

## Positivity and bubbles in overlapping generations models

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### *Abstract*

Bubbles, such as money, cannot be valued in efficient equilibria in overlapping generations models (a borderline case aside). Analysts frequently attribute this result to the fact that if bubbles were valued, the bubble must eventually exceed the endowment of the young. This implies negative consumption by the young, invalidating the equilibrium path. This argument is misleading because it depends on the assumption that negative consumption is infeasible. But negative consumption is admissible under some utility functions, raising questions about the generality of the argument. To investigate this, we characterize equilibrium in an overlapping generations model with negative exponential utility, which allows negative consumption. We show that if the endowment allocation is Pareto optimal, equilibrium paths incorporating valued money at some date eventually reach a point at which the equilibrium path has no continuation.

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In analyses of bubbles in overlapping generations models—money is the outstanding example—it is pointed out that these can occur only in dynamically inefficient equilibria (a borderline case aside). The argument for this proposition is that along efficient paths the interest rate is positive (in stationary settings), implying that a valued bubble would eventually necessarily exceed the endowment of the young, therefore presumably invalidating the assumed equilibrium path.

This argument is at best incomplete. It appears to rely on the assumption that consumption is positive, since otherwise there is no difficulty if the value of the bubble exceeds the endowment of the young. Of course, positive consumption is presumed in the logarithmic utility that is virtually universally specified in applied analyses of overlapping generations models.

It is true that negative consumption is difficult to interpret. However, it is unsatisfying to have the existence or nonexistence of valued bubbles apparently linked to whether negative consumption is admissible, an unrelated specification.

Seen from a different angle, it is evident that the nonexistence of bubbles in efficient equilibria cannot depend on the assumption that consumption is positive-valued. In efficient equilibria the aggregate endowment is finite-valued, implying (by Walras' Law) that the aggregate endowment has the same value as aggregate consumption. Therefore no bubble is possible. It follows that trajectories which incorporate a bubble cannot be feasible as equilibria over infinite time, and this is true whether or not negative consumption is admitted. What is it that renders these paths nonfeasible when negative consumption is admitted?

This matter is easy to investigate: many utility functions are well defined under negative consumption, unlike the logarithmic utility function. It is instructive to determine conditions under which bubbles are valued in overlapping generations under negative exponential utility, which can take negative as well as positive numbers as an argument.

Consider an overlapping generations model that is standard in every way except that generation  $t$  has utility function

$$U^t(x_t^t, x_{t+1}^t) = -e^{-x_t^t} - e^{-x_{t+1}^t}, \quad (1)$$

where  $x_t^t$  and  $x_{t+1}^t$  are consumption of generation  $t$  when young and old. Endowments of each generation are  $\bar{x}^y$  and  $\bar{x}^o$ ; these endowments are not necessarily positive, although it is convenient to think of them as positive. An initial generation is born at date 0 and is endowed with one unit of money. Agents can trade only when they are alive (implying that they cannot arbitrage away valued money).

Assuming that money is not disposable, so that it can take on negative values, the model to be presented is completely symmetric as to positive and negative levels of consumption and the price level. For example, the behavior of equilibrium paths of the price level  $p_t$  is qualitatively similar when  $p_t$  is positive and  $\bar{x}^y > (<)\bar{x}^o$  and when  $p_t$  is negative and  $\bar{x}^y < (>)\bar{x}^o$ . Therefore without loss of generality we can restrict ourselves to the case in which  $p_t$  is positive.

The budget constraints of generation  $t$  are

$$p_t \bar{x}^y = p_t x_t^t + m_t^t \quad (2)$$

$$p_{t+1} \bar{x}^o + m_t^t = p_{t+1} x_{t+1}^t. \quad (3)$$

Maximizing (1) subject to (2) and (3) and using the market-clearing condition  $m_t^t = 1$  results in

$$y(p_t) = o(p_{t+1}), \quad (4)$$

where the functions  $y(p)$  and  $o(p)$  (referring to “young” and “old”) are defined by

$$y(p) \equiv \frac{e^{1/p-\bar{x}^y}}{p}, \quad o(p) \equiv \frac{e^{-1/p-\bar{x}^o}}{p}. \quad (5)$$

These functions are defined on negative as well as positive  $p$  but, as noted, we will restrict analysis to the case of positive  $p$ . The function  $y(p)$  is monotonically decreasing, while  $o(p)$  increases for  $p < 1$  and decreases for  $p > 1$ , approaching an asymptote of zero as  $p$  approaches  $\infty$ .

We assume that  $\bar{x}^y < \bar{x}^o$  so that the endowment allocation is Pareto efficient and has finite date-0 value. There exists no stationary equilibrium with a valued bubble in this case (that is, there is no positive value of  $p$  that satisfies  $y(p) = o(p)$ ).

To examine trajectories with valued money, assume an initial value for the date-0 price level,  $p_0$ . As the figure indicates, the price level will drop, eventually reaching a value  $p_T$  such that  $y(p_T) > \max_z o(z)$ . In Figure 1 this occurs at  $T = 2$ . At that point the economy cannot continue. Assuming rational expectations, this invalidates the assumed initial condition  $p_0$ . Therefore there exist no trajectories with valued money that continue for infinite time. This argument has nothing to do with the value of money eventually exceeding the endowment of the young.

If the endowment allocation is not Pareto-optimal ( $\bar{x}^y > \bar{x}^o$ ), there do exist trajectories with valued money that continue over infinite time.

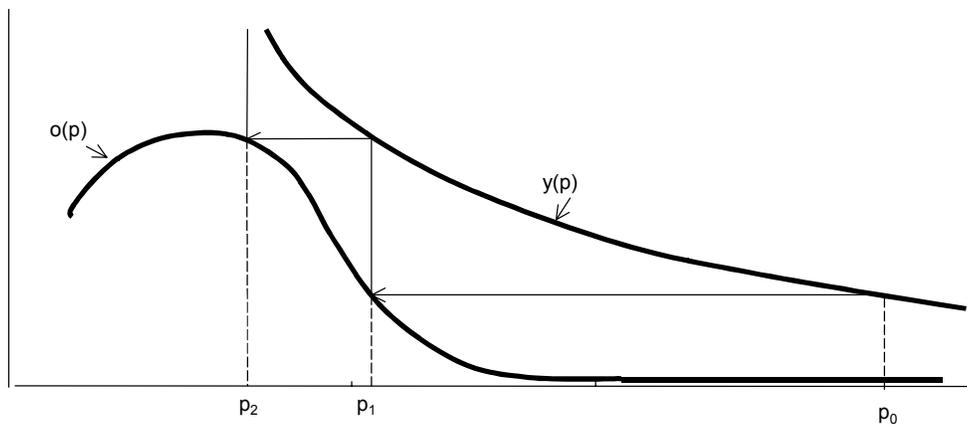


Figure 1: