E C O N O M I C S   B U L L E T I N

Asymmetric adjustment of the equilibrium relationship between the nominal interest rate and inflation rate

Daiki Maki
Graduate School of Economics, Doshisha University

Abstract

This paper investigates the equilibrium relationship between the nominal interest rate and inflation rate in Japan using a threshold cointegration test, which allows for asymmetric adjustment. While the Engle−Granger method assuming symmetric adjustment cannot obtain the result of cointegration, a threshold cointegration approach provides clear evidence of the cointegration relationship characterized by asymmetric adjustment toward equilibrium. This shows that the long−run equilibrium relationship between the nominal interest rate and inflation rate is stable with asymmetric adjustment.

The author would like to thank Shigenori Shiratuka, Shin−ichi Kitasaka, and Yoshitomo Kiyokawa for helpful comments and discussions.
Citation: Maki, Daiki, (2005) "Asymmetric adjustment of the equilibrium relationship between the nominal interest rate and inflation rate." *Economics Bulletin,* Vol. 3, No. 9 pp. 1−8
Submitted: February 23, 2005. **Accepted:** February 23, 2005.
URL: http://www.economicsbulletin.com/2005/volume3/EB−05C20006A.pdf

1. Introduction

The long-run equilibrium relationship between the nominal interest rate and expected inflation rate, represented by the Fisher equation, has been discussed by unit root and cointegration tests. For example, as a recent study, Rapach and Weber (2004) has supported the results of Rose (1988), who concluded that the real interest rate was nonstationary (the equilibrium relationship did not exist), using a more powerful GLS approach for unit root tests (Ng and Perron, 2001) and cointegration tests (Perron and Rodriguez, 2001). The results shown by Rapach and Weber (2004) mean that the equilibrium relationship does not exist in symmetric adjustment because representative unit root and cointegration tests including the GLS approach assume only symmetric adjustment.

However, there is no reason to pre-suppose that the adjustment process toward equilibrium is symmetric. As shown by Ramsey and Rothman (1996), economic variables such as nominal interest rates and inflation have asymmetric adjustment. In addition, Murchison and Siklos (1999) found asymmetries in interest rate changes to inflation shocks. Moreover, as pointed out by Balke and Fomby (1997), the power of representative cointegration tests fall under an asymmetric adjustment process. Therefore, it is possible that asymmetric adjustment leads to poor results of the equilibrium relationship because conventional cointegration tests do not take into account asymmetric adjustment.

The purpose of this paper is to investigate the equilibrium relationship between the nominal interest rate and inflation rate in Japan using the threshold cointegration approach, which allows for asymmetric adjustment, introduced by Enders and Siklos (2001). We show that while Engle-Granger's test assuming only symmetric adjustment does not obtain the results of the cointegraion relationship, the threshold cointegration test provides clear evidence of the equilibrium relationship with asymmetric adjustment.

The organization of this paper is as follows. Section 2 outlines the threshold cointegration test along with the Fisher equation. Section 3 presents the data and compares the results. Section 4 provides a summary of the paper.

2. The Fisher equation and threshold cointegration test

The equilibrium relationship between the nominal interest rate and expected inflation rate is defined by the Fisher equation:

$$i_t = r_t^e + \pi_t^e, \tag{1}$$

where i_t is the nominal interest rate on period t, r_t^e is the *ex ante* real interest rate, and π_t^e is the *ex ante* expected inflation rate. It is assumed that the *ex ante* expected inflation rate is the sum of the actual inflation rate π_t and a stationary error u_t (see Rose, 1988):

$$\pi_t^e = \pi_t + u_t. \tag{2}$$

Therefore, (1) is expressed as follows:

$$i_t = r_t^e + \pi_t + u_t. \tag{3}$$

Although i_t and π_t have to be cointegrated with vector $\beta' = [1, -1]$ in order to interpret the equation accurately when i_t and π_t are nonstationary, the absence of such a proportional relationship does not necessarily invalidate the control and prediction of the future inflation rate (e.g., Moosa and Kwiecien, 2002). If the relationship is stable, that is, cointegrated with symmetric or asymmetric adjustment, the control and prediction of the inflation rate are possible. Therefore, it is required that we test for cointegration with unspecified vector.

