E C O N O M I C S   B U L L E T I N

Opting Out in a War of Attrition

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Abstract

This paper analyzes a War of Attrition where players enjoy private information about their outside opportunities. The main message is that uncertainty about the possibility that the opponent opts out increases the equilibrium probability of concession.

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1. Introduction

The aim of this paper is to study the role played by outside options in negotiations when there is incomplete information about their existence. For this purpose we focus our analysis on the War of Attrition since this is the simplest model of conflict that yields inefficient equilibria under conditions of complete information. It is well known that, in a symmetric War of Attrition without outside options, the unique symmetric equilibrium consists of players randomizing at a constant probability between conceding and not conceding, a very inefficient outcome indeed[1]. We show that the presence of uncertain outside options increases the equilibrium probability of concession.

The relevance of outside opportunities available to the players on the outcome of a negotiation has been well established in models of bargaining with complete information (Shaked and Sutton (1984), Binmore et al. (1986), Shaked (1987), and Ponsati and Sákovics (1998)). Considering uncertainty about outside options is a natural extension of this literature that deserves attention[2]. In this paper we present a model where players enjoy private information about their possibilities of opting out, but they do not know their opponent's opportunities.

We show that introducing the possibility of opting out in a War of Attrition has a significant effect on the outcomes. We find that, on the one hand, if the probability of facing an opponent who has an outside option is sufficiently high, in equilibrium, the negotiation will surely end at some future date. On the other hand, if the likelihood of that kind of opponent is low, types that have outside opportunities eventually opt out with probability 1, leaving types with no outside options to play, from that time on, the symmetric inefficient equilibrium of the complete information War of Attrition. Even in this case, the probability of concession increases in the uncertainty phase of the equilibrium play.

The following section presents the model and characterizes equilibria of this game. Conclusions are presented in the last section.

2. The model

Two players bargain about how to share one unit of surplus that will be available only when they reach an agreement. An agreement is denoted by x, where x indicates the portion of the surplus assigned to player 1. There are only two possible agreements; either $x = 1 - a$ or $x = a$ with $0 < a < \frac{1}{2}$.

[1]See Hendricks, K., A. Weiss and C. Wilson (1988).
[2]See Vislie (1988) and Ponsatí and Sákovics (2001).

Players may also decide to break the negotiation by opting out, in which case, they receive a payoff b_i $i = 1, 2$.

The game is played in discrete time, starting at $t = 0$. At each time (a stage), both players decide simultaneously either: (i) to propose her preferred agreement, or (ii) to concede by proposing her opponent's favorite agreement or (iii) to leave the negotiation and opt out. The game ends whenever a player or both, at the same time, concedes or opts out. Otherwise, disagreement occurs, discounting applies and the game proceeds to a new stage.

Players are assumed to be risk neutral and impatient. Their impatience is modeled by a common discount factor, normalized to be δ per unit of time. And the payoffs are as follows: if players perpetually disagree, they both receive zero payoff. If only player i concedes at time t, then player i gets $a\delta^t$ and player j gets $(1-a)\delta^t$. If both players concede at the same time[3]each player gets $a\delta^t$. And if either or both players opt out, payoffs are $b_i\delta^t$ for $i = 1, 2$.

Each player i has private information about the value of her outside opportunity, which can be either $b_i = 0$ or $b_i = b$, $a < b < 1-a$. A player with no outside option (or whose outside option is 0) is a weak type, denoted as W, and a player with an outside option $b > 0$ is a strong type, denoted as S. Strong types always prefer opting out rather than conceding and weak types prefer conceding rather than opting out. The players entertain beliefs about each other's type and they are represented by an initial probability $0 < \pi_0^i < 1$, that is, the probability that player i is weak. We assume that these probabilities are common knowledge and we set $\pi_0^i = \pi_0$ for simplicity.

