

## Estimating threshold cointegrated systems

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### *Abstract*

Using simulations, the paper shows that there is a trade-off in using CLS and 2SLS on the one hand and ML on the other when estimating the parameters of a bivariate threshold vector equilibrium correction model with regime-specific cointegration vectors.

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# 1 Introduction

Consider a bivariate  $I(1)$  time series  $\mathbf{Y}_t = (Y_{1,t}, Y_{2,t})'$  with a  $2 \times 1$  cointegration vector  $\boldsymbol{\beta}$ . Further, let  $Z_t = \boldsymbol{\beta}'\mathbf{Y}_t$  be a stationary, or  $I(0)$ , time series. Then a two-regime threshold vector equilibrium correction model (TVECM) of order  $p$  can be written as

$$\Delta \mathbf{Y}_t = \sum_{j=1}^2 \left\{ \mathbf{a}_0^{(j)} + \boldsymbol{\alpha}^{(j)} \boldsymbol{\beta}' \mathbf{Y}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Phi}_i^{(j)} \Delta \mathbf{Y}_{t-i} + \mathbf{u}_t^{(j)} \right\} I(Z_{t-d} \in R_j), \quad (1)$$

where  $\Delta = 1 - L$  with  $L$  the lag operator,  $I(\cdot)$  is the indicator function,  $R_j = (r^{(j-1)}, r^{(j)})$  with  $-\infty = r^{(0)} < r^{(1)} < r^{(2)} = \infty$ .  $Z_{t-d}$  is the transition variable with, fixed known, delay  $d \in \{1, 2, \dots\}$ ;  $r^{(j)}$  are the thresholds;  $\mathbf{a}_0^{(j)} = (a_{1,0}^{(j)}, a_{2,0}^{(j)})'$  and  $\boldsymbol{\alpha}^{(j)} = (\alpha_1^{(j)}, \alpha_2^{(j)})'$  are  $2 \times 1$  vectors;  $\boldsymbol{\Phi}_i^{(j)}$  are matrices of coefficients. The sequences  $\{\mathbf{u}_t^{(j)} = (u_{1,t}^{(j)}, u_{2,t}^{(j)})'\}$  ( $j = 1, 2$ ) are bivariate *iid* random sequences with mean  $\mathbf{0}$  and covariance matrices  $\boldsymbol{\Sigma}^{(j)}$  that are independent across different regimes.

Model (1) assumes that there is one common cointegrating vector  $\boldsymbol{\beta}$  in all regimes. This assumption is often unnecessarily restrictive. De Gooijer and Vidiella-i-Anguera (2004) proposed an alternative representation that allows the equilibrium error correction process to be different in each regime, i.e.  $\boldsymbol{\beta}$  in (1) is replaced by a  $2 \times 1$  vector  $\boldsymbol{\gamma}^{(j)}$   $j = 1, 2$ . The resulting two-regime model is called level TVECM (LTVECM). Parameter estimation in (L)TVECMs can be done by using recursive conditional least squares (OLS), by conditional two-stage least squares (2SLS), or by maximum likelihood (ML). The objective of the paper is to address the question how best to proceed in estimating the cointegration parameters of (L)TVECMs through one of the three estimation methods mentioned above. As such the present study extends Gonzalo's (1994) work for linear VECMs to the case of vector threshold specifications.

# 2 Estimation

Consider an LTVECM with  $d$  known. Assume for identification purpose that one element in both  $\boldsymbol{\gamma}^{(j)}$  and  $\boldsymbol{\beta}$  are set at unity. Then, given  $T$  observations, the parameters  $(\boldsymbol{\beta}, \boldsymbol{\gamma}^{(1)}, \boldsymbol{\gamma}^{(2)}, \boldsymbol{\Sigma}^{(1)}, \boldsymbol{\Sigma}^{(2)}, r)$  can be estimated in two steps. First, for given  $r^{(1)} \equiv r$  and  $\boldsymbol{\beta}$ , model (1) reduces to two separate linear regressions from which the OLS estimates of  $\boldsymbol{\gamma}^{(j)}$  and  $\boldsymbol{\Sigma}^{(j)}$  are readily available. The estimates are