This paper specifically employs the threshold cointegration approach introduced by Enders and Siklos (2001) to test for a cointegration relationship with asymmetric adjustment. As the assumption of tests for threshold cointegration, let $\{x_{it}\}_1^T$ denote observable random variables integrated of order one, denoted by $I(1)$. The long-run equilibrium relationship is given by:

$$x_{1t} = \hat{\beta}_0 + \hat{\beta}_2 x_{2t} + \ldots + \hat{\beta}_n x_{nt} + e_t, \tag{4}$$

where $\hat{\beta}_i$ are estimated parameters, and e_t is the disturbance term. The existence of the long-run equilibrium relationship involves stationary e_t. To accept stationarity of e_t, we have to obtain $-2 < \rho < 0$ in the second step procedure given by:

$$\Delta e_t = \rho e_{t-1} + \epsilon_t, \tag{5}$$

where ϵ_t is the white-noise disturbance. If $-2 < \rho < 0$, the long-run equilibrium relationship (4) characterized by symmetric adjustment (5) is accepted.

However, the standard cointegration framework in (5) is misspecified if the adjustment process is asymmetric. Therefore, Enders and Siklos (2001) proposed the following asymmetric adjustment, called the threshold autoregressive (TAR) model:

$$\Delta e_t = I_t \rho_1 e_{t-1} + (1 - I_t)\rho_2 e_{t-1} + \epsilon_t, \tag{6}$$

where I_t is the indicator function such that

$$I_t = \begin{cases} 1 & \text{if } e_{t-1} \geq \tau \\ 0 & \text{if } e_{t-1} < \tau, \end{cases} \tag{7}$$

and τ is the threshold value. As an alternative adjustment process, the momentum threshold (MTAR) model is as follows:

$$\Delta e_t = M_t \rho_1 e_{t-1} + (1 - M_t)\rho_2 e_{t-1} + \epsilon_t, \tag{8}$$

$$M_t = \begin{cases} 1 & \text{if } \Delta e_{t-1} \geq \tau \\ 0 & \text{if } \Delta e_{t-1} < \tau, \end{cases} \tag{9}$$

The MTAR can capture the properties such that the threshold depends on the previous period's change in e_t. When the adjustment process (6) and (8) are serially correlated, (6) and (8) are re-written [1]:

$$\Delta e_t = I_t \rho_1 e_{t-1} + (1 - I_t)\rho_2 e_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta e_{t-i} + \epsilon_t, \tag{10}$$

$$\Delta e_t = M_t \rho_1 e_{t-1} + (1 - M_t)\rho_2 e_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta e_{t-i} + \epsilon_t. \tag{11}$$

To test for threshold cointegration, Enders and Siklos (2001) proposed two types of tests, called the Φ and *t-Max* statistics. The Φ statistic using a F statistic involves procedure testing for the null hypothesis $\rho_1 = \rho_2 = 0$, and the *t-Max* statistic employing a t statistic requires the test for the null hypothesis with the largest $\rho_i = 0$ between ρ_1 and ρ_2. The threshold parameter τ, which is restricted to the ranges of the remaining 70% of e_t or Δe_t when the largest and smallest 15% values are discarded, is selected as an unknown value so as to minimize the sum of the squared residuals obtained from (6) and (8) [2]. If the null hypothesis of no cointegration is rejected, we can test for the null hypothesis $\rho_1 = \rho_2$ by a standard F statistic because the system is stationary.

3. The data and empirical results

In this paper, as the nominal interest rate, we used the call rate, which is a primary indicator of monetary policy in Japan. As the expected inflation rate, we employed

[1] Of course, although it is possible that γ_i is asymmetric, we do not consider this case for the sake of simplicity, similar to Enders and Siklos (2001).

[2] Although Enders and Siklos (2001) proposed tests for when the threshold parameter is known ($\tau = 0$) and unknown, we employ an unknown threshold parameter because we do not have an *a priori* reason to believe that τ is known.

the actual inflation rate using the consumer price index (CPI) [3]. The quarterly data obtained from the International Monetary Fund's International Financial Statistics CD-ROM consisted of 156 periods from 1963:2 to 2002:1.

Table 1 displays the results of unit root tests for the null hypothesis $I(1)$. ADF^{GLS} is Ng and Perron's (2001) test. We chose the lag order employing the data dependent method, denoted by *t-sig*, to control for possible size distortions for ADF-type tests, proposed by Ng and Perron (1995) [4]. We set the maximum lag $k_{max} = 8$. BR is Breitung's (2002) nonparametric test using a variance ratio as a test statistic. The BR test allows for a general mean-reverting process under the alternative hypothesis. Each test includes a constant. Since neither level variable is significant even at a 10% level, the results show that two variables have a unit root process.

Table 1: Unit root tests

	ADF^{GLS}		BR	
	Levels	First differences	Levels	First differences
IFR	-1.548	-2.780**	0.03052	0.0001**
NIR	-1.168	-5.090**	0.05745	0.0006**

IFR and NIR denote the inflation rate and nominal interest rate, respectively.
(†) Significant at a 10% level. (*) Significant at a 5% level. (**) Significant at a 1% level.

Tables 2 and 3 illustrate the results of cointegration tests. Each cointegration test includes only a constant as a deterministic component since the theory of the Fisher equation does not support a trend. For the estimation of e_t, we chose the lag order by *t-sig* [5]. In Tables 2 and 3, E-G and P-R denote Engle and Granger's (1987) and Perron and Rodriguez's (2001) methods assuming only symmetric adjustment,

[3] Rose (1988) and Rapach and Weber (2004) also used the actual inflation rate as the expected inflation rate. More accurately, following their studies, we employed $400 * \{ln(CPI_t) - ln(CPI_{t-1})\}$ as the actual inflation rate on period t.

[4] *t-sig* selects the lag order k via top-down testing. To begin with, we estimate the equation with the maximum lag (here, the maximum lag k_{max}=8). We use the lag order if the t-statistic of the parameter of the maximum lag is significant. If the t-statistic is not significant, we estimate the equation with the lag=$k_{max} - 1$. That is, when the t-statistic of the parameter of the lag=$k_{max} - q$ is significant at a conventional level, we employ the lag order.

[5] We also tested for cointegration using AIC (Akaike Information Criterion, for GLS, Modified AIC). The lag length of Engle-Granger methods was equal between AIC and *t-sig*. For the threshold model, the results of AIC selected shorter lags than *t-sig*.

respectively. The results show that the cointegration relationship by symmetric adjustment is not obtained in both cases, i.e., whether the dependent variable is the nominal interest rate or the inflation rate.

Table 2: Cointegration tests (dependent variable: nominal interest rate)

	E-G	P-R	TAR	MTAR
ρ_1	-0.1239(-1.904)	-0.1057(-1.623)	-0.4430(-4.089)	-0.1092(-1.655†)
ρ_2	NA	NA	-0.1098(-1.641†)	-0.4776(-4.280)
Φ	NA	NA	8.673*	9.452**
F	NA	NA	8.459**	9.929**

Parentheses show t statistics. For TAR and MTAR, we employ *t-Max* statistics.
Φ denotes the tests for the null hypothesis $\rho_1 = \rho_2 = 0$. F shows the tests for symmetry $\rho_1 = \rho_2$.
(†) Significant at a 10% level. (*) Significant at a 5% level. (**) Significant at a 1% level.
Estimated threshold values τ are 2.708 and -1.877 for TAR and MTAR, respectively.