Since we are interested in the role played by outside options on the outcome of the War of Attrition, we find appropriate to examine the Symmetric Perfect Bayesian Equilibria (SPBE) of this game given that inefficiency arises in a War of Attrition when players are constrained to use symmetric strategies. A strategy $\sigma_i(\tau)$ of player i with type $\tau = W, S$ is defined as a pair of sequences $\sigma_i(\tau) = \left\{\alpha_t^i(\tau), \beta_t^i(\tau)\right\}_{t=0}^{\infty}$ where $\alpha_t^i(\tau)$ is the probability of conceding at t and $\beta_t^i(\tau)$ is the probability of opting out at t, given that no player yields before that time. Symmetry in strategies implies that $\alpha_t^i = \alpha_t^j = \alpha_t$ and $\beta_t^i = \beta_t^j = \beta_t$. We denote as π_t^i the belief that player i is weak given that no player either concedes or opts out before t.

Since in a SPBE a weak type will never opt out and a tough type will

[3]This assumption is computationally convenient. Results do not change substantially if we assume that, when both players concede at the same time, a lottery is used to decide the outcome.

never concede, in an abuse of terminology, we will identify the probabilities of conceding α_t with the strategy of the weak type, and the probabilities of opting out β_t with the strategy of the strong type. The first result is quite straightforward. All detailed proofs are relegated to the appendix.

There is no SPBE in pure strategies.

We next turn attention to profiles where players randomize. In a SPBE in mixed strategies, it must be true that the payoff of conceding at t, conditional on the opponent not having conceded or opted out previously, must be equal to the expected payoff of conceding at $t+1$. At the same time, the payoff of opting out at t, conditional on the opponent not having yielded before, must be equal to the expected payoff of opting out at $t+1$:

$$\begin{aligned} a &= (1-a)\pi_t\alpha_t + a\delta(1-\pi_t\alpha_t-(1-\pi_t)\beta_t), \\ b &= (1-a)\pi_t\alpha_t + b(1-\pi_t)\beta_t + b\delta(1-\pi_t\alpha_t-(1-\pi_t)\beta_t), \end{aligned} \quad (1)$$

and the beliefs are obtained from the equilibrium strategies using Bayes' rule:

$$\pi_t = \frac{\pi_{t-1}(1-\alpha_{t-1})}{\pi_{t-1}(1-\alpha_{t-1})+(1-\pi_{t-1})(1-\beta_{t-1})}. \quad (2)$$

Next lemma points out that, in a SPBE it is not possible to have both types yielding at the same time with probability 1. And if the equilibrium is such that weak types concede with probability 1 at some t, then strong types certainly opt out at $t+1$.

If $\{\alpha_t\}_0^\infty$ and $\{\beta_t\}_0^\infty$ are SPBE, then:

(i) there is no t such that $\alpha_t = \beta_t = 1$

(ii) If $\alpha_t = 1$ and $0 < \beta_t < 1$ then $\beta_{t+1} = 1$.

In this game, the equilibrium strategies are characterized by the pair of difference equations (1). To simplify notation let,

$$\begin{aligned} H &= \frac{ab(1-\delta)}{a\delta(1-a-\delta b)+b(1-\delta)(1-a-\delta a)} \\ G &= \frac{(1-\delta)(1-a)(b-a)}{a\delta(1-a-\delta b)+b(1-\delta)(1-a-\delta a)}. \end{aligned}$$

Substituting these expressions on (1) and (2), and solving the corresponding difference equation with the initial condition π_0,

$$\pi_t = \frac{H}{H+G} + (\pi_0 - \frac{H}{H+G})\left(\frac{1}{1-H-G}\right)^t. \quad (3)$$

In what follows, we analyze different profiles that can be sustained as equilibria.

