left(1-u\right)wE^{\theta}\bar{L}^{1-\theta} - c - T\right] \\ &\quad + q[\delta\left(1-u\right)E^{\theta}\bar{L}^{1-\theta}], \end{aligned}$$

where p and q denote shadow values of physical capital and the stock of knowledge obtained by education, respectively. We find that the optimization conditions include:

$$\max_c H \Rightarrow c^{-\sigma} = p, \tag{7}$$

$$\max_u H \Rightarrow \left(1-s\right)wp = \delta q, \tag{8}$$

$$\dot{p} = p\left(\rho - r\right), \tag{9}$$

$$\dot{q} = q\rho - \frac{\delta\theta}{1-s}\left(\frac{E}{\bar{L}}\right)^{\theta-1}. \tag{10}$$

Note that in deriving (10) we use (8).

Since the number of households is one, the symmetric equilibrium condition requires that $\bar{L} = L$. In the competitive equilibrium, the rate of return to physical capital and real wage rate respectively given by $r = \partial y/\partial k = \alpha A\left(k/uh\right)^{\alpha-1}$ and $w = (1-\alpha)\,A\left(k/uh\right)^{\alpha}$. Thus (8) gives $w = A\left(1-\alpha\right)\left(k/uh\right)^{\alpha} = \delta q/\left(1-s\right)p$, implying that

$$\frac{k}{uh} = \left[\frac{\delta q}{A\left(1-\alpha\right)\left(1-s\right)p}\right]^{\frac{1}{\alpha}}. \tag{11}$$

As a result, (9) can be written as

$$\frac{\dot{p}}{p} = \rho - \alpha A\left[\frac{\delta}{A(1-\alpha)\left(1-s\right)}\frac{q}{p}\right]^{\frac{\alpha-1}{\alpha}}. \tag{12}$$

Now let us define:

$$x = E/L, \qquad z = q/p, \qquad v = c/k.$$

From (3), (4) and $\dot{x}/x = \dot{E}/E - \dot{L}/L$, it holds that

$$\frac{\dot{x}}{x} = \delta\left(1-u\right)x^{\theta-1} - \eta u x^{\theta}. \tag{13}$$

Equation (10) becomes $\dot{q}/q = \rho - \left[\delta\theta/\left(1-s\right)\right]x^{\theta-1}$. This and (12) present

$$\frac{\dot{z}}{z} = \alpha A\left[\frac{\delta z}{A(1-\alpha)\left(1-s\right)}\right]^{\frac{\alpha-1}{\alpha}} - \frac{\theta\delta}{1-s}x^{\theta-1}. \tag{14}$$

Next, (5) yields

$$\frac{\dot{k}}{k} = A\left(\frac{\delta z}{A(1-\alpha)\left(1-s\right)}\right)^{\frac{\alpha-1}{\alpha}} - v. \tag{15}$$

Using (7) and (12), we obtain

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}\alpha A\left(\frac{\delta z}{A(1-\alpha)\left(1-s\right)}\right)^{\frac{\alpha-1}{\alpha}} - \rho. \tag{16}$$

Thus from (15) and (16) we derive the following:

$$\frac{\dot{v}}{v} = \frac{1}{\sigma}\left[\alpha A\left(\frac{\delta z}{A(1-\alpha)\left(1-s\right)}\right)^{\frac{\alpha-1}{\alpha}} - \rho\right] - A\left(\frac{\delta z}{A(1-\alpha)\left(1-s\right)}\right)^{\frac{\alpha-1}{\alpha}} + v. \tag{17}$$

Finally, in order to derive the dynamic equation of u, notice that logarithmic differentiation of both sides of (11) yields $\alpha(\dot{k}/k - \dot{u}/u - \dot{h}/h) = \dot{z}/z$. Hence, noting that $\dot{h}/h = \theta\left(\dot{E}/E\right) + (1-\theta)\left(\dot{L}/L\right)$ and using (3), (4) and (14), we find that u changes according to

$$\frac{\dot{u}}{u} = -\frac{1}{\alpha}\left[\alpha A\left(\frac{\delta z}{A(1-\alpha)(1-s)}\right)^{\frac{\alpha-1}{\alpha}} - \delta\theta x^{\theta-1}\right] + A\left(\frac{\delta z}{A(1-\alpha)(1-s)}\right)^{\frac{\alpha-1}{\alpha}}$$
$$-v - \theta\delta(1-u)x^{\theta-1} - \eta(1-\theta)ux^{\theta}. \quad (18)$$

In sum, we have derived a complete dynamic system consisting of (13), (14), (17) and (18) that describe the dynamic behaviors of $x\,(=E/L)$, $z\,(=q/p)$, $v\,(=c/k)$ and u.

