E C O N O M I C S   B U L L E T I N

Public vs private schooling in an endogenous growth model

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Abstract

I present an overlapping generations model, with formal education as the engine of growth, close to Glomm and Ravikumar (1992). Contrary to Glomm and Ravikumar, I show that public schooling, when compared to a private system, may stimulate economic growth.

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1 Introduction

The question addressed in this paper is whether public schooling is growth enhancing or growth inhibiting when compared to a private system. In my model the engine of growth is formal schooling. It is known from the literature, see Glomm and Ravikumar (1992) and Zhang (1996) as the leading references — also see Bräuninger and Vidal (2000), that one of the effects of the public system is the reduction in inequality. This happens because all agents face the same quality of education, while in a private system richer families have better schooling. So, on distributional grounds, there is a consensus that a public system is superior to a private system, at least if one considers equality as a goal.

A more intriguing result is that they conclude that a public education system reduces economic growth. Even when in the presence of homogenous agents this result is true due to a negative fiscal externality. According to Glomm and Ravikumar (1992), students study more in a private system and hence accumulate more human capital. This happens because agents include the money they invest on the education of their children in their utility function. This effect is internalized only in a private system.

In this paper, I argue that if altruistic behavior of the parents is not allowed, the results are reversed, with public schooling becoming growth enhancing relative to private schooling.

2 The Model

I consider a basic Overlapping Generations (OG) model in which agents maximize utility over a life time of two periods. Each generation consists of a continuum of agents. In each period we have a generation of old agents and a generation of newborn agents.

Contrary to Glomm and Ravikumar (1992) and Zhang (1996), I consider the young generation to be responsible for financing their education. I do not introduce gifts in the utility function of the parents. Hence, the decision is fully internal. Young agents have access to international capital markets and can use their human capital as a collateral to finance their education spending.

To simplify, I assume, as in Galor and Zeira (1993), a small open economy with perfect access to the international capital markets. Agents can borrow at an exogenous interest rate r. I assume $r = 0$. We can interpret this assumption as capturing the student loans at subsidized interest rates, common in some countries (although assuming $r > 0$ does not change the results).

I assume that agents only consume in their second period of life. The utility function of the agents born in period t is given by:

$$u\left(c_{t,t}, c_{t,t+1}\right) = c_{t,t+1} \tag{1}$$

In the first period of their lives, agents have to decide whether to go to the university or not and how much money and time they spend on education. The human capital of an agent born in t depends on these choices and also on the human capital of the parent and on the average human capital of the old agents in the economy:

$$h_t = \theta_t \left(e_t\right)^{\alpha} \left(H_{t-1}\right)^{\beta} \left(h_{t-1}\right)^{\gamma}, \qquad 0 < \lambda, \alpha, \beta, \gamma < 1 \quad \textit{and} \quad \alpha + \beta + \gamma = 1 \tag{2}$$

where h_{t-1} is the human capital of the young agent's parent, H_{t-1} is the average human capital at time t of the generation born in $t-1$, $e_t \geq 0$ is the money invested in education and $\theta_t \in [0,1]$ is the time spent in School. One can interpret the influence of e_t and H_{t-1} as measuring the quality of the school.

In a private system, if an agent wants to study she chooses e_t. In a public system, the agent pays nothing when studying, and the total spending on education in each period is financed by taxes raised on all agents (young and old) working in the next period. The money spent per student in education, and hence the necessary tax rate to support that expenditure, is decided by majority voting.

We can interpret this model as one in which basic education, say at a high school level, is guaranteed (for example, through compulsory schooling), but above that it is a private decision.

On the production side, I assume a linear technology (production of each worker is equal to her human capital), and hence wages coincide with the human capital.

To capture the heterogeneity of the agents, I assume that old agents in the same generation are differentiated by their stock of human capital according to some distribution function G_t.

I assume no fixed costs in education and I am considering education to be a pure rival good. Relaxing these assumptions would make the case for public education stronger.

