E C O N O M I C S   B U L L E T I N

Fragmentation in a product cycle model

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Abstract

We develop a simple model in which fragmentation, innovation and imitation take in place simultaneously. Firms in North fragment their business into two parts: assembly and services. A reduction in the cost of services to coordinate fragmented businesses between North and South does not enhance fragmentation. On the contrary, the arrival rate of imitation accelerates and that of innovation slows down. Consequently, the life of Northern goods becomes shorter and that of Southern copies becomes longer. We also derive other results to compare to those in Grossman and Helpman (1991) and in Glass and Saggi (2001).

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1. Introduction

Globalization is the most important keyword in considering world trade in recent years. It entails a wide range of issues in both theory and practice such as increasing wage differentials, unemployment resulting from outsourcing jobs abroad, and so forth. Globalization in theory is also called fragmentation, outsourcing or (economic) geography, etc[1]. We are especially interested in such a situation that a production process is fragmented, which incurs additional costs to coordinate internationally fragmented processes as pointed out in Jones and Kierzkowski (2001), so we use the term *fragmentation* to describe the phenomenon.

Some authors have tackled the problem of fragmentation and growth. Above all, Glass and Saggi (2001) [GS, hereafter] and Grossman and Helpman (1991) [GH] are closely related to our analysis. GS analyzes fragmentation and innovation but imitation is absent. GH extensively analyzes a product cycle model although fragmentation is absent. Glass (2004) extends GS to incorporate costless imitation that is exogenously given. None of these analyses considers coordination costs so we barely know how fragmentation and a product cycle affect each other. Therefore, our task is to shed light on this issue by constructing a framework of fragmentation in a model of endogenous product cycles.

The three propositions below summarize our results and we compare them to the pioneers' above. We begin with the illustration of our model.

2. Basic Model

There are two countries, North and South, in the world. Each has labor as the only resource in this model. A type of R&D is innovation in North and imitation in South. The former aims to improve the quality of a product and the latter aims to learn how to copy the quality improved.

2.1 Consumers

We use the quality ladders model of GH. Consumers choose a product with the lowest quality-adjusted price in each industry. There are n^N goods Northern firms supply and n^S goods Southern firms supply. There is only one good with the lowest quality-adjusted price in each industry and we normalize the industry measure such as $n^N + n^S = 1$. The world expenditure is also normalized to be one at each point in time so that the interest rate is equal to the common time preference rate ρ everywhere. As a result, the

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[1] See Krugman (1995) on geography. See Jones and Kierzkowski (2001) on various issues of fragmentation.

demand for each good x^j is equal to $1/p^j$ for $j = N,S$. (The details are relegated to Appendix A.)

2.2 Firms

A Northern firm has the exclusive right to sell the state-of-the-art good until a Southern firm imitates it. A representative firm earns profits of

$$\pi^N = \left(p^N - w^S a_F\right)x^N - w^N c_F = \left(1 - a_F / \lambda\right) - w^N c_F . \tag{1}$$

Any Northern firm fragments its business into two parts, assembly and services, and outsources the former to South and engages in the latter by itself. The unit labor requirement of assembly is a_F in units of Southern labor. Services that coordinate the fragmented businesses between North and South cost c_F in units of Northern labor as fixed costs. We assume any innovative project targets copies so that a successful innovator charges $p^N = \lambda w^S$, where $\lambda > 1$ is the quality difference between the new good and the old copy whose marginal cost is one in units of Southern labor[2]. By limit pricing, Northern firms always win the price competition until an imitation succeeds.

We also assume that any imitative project in South targets a state-of-the-art good. A successful imitator earns

$$\pi^S = \left(p^S - w^S\right)x^S = 1 - 1/a_F . \tag{2}$$

By limit pricing, it sets $p^S = w^S a_F$ to win the competition. For both types of firms to coexist, $\lambda > a_F > 1$ is required. Fragmentation transfers technology from North to South. The production of state-of-the-art goods entails some information or knowledge South has not mastered. Therefore, $a_F > 1$ implies that the productivity rises once South masters the required technology. On the other hand, $\lambda > a_F$ implies that innovation should be sufficiently profitable.

2.3 Free Entry and No Arbitrage

A unit of R&D activities requires one unit of labor per unit of time in each country. If innovators undertakes R&D targeting one good at intensity ι, the probability of success is equal to ι. As for an imitator, the counterpart is

[2] This corresponds to the case of inefficient followers in GH. Many cases are possible, for example, a Northern innovator also targets a Northern good, and so on. We confine ourselves to the simplest case required to show the effects of fragmentation.

μ.

Free entry into R&D races implies that the expected reward for successful R&D v^j must be equal to the R&D cost in each region[3]:

$$v^j = w^j \ for \ j = N, S, \tag{3}$$

No arbitrage remains, $\pi^j / v^j + \dot{v}^j / v^j = \rho + risk$, where the risk is ι for South and is μ for North. The steady state we investigate requires $\dot{v}^j / v^j = 0$. Using (1), (2) and (3), we have a reduced form of no-arbitrage conditions as follows:

$$\left(1 - a_F / \lambda\right) / w^N - c_F = \rho + \mu, \tag{4}$$

$$\left(1 - 1 / a_F\right) / w^S = \rho + \iota. \tag{5}$$

2.4 Labor Market

Northern labor engages in R&D and services:

$$\iota n^S + n^N c_F = L^N. \tag{6}$$

Southern labor engages in R&D and the assembly of both imitative and innovative goods:

$$\mu n^N + n^S / w^S a_F + n^N a_F / \lambda w^S = L^S. \tag{7}$$

3. Steady State

We investigate a steady state where the resource allocation is constant: the reward for R&D success, R&D intensity, the fraction of each type of firms and the wage in both regions should be constant. We add the following conditions:

$$n^N + n^S = 1, \tag{8}$$

$$g \equiv \iota n^S = \mu n^N. \tag{9}$$

The latter requires that the inflow should meet the outflow of each type of goods. Therefore, g denotes the rate of innovation and imitation and they are equalized in the steady state[4].

It immediately follows from (6) and (9) that

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[3] At intensity μ, for example, an imitator succeeds with probability μ for a time interval dt. Then, max $(v^S - w^S)\,\mu dt$ leads to (3).

[4] This is inherent in the quality ladders model.

$$g = L^N - c_F n^N, \tag{10}$$

where $g' < 0$ and $g'' = 0$ w. r. t. n^N. We also manipulate (5), (7), (8) and (9) to get

$$g = \frac{1-n^N}{1-\left(1-a_F/\lambda\right)n^N}\left[\left(1-\frac{1}{a_F}\right)L^S - \rho\left(\frac{1-n^N}{a_F}+\frac{a_F n^N}{\lambda}\right)\right], \tag{11}$$

where g' w. r. t. n^N can be positive or negative (Appendix B). In Fig. 1, we draw (10) as NN and (11) as SS. A steady state exists if Southern labor is sufficiently large:

$$\left(1-1/a_F\right)L^S > L^N + \rho/a_F. \tag{12}$$

Then, we have a unique steady state since g is positive in (10) and is zero in (11) at $n^N = 1$. We assume these parameters satisfy the condition[5]. Note an angle of the triangle that contains the origin and steady state point represents ι or μ (see (9)).

4. Comparative Steady States and Policy Implications

4.1 An Expansion of Resources

An increase in L^N shifts NN upward and SS is unaffected. The steady state moves from point E to F along SS as illustrated in Fig. 2: g rises and n^N falls. As its labor increases, North allocates more to R&D, which leads to an increase in the Southern allocation to R&D. We thus have a higher rate of innovation and imitation. Since SS has a negative slope, an increase in g is always associated with a decrease in n^N. Consequently, n^S increases.

As the steady state moves from point E to F, the lower-right angle gets flatter and the lower-left angle gets steeper (Fig. 1, 2). Therefore, an increase in n^S overwhelms an increase in g so the arrival rate of innovation ι decreases. On the other hand, the arrival rate of imitation μ increases due to a decrease in n^N. Since g and n^S increase, the demand for labor increases in South. This raises w^S (see (5) with a smaller ι). In contrast, an increase in the Northern labor supply lowers its wage w^N (see (4) with a larger μ). Alternatively, an increase in L^S moves the steady state southeast along NN. The effects are opposite to the case above.

We summarize these results in the following proposition, where we

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[5] Otherwise, we would have no or two possible steady-state points. Then, in the latter case, we should consider which one is stable or relevant in practice, which might alter the results below. We leave this argument for interested readers.

measure the extent of fragmentation by n^N since all Northern firms engage in fragmentation.

Proposition 1. An expansion of the Northern [Southern] labor force will
i) enhance the rate of innovation and imitation,
ii) raise [slow down] the arrival rate of imitation,
iii) slow down [raise] the arrival rate of innovation,
iv) discourage [encourage] fragmentation,
v) decrease [increase] the fraction of Northern products,
vi) increase [decrease] the fraction of Southern products, and
vii) reduce [raise] the relative wage of North.