3 Education and Growth in the Long Run

In the steady state that satisfies $\dot{x} = \dot{v} = \dot{z} = \dot{u} = 0$, it holds that

$$\frac{\dot{k}}{k} = \frac{\dot{h}}{h} = \frac{\dot{c}}{c} = \frac{\dot{E}}{E} = \frac{\dot{L}}{L} = -\frac{1}{\sigma}\frac{\dot{p}}{p} = -\frac{1}{\sigma}\frac{\dot{q}}{q} = g,$$

where g denotes the balanced-growth rate. The stationary conditions in (13), (14), (17) and (18) respectively give the following:

$$\delta(1-u)x^{\theta-1} - \eta u x^{\theta} = 0, \quad (19)$$

$$\frac{1}{\sigma}\left[\alpha A\left(\frac{\delta z}{A(1-\alpha)(1-s)}\right)^{\frac{\alpha-1}{\alpha}} - \rho\right] - A\left(\frac{\delta z}{A(1-\alpha)(1-s)}\right)^{\frac{\alpha-1}{\alpha}} + v = 0, \quad (20)$$

$$\alpha A\left(\frac{\delta z}{A(1-\alpha)(1-s)}\right)^{\frac{\alpha-1}{\alpha}} - \frac{\theta\delta}{1-s}x^{\theta-1} = 0, \quad (21)$$

$$A\left(\frac{\delta z}{A(1-\alpha)(1-s)}\right)^{\frac{\alpha-1}{\alpha}} - v - \eta u x^{\theta} = 0. \quad (22)$$

By use of (19), we obtain $u = \delta x^{\theta-1}/\left(\delta x^{\theta-1} + \eta x^{\theta}\right) = \delta/(\delta + \eta x)$, which gives the following:[5]

$$x = \frac{\delta}{\eta}\frac{1-u}{u}. \quad (23)$$

[5]Equation (23) means that

$$x \equiv \frac{E}{L} = \frac{\delta(1-u)h}{\eta u h}\left(= \frac{\dot{E}}{\dot{L}}\right).$$

Namely, the factor intensity in human capital formation on the balanced-growth path depends on the relative speed of leraning, δ/η, as well as on the relative allocation of human capital, $(1-u)/u$, to education and working.

Using this relation, we see that the condition $\dot{E}/E = g$ yields

$$g = \delta^{\theta}\eta^{1-\theta}(1-u)^{\theta}u^{1-\theta} \equiv F(u). \tag{24}$$

In view of (16), (21) and (24), we obtain

$$g = \frac{\rho(1-s)(1-u)}{\theta - \sigma(1-s)(1-u)} \equiv G(u). \tag{25}$$

Equations (24) and (25) simultaneously determine the balanced-growth rate, g, and the steady-state rate of human-capital allocation to production, u.

First, notice that the original Lucas model assumes that $\theta = 1$ (so that $h = E$). If $\theta = 1$, equation (24) reduces to $g = \delta(1-u)$. Using this and (25) with $\theta = 1$, we obtain $g = (1/\sigma)\left(\frac{\delta}{1-s} - \rho\right)$. Consequently, if ρ or σ decreases, then both g and $1-u$ increase. If the rate of education subsidy, s, increases, then g and $1-u$ rise as well. Note that the steady-state value of u is given by $u = 1 + (\rho/\sigma\delta) - (1/\sigma)(1-s)$, and thus an increase in δ also raises g and $1-u$. As a consequence, in the Lucas model, any parameter change that raises the balanced-growth rate is associated with an increase in the human capital allocation to education, $1-u$.

If $0 < \theta < 1$, the the graph of $F(u)$ is inverse U-shaped and its maximum level is attained where $u = 1-\theta$. Additionally, $F(0) = F(1) = 0$. Function $G(u)$ is monotonically decreasing with $G(0) = \rho(1-s)/[\theta - \sigma(1-s)]$ and $G(1) = 0$. Therefore, provided that $\theta > \sigma(1-s)$, it is easy to see that there is a unique steady state with $g > 0$ and $u \in (0,1)$. Figure 1 depicts the graphs of $F(u)$ and $G(u)$. In panel (a) the steady-state value of u is less than $1-\theta$, while it exceeds $1-\theta$ in panel (b).

As an example of the long-run comparative statics, consider the effects of a change in the education subsidy rate, s. When s increases, $G(u)$ curve shifts downward. Hence, the steady-state value of u decreases, while the balanced growth rate decreases (increases) if the initial level of u is smaller (larger) than $1-\theta$. That is, a higher education subsidy promotes a larger devotion of human capital to education, i.e. an increase in $1-u$, but it accelerates long-run growth only when the initial allocation of human capital to education satisfies $1-u < \theta$.[6] A rise in the education subsidy reallocates human capital from production to education, and thus it has a negative impact on human capital formation through learning by doing on the job. If this negative effect on the leaning effect in production dominates the positive growth effect of education enhancement, human capital formation is decelerated and the long-term growth rate will be lowered. In contrast, a higher efficiency of education (that is a larger value of δ) produces an upward shift of $F(u)$ curve. Thus a rise in δ presents a higher long-term growth rate with a higher level of human capital allocation to education.