2.1 Equilibrium in the private system

The agent's optimization problem is to choose $\theta_t, e_t, c_{t,t+1}$ to maximize

$$u\left(c_{t,t}, c_{t,t+1}\right) = c_{t,t+1}$$

subject to

$$c_{t,t+1} = \theta_t \left(e_t\right)^{\alpha} \left(H_{t-1}\right)^{\beta} \left(h_{t-1}\right)^{\gamma} + \left(1-\theta_t\right) \theta_t \left(e_t\right)^{\alpha} \left(H_{t-1}\right)^{\beta} \left(h_{t-1}\right)^{\gamma} - e_t \theta_t$$

The first term is the value of the human capital of the agent at time $t+1$, the second term is the value of the human capital, times the time spent working in period t, so it represents the wages earned in the first period. The last term is the money the agent borrowed to finance her education. For a strictly positive level of e_t, the optimal time spent on studying is given by:

$$\theta_t = 1 - \frac{e_t^{1-\alpha}}{2H_{t-1}^{\beta} h_{t-1}^{\gamma}} \tag{3}$$

For $e_t = 0$ the agent is indifferent between any choice of θ_t. I assume that in case of indifference the agent choose $\theta_t = 1 - \frac{e_t^{1-\alpha}}{2H_t^{\beta} h_t^{\gamma}}$.

With this information we can find the optimal level of e_t:

$$e_t = \left(\frac{2\alpha}{2-\alpha}\right)^{\frac{1}{1-\alpha}} H_{t-1}^{\frac{\beta}{1-\alpha}} h_{t-1}^{\frac{\gamma}{1-\alpha}} \tag{4}$$

Plugging this result in equation 3:

$$\theta_t = 2\frac{1-\alpha}{2-\alpha}$$

So the human capital of this individual is:

$$h_t = \frac{2-2\alpha}{2-\alpha}\left(\frac{2\alpha}{2-\alpha}\right)^{\frac{\alpha}{1-\alpha}} H_{t-1}^{\frac{\beta}{1-\alpha}} h_{t-1}^{\frac{\gamma}{1-\alpha}} \tag{5}$$

2.2 Equilibrium in the public system

Under the public education regime there is a voting process to decide, by majority rule, how much is spent on education, and, simultaneously, the tax rate necessary to support that expenditure.

I solve the individual's optimization problem in two steps. First, I take, as given, the tax rate τ_{t+1} (I assume a steady state tax rate, $\tau_{t+1} = \tau_t = \tau$) and the education expenses e_t and determine the optimal time of schooling:

$$\begin{array}{l} \max\limits_{\theta_t} c_{t,t+1} \\ \text{subject to} \\ c_{t,t+1} = \theta_t (e_t)^{\alpha} (H_{t-1})^{\beta} (h_{t-1})^{\gamma} (1-\tau) + (1-\theta_t)\,\theta_t (e_t)^{\alpha} (H_{t-1})^{\beta} (h_{t-1})^{\gamma} (1-\tau) \end{array}$$

Solving we get $\theta_t = 1$. The optimal tax rate, from this agent's perspective, can be determined by solving the problem:

$$\begin{array}{l} \max\limits_{c_{t,t+1}, e_t, \tau_{t+1}} c_{t,t+1} \\ \text{subject to} \\ c_{t,t+1} = \theta_t (e_t)^{\alpha} (H_{t-1})^{\beta} (h_{t-1})^{\gamma} (1-\tau) \\ e_t = \tau \int (e_t)^{\alpha} (H_{t-1})^{\beta} (h_{t-1})^{\gamma} dG_{t-1} \end{array}$$

where the second restriction is just a balanced government budget restriction, which can be solved with respect to τ:

$$\tau = \frac{(e_t)^{1-\alpha}}{(H_{t-1})^{\beta} \int (h_{t-1})^{\gamma} dG_{t-1}}$$

Given this result, the optimal e_t is:

$$e_t = \left(\alpha \left(H_{t-1}\right)^{\beta} \int \left(h_{t-1}\right)^{\gamma} dG_{t-1} \right)^{\frac{1}{1-\alpha}} \tag{6}$$

which implies:

$$\tau = \alpha \tag{7}$$

The preferred tax rate and the education expenditure are independent of the human capital of the agent's parent, and hence we have unanimous voting among the young. The old at time t will be indifferent about the taxes at $t+1$, and hence they will not veto this tax rate. In a steady state equilibrium, the tax rate is α and the education expenditure is given by 6.