GH concludes an increase in the size of a region increases the fraction of products manufactured there in contrast to (v), (vi), and that the effects on the relative wage is ambiguous. On the other hand, GS concludes a decrease in Northern labor [an increase in Southern labor] will enhance [enhance] fragmentation, decrease [increase] the rate of innovation, and have no effect on the relative wage. Our results thus bridge the gap between these two models.

4.2 A Reduction in Coordination Costs

The recent decline in communication costs such as the ICT or IT revolution seems to help fragmentation by reducing coordination costs. We consider the effects of a reduction in the coordination cost.

A reduction in the coordination cost rotates NN counterclockwise so that the steady state moves northwest along SS. The effects are straightforward other than one on the Northern wage.

Proposition 2. A reduction in the coordination cost will
i) enhance the rate of innovation and imitation,
ii) raise the arrival rate of imitation,
iii) slow down the arrival rate of innovation,
iv) discourage fragmentation,
v) decrease the fraction of Northern products,
vi) increase the fraction of Southern products, and
vii) raise the wage of South and North. (see Appendix C)

As the coordination cost declines, the expected reward for innovation rises. This attracts more resources to Northern R&D labs, ending up with a smaller extent of fragmentation (Appendix D explains in detail). The Southern government would like to subsidize to FDI from North by bearing a fraction of Northern fixed costs to increase job opportunities and to reduce imitation for a better relationship with North. On the contrary, the attempt will fail as shown in (i), (ii) and (iv).

4.3 R&D Subsidies

Consider that a government in each country subsidizes R&D. This is the same as the effect of an improvement in R&D productivity[6], which attracts more resources to R&D activities.

A subsidy to Northern R&D rotates NN clockwise so that the steady state moves northwest along SS. On the other hand, a subsidy to Southern R&D moves the steady state southeast along NN since SS rotates clockwise.

Proposition 3. A subsidy to innovative [imitative] R&D will
i) enhance [retard] the rate of innovation and imitation,
ii) slow down [raise] the arrival rate of innovation,
iii) raise [slow down] the arrival rate of imitation,
iv) discourage [encourage] fragmentation,
v) decrease [increase] the fraction of Northern products,
vi) increase [decrease] the fraction of Southern products, and
vii) raise [raise] the Southern [Northern] wage although the effect on the Northern [Southern] wage is ambiguous.

An increase in the unit labor requirement of Southern imitation is regarded as tightening of the enforcement of intellectual property rights. Then, Proposition 3 states that it will promote the arrival rate of imitation and discourage fragmentation.

GH concludes a small subsidy to innovative [imitative] R&D, evaluating at the initial state without subsidies, results in an increase [an increase] in the rate of innovation and imitation, a decrease [increase] in the fraction of Northern products, an increase [ambiguity] in the arrival rate of imitation, and an increase in the relative wage of North [South]. Alternatively, GS shows a subsidy to innovation enhances the rate of innovation, fragmentation, and increases the relative wage of North.

5. Conclusion

We developed a model in which fragmentation, innovation and imitation take place simultaneously. North fragments its business into two parts, production and coordination. While South indirectly exports its cheaper labor, North indirectly exports its superior technologies. In sum, North loses its advantages in the global market through changes in favor of its resource constraint.

A reduction in coordination costs or a subsidy to innovative R&D expands the Northern resource constraint. However, the fraction of Northern

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[6] Note the productivity is normalized to be one in (3) for each country. Multiply the right-hand side (RHS) of (3) by the productivity $a_R{}^j$. A reduction in the Northern increases the entire RHS of (10) while that in the Southern increases L^S in (11).

products, hence the extent of fragmentation, decreases as long as we have a steady state on the downward sloping part of SS. As we argue in Appendix D, the Southern wage increases and the extent of fragmentation decreases because the world demands more labor in South.

The outcome that falling coordination costs or subsidies to innovation discourages fragmentation depends on the slope of SS. The price of copies depends on the Southern wage in our model with fragmentation although the price of originals depends on the Southern wage with or without fragmentation because Northern firms face Southern firms as their closest competitor. This is the crucial ingredient for SS to have a negative slope. In contrast, GH that excludes fragmentation has the Northern wage in their Southern labor constraint (15).

We assumed the whole assembly takes place in South. If we allow for partial fragmentation as in GS and Glass (2004), a fraction of the assembly takes place in North so that the marginal cost of Northern goods, hence the price of copies, also depends on the Northern wage. This might alter the slope of SS so our results would reverse.