$$\hat{\boldsymbol{\gamma}}^{(j)}(r, \boldsymbol{\beta}) = \left( \hat{\mathbf{Y}}_{2,j}' \hat{\mathbf{Y}}_{2,j} \right)^{-1} \hat{\mathbf{Y}}_{2,j}' \hat{\mathbf{Y}}_{1,j}, \quad (2)$$

$$\hat{\boldsymbol{\Sigma}}^{(j)}(r, \boldsymbol{\beta}) = (\hat{\mathbf{Y}}_{1,j} - \hat{\mathbf{Y}}_{2,j} \hat{\boldsymbol{\gamma}}^{(j)}(r, \boldsymbol{\beta})) (\hat{\mathbf{Y}}_{1,j} - \hat{\mathbf{Y}}_{2,j} \hat{\boldsymbol{\gamma}}^{(j)}(r, \boldsymbol{\beta}))' / (T_j - k) \quad (j = 1, 2), \quad (3)$$

where  $\hat{\mathbf{Y}}_{i,1}$  corresponds to  $\mathbf{Y}_{i,t-1} I(\boldsymbol{\beta}'\mathbf{Y}_{t-1} \leq r)$  and  $\hat{\mathbf{Y}}_{i,2}$  corresponds to  $\mathbf{Y}_{i,t-1} I(\boldsymbol{\beta}'\mathbf{Y}_{t-1} > r)$  ( $i = 1, 2$ ), respectively,  $T_j$  is the number of observations in regime  $j$ , and  $k$  is the dimension of  $\hat{\mathbf{Y}}_{2,j}$ , satisfying  $k < T_j$ , for  $j = 1, 2$ . Denote the sum of squares of residuals by

$$S(r, \boldsymbol{\beta}) = S_1(r, \boldsymbol{\beta}) + S_2(r, \boldsymbol{\beta})$$

where  $S_j(r, \boldsymbol{\beta})$  denotes the trace of  $(T_j - k) \hat{\boldsymbol{\Sigma}}^{(j)}(r, \boldsymbol{\beta})$ . Next, in step 2, form a grid search over  $(r, \boldsymbol{\beta})$ . Find  $(r_{\text{OLS}}, \boldsymbol{\beta}_{\text{OLS}})$  as the estimates of  $(r, \boldsymbol{\beta})$  on the grid which yield the lowest value of  $S(r, \boldsymbol{\beta})$ , that is

$$(r_{\text{OLS}}, \boldsymbol{\beta}_{\text{OLS}}) = \arg \min_{r, \boldsymbol{\beta}} S(r, \boldsymbol{\beta}).$$

The resulting least squares estimates for the parameters are

$$\gamma_{\text{OLS}}^{(j)} = \hat{\gamma}^{(j)}(r_{\text{OLS}}, \beta_{\text{OLS}}) \quad \text{and} \quad \Sigma_{\text{OLS}}^{(j)} = \hat{\Sigma}^{(j)}(r_{\text{OLS}}, \beta_{\text{OLS}}).$$

To reduce the bias conditional 2SLS may be useful. Similar as above, the estimation of the parameters  $(\beta, \gamma^{(1)}, \gamma^{(2)}, \Sigma^{(1)}, \Sigma^{(2)}, r)$  follows two steps. First, for fixed values of  $r$  and  $\beta$ ,  $\gamma^{(j)}$  is estimated as

$$\tilde{\gamma}^{(j)}(r, \beta) = \left( \hat{\mathbf{Y}}'_{2,j} \hat{\mathbf{P}}_{-1} \hat{\mathbf{Y}}_{2,j} \right)^{-1} \hat{\mathbf{Y}}'_{2,j} \hat{\mathbf{P}}_{-1} \hat{\mathbf{Y}}_{1,j}, \quad (j = 1, 2), \quad (4)$$