2.1. Concession Equilibria

A Concession Strategy Profile is a strategy profile where weak types eventually concede with probability 1. Define $\underline{T}$ as the natural number that solves:

$$\tfrac{H}{H+G} + (\pi_0 - \tfrac{H}{H+G})(\tfrac{1}{1-H-G})^t \quad \leq H \leq \quad \tfrac{H}{H+G} + (\pi_0 - \tfrac{H}{H+G})(\tfrac{1}{1-H-G})^{t-1}.$$

In a Concession Equilibrium, as time passes, players become more pessimistic about their opponents being a weak type. That fact will naturally affect the probability of conceding α_t which increases over time and the probability of opting out β_t which decreases. At some time $t = \underline{T}$ the probability that her opponent is strong is so high that a weak type optimally concedes with probability 1 since the chance of receiving her preferred agreement is too small. And a strong type will opt out at $\underline{T} + 1$ with probability 1, since waiting until period $\underline{T} + 2$ discounts her payoff and provides no additional probability that a weak type will make a concession. The formal statement of this result follows:

If $\pi_0 \in (0, H]$, there is a unique SPBE such that $\alpha_t = \beta_{t+1} = 1$ for all time t and $\beta_0 = \frac{b(1-\delta)-\pi_0(1-a-\delta b)}{b(1-\delta)(1-\pi_0)}$. Conversely, if $\pi_0 \in \left(H, \frac{H}{H+G}\right)$, the unique SPBE is such that:

$$\begin{aligned}
\alpha_t &= \frac{H}{\frac{H}{H+G} + (\pi_0 - \frac{H}{H+G})\left(\frac{1}{1-H-G}\right)^t}, \text{ for } t < \underline{T}, \\
\beta_t &= \frac{G}{1 - \left[\frac{H}{H+G} + (\pi_0 - \frac{H}{H+G})\left(\frac{1}{1-H-G}\right)^t\right]}, \text{ for } t \leq \underline{T}, \\
\alpha_t &= \beta_{t+1} = 1 \text{ for } t \geq \underline{T}
\end{aligned}$$

2.2 Opting Out Equilibria

An Opting Out Profile is characterized by strong types taking their outside opportunities at some time with probability 1, leaving weak types to play as in the complete information War of Attrition from that time on. Define as $\overline{T}$ the natural number that solves

$$\tfrac{H}{H+G} + (\pi_0 - \tfrac{H}{H+G})(\tfrac{1}{1-H-G})^{t-1} \quad \leq 1 - G \leq \quad \tfrac{H}{H+G} + (\pi_0 - \tfrac{H}{H+G})(\tfrac{1}{1-H-G})^t.$$

If $\pi_0 \in \left(\frac{H}{H+G}, 1-G\right)$, the unique SPBE is such that:

$$\alpha_t = \frac{H}{\frac{H}{H+G} + (\pi_0 - \frac{H}{H+G})\left(\frac{1}{1-H-G}\right)^t}, \text{ for } t \le \overline{T},$$

$$\beta_t = \frac{G}{1 - \left[\frac{H}{H+G} + (\pi_0 - \frac{H}{H+G})\left(\frac{1}{1-H-G}\right)^t\right]}, \text{ for } t < \overline{T},$$

$$\beta_t = 1 \text{ and } \alpha_{t+1} = \frac{a(1-\delta)}{1-a-\delta a} \text{ for } t \ge \overline{T}.$$

And if $\pi_0 \in [1-G, 1)$ $\beta_t = 1$ for $t \geqslant 0$ and $\alpha_0 = \frac{a(1-\delta\pi_0)}{(1-a-\delta a)\pi_0}$, $\alpha_t = \frac{a(1-\delta)}{(1-a-\delta a)}$ for $t \ge 1$.