[6] The effects of decreases in ρ and σ have the same effects on g and u as those given by a rise in s.

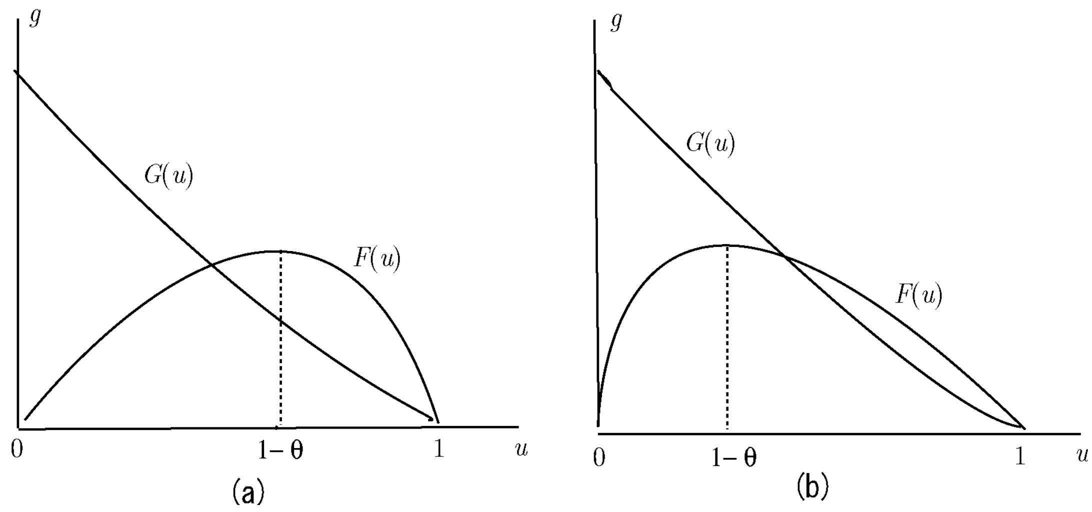

Figure 1:

Similarly, an increase in η also raises g and $1 - u$. Higher learning speeds both in formal education and in working places, therefore, contribute to promoting growth as well as education attainment.

The above discussion shows that the positive relation between long-term growth and education can be obtained if the balanced-growth equilibrium is characterized by panel (b) in Figure 1. Other things being equal, the situation of panel (b) tends to hold, if θ is large or if s is small. A large θ means that knowledge and skills obtained in formal education is relatively important in human capital formation, so that it is easy to establish the growth-enhancing effect of education. A smaller s indicates that education subsidy is not high enough to attains the maximum balanced-growth rate. Given θ and s, the situation like panel (b) can hold if ρ and σ are relatively high or if δ and η are relatively low. In sum, the conditions for establishing the situation in panel (b) are satisfied, if education share of human capital is high, the education subsidy is low, and the speeds of learning both in schools and in work places are not high enough. Conversely, an economy with a highly developed education system and enough government's subsidy for education tends to have the steady state characterized by panel (a). This suggests that the positive relationship between education and long-term growth may not be observed in advanced countries that have already realized high levels of education attainment.

It is to be pointed out that we have focused on the balanced-growth equilibrium and a simple policy experiment. Future research should examine the dynamic behavior of the model economy out of the balanced-growth path and present more general discussion of various policy effects in our setting.

References

[1] Barro, R.J. and Lee, J-W. (2001). International data on educational attainment: updates and implications. Oxford Economic Papers 53, 541-63.

[2] Barro, R. and Sala-i-Martin, X. (2004). Economic Growth. (2nd Edition) Cambridge MA. MIT Press.

[3] Ben-Porath, Y. (1967) The Production of human capital and the life cycle of earnings. Journal of Political Economy 75, 352-365.

[4] Bils, M. and Klenow, P. J. (2000). Does schooling cause growth? American Economic Review 90, 1160-83.

[5] Faig, M. (1995). A simple model with human capital accumulation: Transitional dynamics, technology shocks, and fiscal policies. Journal of Macroeconomics 17, 421-46.

[6] Lucas, R.E., (1988). On the mechanics of economic development. Journal of Monetary Economics 22, 3-42.

[7] Pritchett, L. (2001). Where has all the education gone? World Bank Economic Review 15, 367-91.

[8] Temple, J, (2001). Growth effects of education and social capital in the OECD countries. OECD Economic Studies 33, 57-101.

[9] Uzawa, H. (1965). Optimal technical change in an aggregate model of economic growth. International Economic Review 6, 18-31.

[10] Woessmann, L. (2000). Specifying human capital: a review, some extensions, and development effects. Kiel working paper No.1007.