where  $\hat{\mathbf{P}}_{-1} = \hat{\mathbf{Y}}'_{-1,j} \left( \hat{\mathbf{Y}}'_{-1,j} \hat{\mathbf{Y}}_{-1,j} \right)^{-1} \hat{\mathbf{Y}}_{-1,j}$  with  $\hat{\mathbf{Y}}_{-1,1}$  corresponding to  $\mathbf{Y}_{2,t-2} I(\beta' \mathbf{Y}_{t-1} \leq r)$  and  $\hat{\mathbf{Y}}_{-1,2}$  corresponding  $\mathbf{Y}_{2,t-2} I(\beta' \mathbf{Y}_{t-1} > r)$ , respectively. Let  $\tilde{\Sigma}^{(j)}(r, \beta)$  denote the estimate of  $\Sigma^{(j)}$ . Next, through a grid search over  $(r, \beta)$ , the conditional 2SLS estimates  $(r_{2\text{SLS}}, \beta_{2\text{SLS}})$  are found as the minimizers of the sum of squares of residuals, following a similar approach as in the OLS case. The resulting 2SLS estimates for the parameters are

$$\gamma_{2\text{SLS}}^{(j)} = \tilde{\gamma}^{(j)}(r_{2\text{SLS}}, \beta_{2\text{SLS}}) \quad \text{and} \quad \Sigma_{2\text{SLS}}^{(j)} = \tilde{\Sigma}^{(j)}(r_{2\text{SLS}}, \beta_{2\text{SLS}}).$$

Note that  $\tilde{\gamma}^{(j)}(r, \beta)$  is obtained from replacing first  $\hat{\mathbf{Y}}_{2,j}$  by  $\tilde{\mathbf{Y}}_{2,j} = \hat{\mathbf{P}}_{-1} \hat{\mathbf{Y}}_{2,j}$  and then regressing  $\hat{\mathbf{Y}}_{1,j}$  on  $\tilde{\mathbf{Y}}_{2,j}$  ( $j = 1, 2$ ). In this way  $\tilde{\gamma}^{(j)}(r, \beta)$  is estimated without the simultaneous equation bias that characterizes the OLS estimator  $\hat{\gamma}^{(j)}(r, \beta)$ . Unfortunately, there is still left a bias related to the presence of the unit root term. Thus, (4) is expected to improve (2) only marginally.

For ML estimation a modified version of Hansen and Seo's (2002) ML procedure can be used. In particular, first a grid search over  $\beta$  that maximizes the concentrated likelihood function is applied. Once  $\beta$  is fixed, the  $\gamma^{(j)}$ 's are estimated. For  $\beta$  the grid search region is first calibrated using a consistent estimate of  $\hat{\beta}$ . The mid-point of the region is set at  $\hat{\beta}$  and the width of the region is taken as the standard deviation of  $\hat{\beta}$  times a scaling factor. For near nonstationary systems, we noticed that the standard deviation of  $\hat{\beta}$  obtained by either OLS or 2SLS estimation can be very small. One possible solution is to compute numerically the true standard deviation using formula in Tanaka (1996, Chapter 11). As an alternative, we used a scaling factor big enough to result in a sufficiently large region. Next, given a consistent estimate of  $\beta$ , the empirical support of  $r$  is computed from which the empirical range of  $r$  is obtained.

### 3 Monte Carlo design

Results will be reported for the following LTVECM with  $p = 1$  and  $d = 1$ :

$$\Delta \mathbf{Y}_t = \sum_{j=1}^2 \left\{ \begin{pmatrix} \alpha_{1,0}^{(j)} \\ 0 \end{pmatrix} (1, -\gamma^{(j)}) \mathbf{Y}_{t-1} + \Sigma_j^{1/2} \mathbf{a}_t \right\} I((1, -\beta) \mathbf{Y}_{t-1} \in R_j) \quad (5)$$

where  $\mathbf{a}_t \sim NID(\mathbf{0}, \mathbf{I}_2)$ , and  $\Sigma_j$  is symmetric and positive definite. Model(5) is often used in empirical applications; see, e.g., Clements and Galvão (2004).

Within each simulation all three estimation procedures used the same standard normal distributed random numbers, given a particular set of parameter values. The choice of the true parameter values for the DGPs is such that each regime has approximately an equal number of observations (50%). The true parameter values are:  $\beta = 1.5$ ,  $\gamma^{(1)} = 2$ ,  $\gamma^{(2)} = 1$ , and  $r = 0$ . For  $T = 400$ , results are presented in terms of: i) the parameter  $\theta$ , in  $\Sigma_j = \begin{pmatrix} 1 & \theta\sigma \\ \theta\sigma & \sigma^2 \end{pmatrix}$ ; ii) the

loading parameter  $\alpha \equiv \alpha_{1,0}^{(1)} = \alpha_{1,0}^{(2)}$ ; and iii) the ratio signal-noise  $\sigma$  that measures how big the random walk component of the variables is. The grid sizes for estimating  $\beta$  and  $r$  were set at 100 and 400, respectively. To ensure that there are enough observations in each regime, we used a trimming percentage of 10%.