GS shows a subsidy to innovation enhances outsourcing. Since they do not include imitation, South does not allocate more resources to R&D as North does. Consequently, the relative wage of North rises in their proposition 4. Incorporating imitation in South therefore makes South reallocate its resources between R&D and production although incorporating partial fragmentation with uncertain probability would change our results to such as GS.

On the other hand, we ignore the possibility North also targets Northern goods as in GH, GS and Glass (2004). Allowing for this possibility opens the channel of the reallocation of resources between two types of R&D in North. Incorporating costless imitation as in Glass (2004) alters the analysis in this direction rather than considering the reallocation effect in South.

Our focus is fragmentation that incurs coordination costs. If North takes advantage of lower costs in South, the price of copies inevitably depends on the Southern wage to a large extent. Then, falling coordination costs encourage innovation and imitation so that the demand for labor in South increases. This raises the Southern wage, which gets rid of the advantage of fragmentation and resources shift from production to R&D in South. Therefore, falling coordination costs discourages fragmentation.

As GS argues, an expansion of Southern resources leads to a greater extent of fragmentation and decreases the relative wage of South. Our analysis suggests that the recent expansion of fragmentation may be caused mainly by an increase in the Southern labor supply rather than reduced coordination costs.

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Appendix A. Consumers

The intertemporal utility is given by

$$U_t = \int_t^{\infty} e^{-\rho(\tau - t)} \log u_\tau d\tau \,. \tag{A.1}$$

The instantaneous utility is given by

$$\log u_t = \int_0^1 \log\left[\sum\nolimits_m q_m(i) x_{mt}(i)\right] di \,, \tag{A.2}$$

where $x_{mt}(i)$ denotes consumption of quality level m in industry $i \in [0,1]$, and $q_m(i) = \lambda^m$. By symmetry, the demand for each good is

$$x^j = E / p^j \tag{A.3}$$

where E denotes the world expenditure on consumption and p^j is the lowest quality-adjusted price. The optimal path of the world expenditure obeys

$$\dot{E} / E = r - \rho \,. \tag{A.4}$$

By normalization, we obtain $r = \rho$.

Appendix B. Slope of SS

Define g given in (11) by $g(n^N)$. Then, $g(0) = \left(1 - 1/a_F\right)L^S - \rho / a_F$ and $g(1) = 0$. By construction, in the range of $n^N \in (0,1)$,

$$\left(1-\frac{1}{a_F}\right)L^S > \rho\left(\frac{1-n^N}{a_F}+\frac{a_F n^N}{\lambda}\right). \tag{B.1}$$

Suppose $a_F/\lambda \geq 1/a_F$, in other words, $(a_F)^2 \geq \lambda$, each quality improvement is not so large. Then, (B.1) implies $\left(1-1/a_F\right)L^S > \rho a_F/\lambda$. Alternatively, suppose $a_F/\lambda < 1/a_F$ or $\lambda > \left(a_F^S\right)^2$. Then, (B.1) implies $\left(1-1/a_F\right)L^S > \rho/a_F$. In either case, $g(0)$ must be positive. Manipulate (11) to get

$$\alpha g(n^N) = \underbrace{\left(\frac{\tilde{L}^S}{1-\alpha}+C\right)+\frac{\alpha C}{1-\alpha}n^N}_{f_1} - \underbrace{\frac{\tilde{L}^S+C}{1-\alpha n^N}}_{f_2}, \tag{B.2}$$

where $\alpha \equiv 1-a_F/\lambda \in (0,1)$, $C \equiv (\rho/\alpha)\left(1/a_F - a_F/\lambda\right)$ and $\tilde{L}^S \equiv \left(1-1/a_F\right)L^S$ $-\rho/a_F > 0$. Note $f_2' > 0$ and $f_2'' > 0$. If $(a_F)^2 \geq \lambda$, or $C \leq 0$, $g'(n^N) < 0$ and $g''(n^N) < 0$ result directly from (11). If $\lambda > \left(a_F^S\right)^2$, or $C > 0$, SS can have a positive slope for a small n^N due to $f_2' > 0$, $f_2'' > 0$. (B.2) gives us

$$g'(n^N) = \frac{C}{1-\alpha} - \frac{\tilde{L}^S + C}{(1-\alpha n^N)^2}. \tag{B.3}$$

Evaluating at $n^N = 0$, $g'(0) > 0$ holds if

$$\frac{\rho\left(1/a_F - a_F/\lambda\right)}{1-\left(1-a_F/\lambda\right)} + \frac{\rho}{a_F} > \left(1-\frac{1}{a_F}\right)L^S. \tag{B.4}$$

Together with (B.1),

$$\frac{\rho\left(1/a_F - a_F/\lambda\right)}{1-\left(1-a_F/\lambda\right)} + \frac{\rho}{a_F} > \left(1-\frac{1}{a_F}\right)L^S > \frac{\rho}{a_F}. \tag{B.5}$$

This is rather restrictive. As long as we confine ourselves to a unique steady state rather than multiple steady states, SS and NN meet in the range where SS is sloping downward (see Fig. 3). This requires (12) again. Therefore, the propositions in the main text do not alter.