The law of motion of the individual's human capital is:

$$h_t = \alpha^{\frac{\alpha}{1-\alpha}} \left(H_{t-1}\right)^{\frac{\beta}{1-\alpha}} \left(h_{t-1}\right)^{\gamma} \left(\int \left(h_{t-1}\right)^{\gamma} dG_{t-1} \right)^{\frac{\alpha}{1-\alpha}} \tag{8}$$

3 Comparisons between the two systems

3.1 Homogeneous agents

Consider the case of homogeneous agents. This hypothesis helps us to understand the growth implications of each of the systems by abstracting from inequality issues.

The distribution function G_{t-1} is degenerate and $E_t \left(h_{t-1}\right)^{\gamma} = \left(h_{t-1}\right)^{\gamma}$. Using 8 we have the law of motion for human capital under the public system:

$$h_t^{public} = \alpha^{\frac{\alpha}{1-\alpha}} \left(H_{t-1}\right)^{\frac{\beta}{1-\alpha}} \left(h_{t-1}^{public}\right)^{\frac{\gamma}{1-\alpha}}$$

For the private system we have:

$$h_t^{private} = \frac{2-2\alpha}{2-\alpha} \left(\frac{2\alpha}{2-\alpha}\right)^{\frac{\alpha}{1-\alpha}} H_{t-1}^{\frac{\beta}{1-\alpha}} \left(h_{t-1}^{private}\right)^{\frac{\gamma}{1-\alpha}}$$

Proposition 1 *if $h_0^{public} = h_0^{private}$ and $\alpha < 0.615$ then for $t \geq 1$ we always have $h_t^{public} > h_t^{private}$*

Proof. To prove this result I only need to show that

$$\alpha^{\frac{\alpha}{1-\alpha}} > \tfrac{2-2\alpha}{2-\alpha} \left(\tfrac{2\alpha}{2-\alpha}\right)^{\frac{\alpha}{1-\alpha}}$$
$$\Leftrightarrow 0 < \alpha < 0.615018$$

The last step was solved numerically using Maple V. ■

This is the key result of the paper. Contrary to Glomm and Ravikumar (1992) and Zhang (1996), the public school system may be growth enhancing relatively to a purely private system. To know which of the systems is actually better for growth, one has to estimate the elasticity of private income with respect to education spending. It is unlikely that the elasticity is larger than 60%, so, for the rest of the paper, I assume that $0 < \alpha < 0.615018$.

3.2 Heterogeneous agents

As in Glomm and Ravikumar (1992) and Zhang (1996), I assume that G_t is lognormal with parameters (μ_t, σ_t^2). The evolution of these parameters can be calculated using the human capital motion equations and the properties of the lognormal distribution.

For the public schooling, using 8:

$$\begin{aligned} h_t &= \alpha^{\frac{\alpha}{1-\alpha}} (H_{t-1})^{\frac{\beta}{1-\alpha}} (h_{t-1})^{\gamma} \exp\left(\left(\gamma\mu_{t-1} + \gamma^2\sigma_{t-1}^2\right)\frac{\alpha}{1-\alpha}\right) \\ \mu_t &= \frac{\alpha}{1-\alpha}\ln\alpha + \left(\frac{\beta+\gamma}{1-\alpha}\right)\mu_{t-1} + \frac{\alpha\gamma^2(1-\alpha)+\beta^2}{(1-\alpha)^2}\frac{\sigma_{t-1}^2}{2} \\ \sigma_t^2 &= \gamma^2\sigma_{t-1}^2 \end{aligned}$$