## 4 Results

### 4.1 Restricted model

Table 1 shows estimation results for  $\beta$ ,  $\gamma^{(1)}$  and  $\gamma^{(2)}$  when  $r$  is known to be equal zero. The bias in median in estimating these parameters is about the same for all three estimators. Similar results were obtained when  $\beta$  is known. When both  $r$  and  $\beta$  are known, estimators of  $\gamma^{(1)}$ , and  $\gamma^{(2)}$  are more accurate than when less information is used in the estimation of these parameters. The largest improvement in median bias occurs with the ML estimator. In terms of sample dispersion, as measured by the 50% interquartile range (IQR), the difference in performance between estimating regime-specific threshold parameters  $\gamma^{(1)}$  and  $\gamma^{(2)}$  by OLS and 2SLS on the one hand and ML on the other is somewhat mixed. In particular, if  $\sigma = 2$  and irrespective of the parameter values  $\theta$  and  $\alpha$ , OLS and 2SLS have considerably less dispersion than ML. Thus, for high signal-noise ratios the ML does not perform so well.

For fixed  $\alpha$  and  $\sigma$ , different values of  $\theta$  do not have a big impact on the estimation results. This is consistent with Hansen and Phillips (1990) who observed, in linear models, that  $\sigma$  appears to be a more relevant factor than the degree of long-run endogeneity. Given fixed values of  $\theta$  and  $\sigma$ , the bias in median of all three estimators of  $\beta$ ,  $\gamma^{(1)}$  and  $\gamma^{(2)}$  depends on the value of  $\alpha$ . In particular this is the case for OLS and 2SLS. Similar results on the influence of  $\alpha$  in linear models are found by Banerjee, Dolado, Hendry and Smith (1986). The bias of the ML estimator seems to be less sensitive to different values of  $\alpha$ . Finally, we noticed that all three estimators of  $r$  perform the same.

### 4.2 Unrestricted model

Table 2 shows estimation results for the unrestricted LTVECM. For fixed values of  $\theta$  and  $\sigma$  OLS performs very similarly to 2SLS when estimating  $\gamma^{(1)}$  and  $\gamma^{(2)}$  with high bias in median. With respect to sample dispersion, we see from the IQR values that for  $\beta$  2SLS values are often smaller than those reported for ML. In particular in the case  $\sigma = 2$ , 2SLS is doing well. IQR values for OLS are also smaller in the case  $\sigma = 2$  than in the case  $\sigma = 0.25$  but to a lesser degree than the decrease in IQR values for 2SLS. For both  $\gamma^{(1)}$  and  $\gamma^{(2)}$  the IQR values reported for OLS and 2SLS are much smaller than those for ML in almost all cases. As the signal-noise ratio increases the sample dispersion of these two estimators decreases. On the other hand, the IQR values of the ML estimator of  $\gamma^{(1)}$  and  $\gamma^{(2)}$  are much larger than those reported for OLS and 2SLS. Thus, the favourable bias in median results for the ML estimator do not show up so clearly in terms of IQR. Again, the IQR values of  $r$  do not seem to support one single best estimation method. In most cases all three estimators perform the same. Only when  $\alpha = 0.8$ , OLS is doing better than 2SLS and ML. Finally, for  $\beta$  the 2SLS values of  $\text{Prob}(|\hat{\beta} - \beta| < 0.05)$  are often much higher than those reported for ML and OLS. For  $\gamma^{(1)}$  and  $\gamma^{(2)}$  ML is better than the other two estimators, though still very poor. Note, that for  $r$  all estimators behave poorly.

As compared with the results in Subsection 4.1, it is obvious that the performance of the three estimators for estimating  $\beta$ ,  $\gamma^{(1)}$  and  $\gamma^{(2)}$  is less good than for the restricted LTVECM. For the unrestricted LTVECM, the ML results are rather mixed. ML has the smallest bias in median. On the other hand, OLS and 2SLS have much smaller sample dispersion than ML, as measured by IQR. Thus, given a choice for one of these two accuracy measures, there seems to

be a trade-off between the ML and OLS or 2SLS method for estimating threshold cointegrated parameters. Similar results were observed for  $T = 250$ . More extensive tables are available upon request.