Appendix C. Proof of $dw^N / dc_F > 0$

Totally differentiate (4) w. r. t. c_F to obtain

$$\frac{1 - a_F / \lambda}{(w^N)^2} \frac{dw^N}{dc_F} = -\frac{d\mu}{dc_F} - 1 .$$

It is sufficient to show $d\mu / dc_F < -1$. From (10) we have

$$\frac{dg}{dc_F} = -\left(c_F \frac{dn^N}{dc_F} + n^N \right). \tag{C.1}$$

On the other hand, using (C.1), (9) says

$$\frac{d\mu}{dc_F} = -\left[\mu \left(\frac{c_F}{g} + \frac{1}{n^N} \right) \frac{dn^N}{dc_F} + 1 \right] < 0 ,$$

where the inside of the square brackets exceeds one due to $dn^N / dc_F > 0$. ▌

Since the wage increases in both countries, the effect on the relative wage is ambiguous.

Appendix D. Effects of the Reduction in the Coordination Cost

The results in the proposition depend on the shape of SS. (7) translates into

$$g + \frac{1}{w^S} \left(\frac{1 - n^N}{a_F} + \frac{a_F n^N}{\lambda} \right) = L^S .$$

For a given w^S an increase in g according to an expansion of the Northern resource constraint, which stems from a reduction in the coordination cost or an increase in the Northern labor, requires a reduction of resources devoted to the production in South so that n^N must decrease if $1 / a_F < a_F / \lambda$, or $n^S (= 1 - n^N)$ must decrease if $1 / a_F > a_F / \lambda$.

In either case, however, keeping w^S constant also needs a change in n^N according to (5) and (9):

$$\frac{1}{w^S} = \frac{\rho + \iota}{1 - 1 / a_F} = \frac{1}{1 - 1 / a_F} \left(\rho + \frac{g}{1 - n^N} \right).$$

This implies, for a constant L^S, n^N must decline as g grows. The total effect is a rise in w^S, which in turn implies that n^N declines to a large extent.

Figure 1. Steady State at Point E

The lower-left [lower-right] angle is equal to μ [ι].

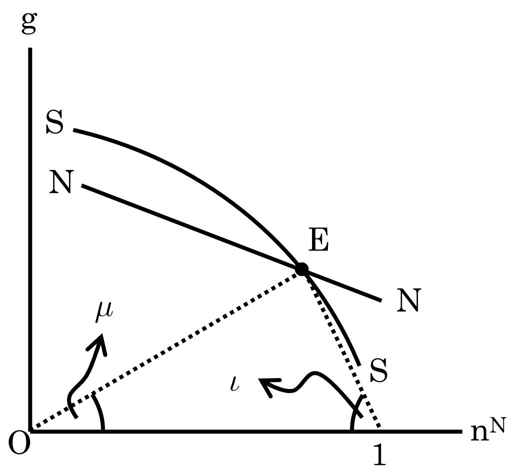

Figure 2. An Increase in L^N

NN shifts upward (not drawn) and SS is unaffected. The steady state moves from point E to F along SS.

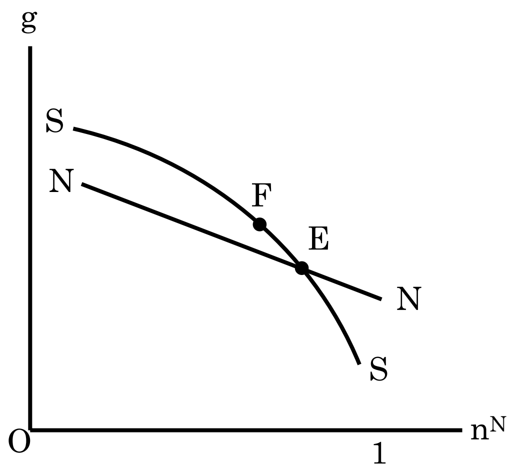

Figure 3. Shape of SS on condition that (B.5) holds.

It is possible for SS to have a positive slope for a small n^N.