E C O N O M I C S B U L L E T I N

Alternate contracts for side payments

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Abstract

We characterize efficient equilibrium outcomes of two−player games that remain equilibrium outcomes even when the two players may alternately make binding offers of strategy contingent side payments before the game is played. Our characterization result implies that alternately contracting for side payments has more efficiency of a certain type in equilibria than simultaneously side contracting which is analyzed by Jackson and Wilkie (2005).

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1. Introduction

Jackson and Wilkie (2005) explored two-stage games when players may *simultaneously* make binding offers of strategy contingent side payments before choosing actions. Among their various findings, there is a fact for two-player games that given a profile of efficient actions which constitutes an equilibrium in the underlying game (the second stage game without side contracts), the equilibrium payoff distribution under the actions remains an equilibrium outcome even in the two-stage game if and only if each player's payoff in the distribution is no less than what is called his *solo payoff*.

We study two-player three-stage games where players may *alternately* make binding offers of strategy contingent side payments in the first and second stages before choosing actions in the final stage. It is shown in our analysis that given a profile of efficient actions which constitutes an equilibrium in the underlying game (the third stage game without side contracts), the equilibrium payoff distribution under the actions remains an equilibrium outcome even in the three-stage game if and only if *the second transfer-offerer's payoff* in the distribution is no less than his solo payoff, no matter how much payoff his counterpart enjoys in the distribution.

Jackson and Wilkie (2005) also discussed about timing problems, and doubted that players' alternating in announcing their transfer schemes would generally improve efficiency.[1] Indeed their assertion might be true, but at the same time our characterization result tells that alternately side contracting might have more efficient actions of a certain type in equilibria than simultaneous side contracting.

In what follows we present the model in Section 2 and the analysis in Section 3. Our concluding remarks appear in Section 4.

2. Model

We consider two-player three-stage games played as follows.

[1] Jackson and Wilkie (2005) concluded: "Thus, in order for timing to really be an issue it must either be that some players are restricted not to be able to respond to the contracts of others or else there must be some frictions in timing, for instance in the form of time discounting and some time or effort cost to writing contracts. But note that neither of these situations should generally improve efficiency, and in some cases might harm it" (p. 561).

Stage 1: Player 1 announces a transfer function (transfer scheme), which is assumed to be binding.
Stage 2: Player 2 announces a transfer function, which is assumed to be binding.
Stage 3: Each player chooses an action.

The players *alternately* make side contracts in the first and second stages.

Let i denote any given one of the two players. When a player is denoted by i, let j denote the other player. A player i's finite pure strategy space in the third stage game is denoted by X_i, with $X = X_1 \times X_2$. Let $\Delta(X_i)$ denote the set of mixed strategies for i, and let $\Delta = \Delta(X_1) \times \Delta(X_2)$. We denote by x_i, x, μ_i, and μ generic elements of X_i, X, $\Delta(X_i)$, and Δ respectively. For simplicity, we sometimes use x_i and x to denote μ_i and μ respectively that place probability one on x_i and x. A player i's payoffs in the third stage game are given by a von Neumann-Morgenstern utility function $v_i : X \to \mathbb{R}$.

A transfer function announced by player i in the first or second stage is denoted by t_i, where $t_i : X \to \mathbb{R}_+$ represents i's *nonnegative* promises to j as a function of actions chosen in the third stage. Let T be the universal set of t_i. T contains i's degenerate transfer function $t_i^0 : X \to \{0\}$. Let $t = (t_1, t_2)$.

Given a profile t of transfer functions in the first and second stages, and a play x in the third stage game, the payoff U_i to player i becomes

$$U_i(x,t) = v_i(x) + t_j(x) - t_i(x).$$

It is assumed here that each player could not reject the other's offer of side payments. Thus, the players' contracts for side payments are *unilateral*.[2]

Given a profile t of transfer functions in the first and second stages, and a play μ in the third stage game, the expected payoff EU_i to player i becomes

$$EU_i(\mu,t) = \sum_x \mu_1(x_1) \cdot \mu_2(x_2) \cdot (v_i(x) + t_j(x) - t_i(x)).$$

Let $NE(t)$ denote the set of (mixed) Nash equilibria of the third stage game given t in the first and second stages. Let NE represent the set of (mixed) Nash equilibria of the underlying game (the third stage game without side contracts).