With private schooling, using 5:

$$\begin{aligned} \mu_t &= \ln\left(\frac{2-2\alpha}{2-\alpha}\left(\frac{2\alpha}{2-\alpha}\right)^{\frac{\alpha}{1-\alpha}}\right) + \left(\frac{\beta+\gamma}{1-\alpha}\right)\mu_{t-1} + \left(\frac{\beta}{1-\alpha}\right)^2\frac{\sigma_{t-1}^2}{2} \\ \sigma_t^2 &= \left(\frac{\gamma}{1-\alpha}\right)^2\sigma_{t-1}^2 \end{aligned}$$

Proposition 2 *Consider two economies, with the same education system and the same average human capital in period t. In period $t+1$ the economy with the lowest variance will have a higher average human capital.*

Proof. See proposition 6 of Glomm and Ravikumar (1992). ■

Proposition 3 *Consider two identical economies with different education regimes at time t. If income inequality is not too large then the economy with public schooling will have a higher average human capital at period $t+1$.*

Proof. Let u_t and σ_t^2 characterize the economy with public schooling, and u_t' and $\sigma_t'^2$ characterize the economy with private schooling. Given our assumptions $(\mu_t, \sigma_t^2) = (\mu_t', \sigma_t'^2)$.

$$\begin{aligned} H_{t+1} &= \exp\left(\frac{\alpha}{1-\alpha}\ln\alpha + \left(\frac{\beta+\gamma}{1-\alpha}\right)\mu_t + \frac{(1-\alpha)\gamma^2+\beta^2}{(1-\alpha)^2}\frac{\sigma_t^2}{2}\right) \\ H_{t+1}' &= \exp\left(\ln\left(\frac{2-2\alpha}{2-\alpha}\left(\frac{2\alpha}{2-\alpha}\right)^{\frac{\alpha}{1-\alpha}}\right) + \left(\frac{\beta+\gamma}{1-\alpha}\right)\mu_t + \frac{\beta^2+\gamma^2}{(1-\alpha)^2}\frac{\sigma_t^2}{2}\right) \end{aligned}$$

So $H_{t+1} > H'_{t+1}$ iff $\frac{\alpha}{1-\alpha}\ln\alpha + \frac{(1-\alpha)\gamma^2}{(1-\alpha)^2}\frac{\sigma_t^2}{2} > \ln\left(\frac{2-2\alpha}{2-\alpha}\left(\frac{2\alpha}{2-\alpha}\right)^{\frac{\alpha}{1-\alpha}}\right) + \frac{\gamma^2}{(1-\alpha)^2}\frac{\sigma_t^2}{2}$

Given our assumption that $0 < \alpha < 0.615018$ we have $\frac{\alpha}{1-\alpha}\ln\alpha > \ln\left(\frac{2-2\alpha}{2-\alpha}\left(\frac{2\alpha}{2-\alpha}\right)^{\frac{\alpha}{1-\alpha}}\right)$ and hence, if σ_t^2 is not too large, we will have $H_{t+1} > H'_{t+1}$. ■

We are left, again, with an empirical question. What is the meaning of saying that σ_t^2 is not too large? Consider for example: $\alpha = \frac{1}{3}$ and $\gamma = \frac{1}{3}$. In this case the theorem holds if $\sigma_t < 3.1675$. Taking the United States as a benchmark, and assuming that the income distribution follows a log normal distribution, then, table 3 of Gottschalk and Smeeding (1997) imply a value of σ around 0.6.

4 Conclusion

In this paper, I challenged the conclusion of Glomm and Ravikumar (1992) and Zhang (1996) that public education hinders economic growth when compared to a private education system. The conclusion of Zhang (1996) that public education may be growth enhancing only if income inequality is very big was also contested. The key to revert their results was to fully internalize the investment decisions in education.

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