In an Opting Out Equilibrium, weak types place a small probability of conceding at each period. The posterior of facing a weak type opponent π_t increases over time, but the probability α_t that weak types concede decreases. In equilibrium, there will be some time $t = \overline{T}$ such that the optimal concession probability of the weak types cannot induce strong types to stay in the game beyond $\overline{T}$ since the payoff they get by opting out at that time, b, is greater than the expected payoff of waiting an additional period for $(1-a)$. After $\overline{T}$ the posterior probability of facing a weak opponent is 1. Players that are still at the negotiation table identify themselves as weak types and thus, from that period $\overline{T}$ on, they play the Symmetric Perfect Equilibrium of the complete information War of Attrition without outside options. In this continuation, the equilibrium concession probability remains constant over time at $\frac{a(1-\delta)}{1-a-\delta a}$.

2.3. Pooling Equilibrium

The next proposition establishes the unique combination of parameters for which the SPBE is pooling. Players follow strategies such that both types randomize at the same constant rate between yielding and not yielding.

If $\pi_0 = \frac{H}{H+G}$, the unique SPBE is $\alpha_t = \beta_t = H + G$, for $t \ge 0$.

If the probability of facing a weak opponent is exactly $\pi_0 = \frac{H}{H+G}$, in equilibrium, both types remain indifferent between conceding and opting out at every time. That is, in terms of randomized strategies, each player believes, at each time, that the probabilities (α_t, β_t) that the opponent concedes or opts out at subsequent times are exactly so as to make continuation marginally worthwhile. No information is revealed along this equilibrium. No player updates her beliefs about the weakness of her opponent since if players concede and opt out at each time with the same probability, the posterior π_t is constant over time.

3. Conclusions

In this paper we have explored the effect of the private information about outside options on the outcomes of negotiations. In order to address this issue we analyzed a War of Attrition allowing players to leave the negotiation in order to opt out and we characterized the Symmetric Perfect Bayesian Equilibrium of this game. There are two types of players: a weak type who has a valueless outside option-she always prefers conceding rather than opting out- and a strong type who has a valuable outside option that she prefers to take rather than conceding. We show that uncertainty about the possibility that the opponent opts out increases the equilibrium probability of concession. More precisely, if the probability that the opponent is strong is relatively high, in equilibrium, the negotiation eventually ends with a sure concession. On the other extreme, if the likelihood of a weak opponent is high, strong types stay in the game for a while and eventually leave the negotiation and opt out with probability 1. From that date on, weak types play the (inefficient) symmetric equilibrium of the classical War of Attrition with complete information. Even in this case, the probability of concession by weak types along the uncertainty phase of the equilibrium play increases.

Appendix

Proof of Proposition 1

Define t_τ $\tau = W, S$ as a time at which type τ plans to yield (to concede if she is weak and to opt out otherwise) given than no player yields before that time. Assume first that $t_S \leq t_W$. Thus, strong types know that, in equilibrium, weak types do not concede before they opt out with certainty. Then, it is optimal for a strong type to opt out at period 0, so she avoids any discounting of the payoff. The same happens to a weak type, since she knows she is not going to get any concession from her opponent. Thus, it must be $t_W = t_S = 0$. But this cannot be an equilibrium since $b < (1-a)\pi_0 + b(1-\pi_0)$. The other potential equilibrium is $t_W < t_S$ in which case $t_W = 0$ and $t_S = x$ with $x \geq 1$. If weak types concede in equilibrium at $t = 0$, then it must be true that $a \geq (1-a)\pi_0 + a\delta(1-\pi_0)$ or $\pi_0 \leq \frac{a(1-\delta)}{1-a-\delta a}$. Since $\pi_0 \leq \frac{a(1-\delta)}{1-a-\delta a} < \frac{b(1-\delta)}{1-a-\delta b}$, strong types deviate and opt out at $t = 0$.

Proof of Lemma 2

Statement *(i)* indicates that, in equilibrium, it is not possible that both types yield, at the same time, with probability 1. If the strategy of the opponent is to concede and to opt out at some t with probability 1, then a strong player will have always incentives to wait one period since $b < (1-a)\pi_t + b(1-\pi_t)$, breaking the symmetry of the strategies.