A pure strategy profile $x \in X$ of the third stage game together with a vector $\overline{u} = (\overline{u}_1, \overline{u}_2) \in \mathbb{R}^2$ of payoffs such that $\overline{u}_1 + \overline{u}_2 = v_1(x) + v_2(x)$

[2]For the case when each player could reject the others' offers of side payments, see Yamada (2003).

is *supportable* if there exists a subgame perfect equilibrium of the three-stage game where some t is announced in the first and second stages and x is played in the third stage on the equilibrium path, and $U_i(x,t) = \overline{u}_i$ for each i. A pure equilibrium strategy profile $x \in NE$ of the underlying game is *surviving* in the three-stage game if $(x, v(x))$ is supportable, where $v(x) = (v_1(x), v_2(x))$.

3. Analysis

Let $u_i(t_j) = \sup_{t_i \in T} \left[\min_{\mu \in NE(t_j,t_i)} EU_i(\mu, t_j, t_i) \right]$. $u_i\left(t_j^0\right)$ is particularly called i's *solo payoff.* We obtain the following two results, which characterize efficient equilibrium actions of the underlying game that survive in the three-stage game.

Theorem 1. *$x \in NE$ is surviving only if $v_2(x) \geq u_2(t_1^0)$.*

Proof of Theorem 1. When x is surviving, there exists a subgame perfect equilibrium of the three-stage game where some $t = (t_1, t_2)$ is announced in the first and second stages and x is played in the third stage on the equilibrium path, and $U_i(x,t) = v_i(x)$ for each i. Suppose to the contrary that $v_2(x) < u_2(t_1^0)$, namely $v_2(x) = u_2(t_1^0) - \delta$ for some $\delta > 0$. Let $t_2' \in \left\{ t_2 \in T : \min_{\mu \in NE\left(t_1^0, t_2\right)} EU_2(\mu, t_1^0, t_2) > u_2(t_1^0) - \delta \right\}$. Let $\widehat{t_2} = t_1 + t_2'$. Then, since $NE\left(t_1, \widehat{t_2}\right) = NE(t_1^0, t_2')$, $\min_{\mu \in NE\left(t_1, \widehat{t_2}\right)} EU_2\left(\mu, t_1, \widehat{t_2}\right) = \min_{\mu \in NE\left(t_1^0, t_2'\right)} EU_2(\mu, t_1^0, t_2') > u_2(t_1^0) - \delta = v_2(x)$. That is, Player 2 has an incentive to deviate from t in the second stage. A contradiction. Thus, $v_2(x) \geq u_2(t_1^0)$. ■

Theorem 2. *Let $x \in NE$ such that $v_1(x) + v_2(x) \geq v_1(x') + v_2(x')$ for any $x' \in X$. Then, x is surviving if $v_2(x) \geq u_2(t_1^0)$.*

Proof of Theorem 2. Given t_1, let $t_2(t_1) \in \arg\max_{t_2 \in T} \left[\max_{\mu \in NE(t_1,t_2)} EU_2(\mu, t_1, t_2) \right]$.
Consider the following strategy profile.
Stage 1: Player 1 announces $t_1 = t_1^0$.
Stage 2: If $t_1 = t_1^0$, then Player 2 announces $t_2 = t_2^0$. If $t_1 \neq t_1^0$, then $t_2 = t_2(t_1)$.

Stage 3: If $t = t^0$, then x is chosen. If $t_1 = t_1^0$ and $t_2 \neq t_2^0$, then some $\mu \in \arg \min_{\mu \in NE(t_1,t_2)} EU_2(\mu, t_1, t_2)$ is chosen. If $t_1 \neq t_1^0$, then $\mu \in \arg \max_{\mu \in NE(t_1,t_2)} EU_2(\mu, t_1, t_2)$ is chosen.