## 5 Conclusions

The objective of the paper was compare the finite sample performance of three estimators in estimating threshold cointegrated parameters in LTVECMs. We find that ML is to be preferred over OLS and 2SLS if prior knowledge about the threshold parameter  $r$  and/or the threshold cointegration parameter  $\beta$  is available. However, when no parameter restrictions can be imposed on the LTVECM, we showed that both OLS and 2SLS estimators remain good alternatives to ML. Future work needs to be directed toward developing better methods for estimating the threshold parameter  $r$ . In addition, research needs to be devoted toward developing a distribution theory for *all* parameters of LTVECMs.

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Table 1: Characteristics of the empirical distribution of OLS, 2SLS and ML estimators for (5) with the true value  $r = 0$  known;  $T = 400, 2000$  replications.

$\theta$	$\alpha$	$\sigma$	$\beta$			$\gamma^{(1)}$			$\gamma^{(2)}$		
			OLS	2SLS	ML	OLS	2SLS	ML	OLS	2SLS	ML
Bias in median											
-0.5	0.2	0.25	0.01	-0.02	0.01	-0.55	-0.54	-0.16	0.50	0.51	0.12
-0.5	0.2	2	0.00	-0.00	0.00	-0.46	-0.45	-0.23	0.44	0.45	0.03
0	0.2	0.25	-0.00	-0.00	-0.00	-0.53	-0.52	-0.16	0.52	0.52	0.15
0	0.2	2	0.00	0.00	0.00	-0.45	-0.45	-0.11	0.45	0.45	0.10
-0.5	0.8	0.25	0.02	0.00	0.02	-0.20	-0.16	-0.08	0.16	0.17	0.07
-0.5	0.8	2	-0.13	0.00	-0.01	-0.17	-0.16	-0.00	0.16	0.17	0.01
0	0.8	0.25	-0.00	0.00	-0.00	-0.18	-0.16	-0.07	0.18	0.17	0.08
0	0.8	2	-0.01	0.00	-0.00	-0.17	-0.16	-0.01	0.17	0.17	0.01
IQR <sub>(50)</sub>											
-0.5	0.2	0.25	0.12	0.10	0.13	0.30	0.40	0.31	0.52	0.55	0.33
-0.5	0.2	2	0.01	0.01	0.01	0.02	0.01	0.30	0.01	0.02	0.18
0	0.2	0.25	0.13	0.07	0.14	0.36	0.43	0.34	0.35	0.40	0.35
0	0.2	2	0.02	0.01	0.02	0.01	0.01	0.34	0.01	0.02	0.34
-0.5	0.8	0.25	0.13	0.04	0.12	0.06	0.11	0.15	0.07	0.06	0.16
-0.5	0.8	2	0.09	0.01	0.02	0.00	0.02	0.08	0.00	0.01	0.08
0	0.8	0.25	0.14	0.04	0.13	0.06	0.09	0.16	0.06	0.06	0.17
0	0.8	2	0.29	0.01	0.04	0.00	0.02	0.08	0.00	0.01	0.08
Prob( est. par.-true par.  < 0.05)											
-0.5	0.2	0.25	0.45	0.55	0.45	0.03	0.06	0.13	0.05	0.06	0.14
-0.5	0.2	2	0.96	0.98	0.96	0.00	0.00	0.12	0.00	0.00	0.30
0	0.2	0.25	0.43	0.58	0.43	0.04	0.06	0.10	0.04	0.06	0.11
0	0.2	2	0.96	0.97	0.95	0.00	0.00	0.19	0.00	0.00	0.19
-0.5	0.8	0.25	0.40	0.74	0.43	0.06	0.14	0.28	0.09	0.10	0.31
-0.5	0.8	2	0.07	0.98	0.82	0.00	0.01	0.59	0.00	0.00	0.58
0	0.8	0.25	0.33	0.75	0.37	0.07	0.15	0.29	0.07	0.11	0.27
0	0.8	2	0.04	0.98	0.81	0.00	0.01	0.57	0.00	0.00	0.58

Table 2: Characteristics of the empirical distribution of OLS, 2SLS and ML estimators for the DGP (5);  $T = 400$ , 2000 replications.