Statement *(ii)* establishes that, if the weak type strategy is to concede with probability 1 at some period t, then to opt out at $t+1$ dominates doing so in $t+2$, since waiting until period $t+2$ discounts their payoff and provides no additional probability that a weak type will make a concession.

Proof of Proposition 3

Consider the equation (3) that rules the posterior. If $\pi_0 < \frac{H}{H+G}$, π_t decreases over time and, thus $\alpha_t = \dfrac{H}{\pi_t}$ increases. At some period $\underline{T}$, α_t reaches the value of 1. Then, it must be true that $\pi_{\underline{T}-1}\alpha_{\underline{T}-1} = H$, and $\pi_{\underline{T}}\alpha_{\underline{T}} \leq H$. Since $\alpha_{\underline{T}} = 1$ and $\alpha_{\underline{T}-1} < 1$ then $\pi_{\underline{T}-1} \geq H \geq \pi_{\underline{T}}$. Using the equation (3), $\underline{T}$ is the natural number that solves:

$$\tfrac{H}{H+G} + (\pi_0 - \tfrac{H}{H+G})(\tfrac{1}{1-H-G})^t \leq H \leq \tfrac{H}{H+G} + (\pi_0 - \tfrac{H}{H+G})(\tfrac{1}{1-H-G})^{t-1}.$$

By lemma 2 we know that if $\alpha_{\underline{T}} = 1$ then $\beta_{\underline{T}+1} = 1$. Notice that if $\pi_0 \leq H$, then $\underline{T} = 0$.

Proof of Proposition 4

If $\frac{H}{H+G} < \pi_0 < 1-G$, then π_t increases over time and β_t increases until, at some time $\overline{T}$, it reaches the value of 1. Then it must be the case that $(1-\pi_{T^*_H-1})\beta_{\overline{T}-1} = G$, and $(1-\pi_{\overline{T}})\beta_{\overline{T}} \leq G$. Since $\beta_{\overline{T}} = 1$ and $\beta_{\overline{T}-1} < 1$, then $1-\pi_{\overline{T}-1} \geq G \geq 1-\pi_{\overline{T}}$. Using the equation (3), $\overline{T}$ is the natural number that solves:

$$\frac{H}{H+G} + (\pi_0 - \frac{H}{H+G})(\frac{1}{1-H-G})^{t-1} \leq 1-G \leq \frac{H}{H+G} + (\pi_0 - \frac{H}{H+G})(\frac{1}{1-H-G})^{t}.$$

Since $\beta_{\overline{T}} = 1$, then $\pi_t = 1$ for $t \geq \overline{T}+1$. Players that are still playing are weak types and thus $\alpha_t = \frac{a(1-\delta)}{1-a-\delta a}$ for $t \geqslant \overline{T}+1$.

If $1-G \leqslant \pi_0$, $\overline{T} = 0$ and thus $\pi_t = 1$ for $t \geq 1$. We substitute $\beta_t = 1$ for $t \geq 0$ and $\alpha_0 = \dfrac{a(1-\delta\pi_0)}{(1-a-\delta a)\pi_0}$ $\alpha_t = \dfrac{a(1-\delta)}{1-a-\delta a}$ for $t \geq 1$ in the following equations:

$$\begin{array}{rcl} b &>& \pi_t\alpha_t(1-a) + b(1-\pi_t)\beta_t + b\delta(1-\pi_t\alpha_t-(1-\pi_t)\beta_t) \\ a &=& \pi_t\alpha_t(1-a) + a\delta(1-\pi_t\alpha_t-(1-\pi_t)\beta_t) \end{array}$$

and easily check that are satisfied for all t only if $\pi_0 \geq 1-G$.

Proof of Proposition 5

Substituting $\pi_0 = \frac{H}{H+G}$ in 2 we get that $\pi_t = \pi_0$. Then $\alpha_t = \dfrac{H}{\pi_0} = H+G$ and $\beta_t = \frac{G}{1-\pi_0} = H+G$ for $t \geq 0$.

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