What is chosen in the third stage is an element of $NE(t)$.

As for the second stage, if Player 2 announces $t_2 \neq t_2^0$ when $t_1 = t_1^0$, then his expected payoff is no more than $u_2(t_1^0) \leq v_2(x)$. Hence $t_2 = t_2^0$ is the best response to $t_1 = t_1^0$. Clearly, $t_2 = t_2(t_1)$ is the best reply to $t_1 \neq t_1^0$ by the construction of the third stage strategy profile.

As for the first stage, if Player 1 announces $t_1 \neq t_1^0$, then Player 2 chooses $t_2 = t_2(t_1)$ in the second stage so that Player 2's expected payoff becomes $\max_{\mu \in NE(t_1,t_2(t_1))} EU_2(\mu, t_1, t_2(t_1))$, which must be no less than $v_2(x)$ since Player 2 prefers $t_2 = t_2(t_1)$ where he can choose $t_2 = t_1$ to obtain $\max_{\mu \in NE(t_1,t_2)} EU_2(\mu, t_1, t_2) = \max_{\mu \in NE\left(t_1^0,t_2^0\right)} EU_2(\mu, t_1^0, t_2^0) \geq v_2(x)$. Then, Player 1's expected payoff is no more than $v_1(x)$ since $v_1(x) + v_2(x) \geq v_1(x') + v_2(x')$ for any $x' \in X$. Therefore, Player 1 has no incentive to choose $t_1 \neq t_1^0$ in the first stage.

Thus, the strategy profile constitutes a subgame perfect equilibrium of the three-stage game, where t^0 is announced in the first and second stages and x is played in the third stage on the equilibrium path, and $U_i(x,t) = v_i(x)$ for each i. ■

Corollary 1. *Let $x \in NE$ such that $v_1(x) + v_2(x) \geq v_1(x') + v_2(x')$ for any $x' \in X$. Then, x is surviving if and only if $v_2(x) \geq u_2(t_1^0)$.*

Remark 1. Corollary 1 corresponds to Theorem 1 in Jackson and Wilkie (2005) that is for the case when players may simultaneously make side contracts before choosing actions. In contrast to that theorem, our characterization does not need any condition for Player 1 like $v_1(x) \geq u_1(t_2^0)$. In our three-stage game, Player 2 decides his transfer scheme after knowing what transfer Player 1 promised in the previous stage. Player 2 is even able to propose a transfer scheme which cancels Player 1's offer. Thus, Player 2 can arbitrarily affect the payoff structure of the third stage game by his transfer, no matter what transfer the other promises. Therefore, Player 1's deviation from t_1^0 in the first stage would not reduce Player 2's payoff. Since the deviation only maintains or destroys efficiency, its not reducing Player 2's payoff implies that Player 1 cannot enjoy any additional benefit by the deviation.

This is why the characterization is carried out only by a condition for Player 2.

Remark 2. According to Theorem 1 of Jackson and Wilkie (2005) and our Corollary 1, the set of efficient equilibrium actions of two-player games that survive in the three-stage games with alternate contracts for side payments, includes the set of efficient equilibrium actions that survive in the two-stage games with simultaneous side contracts. This implies that alternately contracting for side payments has more efficient actions of a certain type in equilibria than simultaneous side contracting. This induces us to withhold our full consent to the discussion by Jackson and Wilkie (2005), as mentioned in Introduction.

4. Conclusion

We characterize efficient equilibrium outcomes of two-player games that remain equilibrium outcomes even when the two players may alternately make binding offers of strategy contingent side payments before the game is played. Our characterization result implies that alternate contracts for side payments have more efficiency of a certain kind in equilibria than simultaneously contracting.

To make it clear whether the implication of our result holds more generally, we would try next to characterize efficient outcomes of two-player games that may not be equilibrium ones in the underlying game but are realized in equilibria when such alternate side contracts are allowed.

5. Reference

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