$\theta$	$\alpha$	$\sigma$	$\beta$			$r$			$\gamma^{(1)}$			$\gamma^{(2)}$		
			OLS	2SLS	ML	OLS	2SLS	ML	OLS	2SLS	ML	OLS	2SLS	ML
Bias in median														
-0.5	0.2	0.25	0.00	-0.02	0.01	0.26	0.44	0.21	-0.56	-0.54	-0.34	0.50	0.51	0.27
-0.5	0.2	2	0.01	-0.00	0.01	-0.47	0.11	-0.41	-0.46	-0.46	-0.18	0.45	0.46	0.12
0	0.2	0.25	-0.00	-0.00	-0.00	0.09	-0.07	0.09	-0.54	-0.53	-0.34	0.53	0.53	0.31
0	0.2	2	0.00	0.00	0.00	0.02	0.01	0.03	-0.46	-0.46	-0.17	0.46	0.46	0.16
-0.5	0.8	0.25	0.00	0.00	0.01	0.31	0.56	0.43	-0.24	-0.26	-0.18	0.19	0.23	0.08
-0.5	0.8	2	-0.14	0.00	-0.01	0.48	0.93	0.33	-0.17	-0.17	-0.11	0.17	0.17	0.09
0	0.8	0.25	-0.01	0.00	-0.00	-0.04	0.27	0.02	-0.20	-0.23	-0.14	0.21	0.24	0.12
0	0.8	2	-0.00	0.00	-0.00	-0.07	0.35	-0.03	-0.17	-0.17	-0.10	0.17	0.17	0.09
IQR <sub>(50)</sub>														
-0.5	0.2	0.25	0.10	0.10	0.01	2.69	2.71	2.66	0.30	0.32	0.47	0.57	0.53	0.52
-0.5	0.2	2	0.02	0.01	0.02	1.37	3.00	1.48	0.03	0.04	0.36	0.04	0.05	0.36
0	0.2	0.25	0.12	0.07	0.12	2.73	2.71	2.70	0.36	0.39	0.50	0.43	0.41	0.52
0	0.2	2	0.02	0.01	0.02	2.52	2.91	2.26	0.04	0.04	0.44	0.04	0.05	0.43
-0.5	0.8	0.25	0.09	0.04	0.09	1.99	2.46	2.02	0.23	0.22	0.43	0.17	0.21	0.36
-0.5	0.8	2	0.16	0.01	0.03	2.30	6.47	7.00	0.01	0.03	0.50	0.01	0.02	0.37
0	0.8	0.25	0.11	0.04	0.10	1.96	2.53	2.03	0.21	0.23	0.44	0.19	0.21	0.40
0	0.8	2	0.28	0.01	0.03	2.21	7.58	7.66	0.01	0.03	0.48	0.01	0.03	0.48
Prob( est. par.-true par.  < 0.05)														
-0.5	0.2	0.25	0.49	0.55	0.48	0.01	0.01	0.02	0.03	0.03	0.07	0.05	0.04	0.09
-0.5	0.2	2	0.96	0.98	0.93	0.03	0.06	0.03	0.00	0.00	0.13	0.01	0.01	0.18
0	0.2	0.25	0.47	0.58	0.46	0.01	0.01	0.02	0.03	0.03	0.06	0.04	0.03	0.07
0	0.2	2	0.96	0.97	0.94	0.04	0.08	0.04	0.00	0.00	0.15	0.00	0.01	0.15
-0.5	0.8	0.25	0.51	0.74	0.54	0.02	0.01	0.02	0.08	0.07	0.18	0.13	0.08	0.25
-0.5	0.8	2	0.21	0.98	0.84	0.03	0.98	0.01	0.00	0.00	0.30	0.00	0.00	0.30
0	0.8	0.25	0.31	0.75	0.50	0.35	0.01	0.02	0.07	0.08	0.20	0.07	0.09	0.19
0	0.8	2	0.27	0.98	0.83	0.02	0.01	0.01	0.00	0.00	0.31	0.00	0.00	0.29