Table 3: Cointegration tests (dependent variable: inflation rate)

	E-G	P-R	TAR	MTAR
ρ_1	-0.2641(-2.309)	-0.2612(-2.294)	-0.3185(-2.745**)	-0.2159(-1.659†)
ρ_2	NA	NA	-0.5812(-3.613)	-0.5489(-4.241)
Φ	NA	NA	7.571*	8.994**
F	NA	NA	2.795†	5.417*

Parentheses show t statistics. For TAR and MTAR, we employ *t-Max* statistics.
Φ denotes the tests for the null hypothesis $\rho_1 = \rho_2 = 0$. F shows the tests for symmetry $\rho_1 = \rho_2$.
(†) Significant at a 10% level. (*) Significant at a 5% level. (**) Significant at a 1% level.
Estimated threshold values τ are -2.765 and 3.148 for TAR and MTAR, respectively.

In contrast, the results of threshold cointegration using the Φ statistic provide clear evidence of cointegration at the 5% and 1% critical value in TAR and MTAR, respectively. Symmetry is rejected at the 1%, 5%, or 10% significance level. Table 2 shows that the adjustment process toward equilibrium below the estimated threshold is persistent in TAR, and that there is rapid convergence in MTAR below the estimated threshold. In addition, the estimates of Table 3 suggest that the adjustment process toward equilibrium is persistent above the appropriately estimated threshold. The results mean that the long-run equilibrium relationship between the nominal interest rate and inflation rate is stable with asymmetric adjustment, and imply asymmetries in nominal interest rate changes to inflation shocks or inflation changes to nominal interest rate shocks.

4. Summary

This paper has investigated the equilibrium relationship between the nominal interest rate and inflation rate in Japan using the threshold cointegration method, which allows for asymmetric adjustment. The results have shown that the approach provides clear evidence of the equilibrium relationship with asymmetric adjustment, compared with standard methods assuming only symmetric adjustment: The long-run equilibrium relationship between the nominal interest rate and inflation rate is stable with asymmetric adjustment. This finding is important for monetary policies to control and predict the inflation rate. We plan to undertake a further empirical study of such monetary policies for the recent Japanese economy characterized by zero interest rate and deflation.

References

Balke, N. S., and T. B. Fomby (1997) "Threshold cointegration" *International Economic Review* **38**, 627-643.

Breitung, J. (2002) "Nonparametric tests for unit roots and cointegration" *Journal of Econometrics* **10**, 343-363.

Enders, W., and P. L. Siklos (2001) "Cointegration and threshold adjustment" *Journal of Business and Economic Statistics* **19**, 166-176.

Engle, R. F., and C. W. J. Granger (1987) "Cointegration and error correction: representation, estimation, and testing" *Econometrica* **55**, 251-276.

Moosa, I. A., and J. Kwiecien (2002) "Cross-country evidence on the ability of the nominal interest rate to predict inflation" *Japanese Economic Review* **53**, 478-495.

Murchison, S., and P. L. Siklos (1999) "Central bank reaction functions in inflation and non-inflation targeting economies" Working paper, Wilfred Laurier University.

Ng, S., and P. Perron (1995) "Unit root tests in ARMA models with data-dependent methods for the selection of the truncation lag" *Journal of the American Statistical Association* **90**, 268-281.

Ng, S., and P. Perron (2001) "Lag length selection and the construction of unit root tests with good size and power" *Econometrica* **69**, 1519-1554.

Perron, P. and G. Rodriguez (2001) "Residual based tests for cointegration with GLS detrended data" Manuscript, Boston University.

Ramsey, J. B., and P. Rothman (1996) "Time irreversibility and business cycle asymmetry" *Journal of Money, Credit and Banking* **28**, 1-21.

Rapach, D. E., and C. E. Weber (2004) "Are real interest rate really nonstationary? New evidence from tests with good size and power" *Journal of Macroeconomics* **26**, 409-430.

Rose, A. K. (1988) "Is the real interest rate stable?" *Journal of Finance* **43**, 1095-1112.