E C O N O M I C S   B U L L E T I N

Industry profit maximizing R and D networks

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Abstract

In this paper, we extend the model of R and D network formation by Goyal and Moraga−Gonzàlez (2001) by allowing for imperfect spillovers among linked firms. We show that the complete network maximizes industry profit if spillovers for linked firms are below a threshold level. Furthermore, this threshold level turns out to be high in absolute terms in concentrated markets: when the number of firms is low, small departures from the case of perfect spillover imply that firms' private incentives to form links cannot be excessive with respect to their collective interest. This implies that the Goyal and Moraga−Gonzalez argument, for which excessive private incentives could explain the empirical stylized fact of R and D alliances instability, is no longer valid in these cases.

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1. Introduction

In their seminal paper on R&D network formation, Goyal and Moraga-Gonzàlez (2001) (GM from now on) prove the following result. Assume that firms are Cournot competitors in the product market. In the class of symmetric networks (where all firms have the same number of links), industry profits are maximized at an intermediate level of collaborative activity: that is, industry profits are maximized by a network which is neither empty nor complete (Proposition 7, p. 696).

In Proposition 6 (p. 695), GM show that the complete network is pairwise stable, in the sense that firms do not have unilateral incentives to sever one of their existing links. Although the authors cannot prove that such a structure is in general the unique pairwise stable network, Propositions 6 and 7 together may suggest that firms have individual incentives in forming links that are *excessive* with respect to their collective interest. This could explain, according to the authors, the robust stylized fact on the instability and high rate of failure of collaborative agreements between firms (Kogut 1988, Podolny and Page 1998):

"Our results (…) provide an explanation for why a large number of strategic alliances are unstable or terminated early, and they also explain why some alliances work well. In highly competitive markets, firms would "collectively" prefer not to form many collaborative ties, since in this way they could obtain higher profits. However, a pair of individual firms gain competitive advantage over the rivals by forming a collaboration and thus increase their profits. This implies that firms may have incentives to form too many links, which would lead to poor overall performance." (GM, p. 688)

In this paper, we extend their model of R&D networks, by allowing for imperfect spillovers among linked firms. We show that the complete network maximizes industry profits if spillovers for linked firms are below a threshold level. Furthermore, this threshold level turns out to be high in absolute terms in concentrated markets: when the number of firms is low, small departures from the case of perfect spillover imply that firms' private incentives to form links cannot be excessive with respect to their collective interest. This implies that GM argument is no longer a candidate to explain R&D alliances instability in these cases[1].

The paper is structured as follows. Section 2 describes the model. In section 3 we prove our result. Section 4 concludes.

2. The model

We consider a three-stage game Γ, which coincides with the one presented in Goyal and Moraga-Gonzàlez (2001). In the first stage, firms can form collaborative links, which give raise to a well specified R&D network. Given the network structure, firms choose non-cooperatively their R&D effort. Given the level of R&D efforts, the cost function of each firm is determined. Finally, given costs, firms compete in the market.

Let $N = \{1,..,n\}$ be the set of firms. The R&D network resulting from the first stage is denoted by g. When we write $ij \in g$, this implies that there is a collaborative link between i and j. We define $N_i(g) = \{j \in N \setminus \{i\} : ij \in g\}$ as the set of firms having a collaborative link with i. Also, we indicate with $n_i(g) = |N_i(g)|$ the cardinality of the set of partners for firm i in g.

[1] The model by GM is extended by Song and Vannetelbosch (2005) to analyze the role of government policies. However, they focus on a three firms' industry and do not consider the issue of networks which maximize industry profits.

If g is the network resulting from the first stage, we denote by $\Gamma(g)$ the corresponding subgame. In such a subgame, firms fix their level of R&D expenditures correctly anticipating the Cournot outcome of the last stage. Firm *i*'s action in this stage is given by $e_i \in [0, \bar{c}]$, where e_i is the effort put by firm i in the R&D activity. The cost associated with e_i is given by $C(e_i) = e_i^2$. Consequently, $e = (e_i)_{i \in N}$ is the action profile of $\Gamma(g)$. Given the R&D investments *e*, the unit cost of production for $i \in N$ is determined by:

$$c_i(g,e) = \bar{c} - e_i - \sum_{j \in N_i(g)} \beta e_j \tag{1}$$

This implies that R&D spillovers among non-linked firms are 0 (as in GM when they discuss the general case with *n* firms), while the spillover for linked firms is $0 < \beta \leq 1$. The case discussed in GM implies $\beta = 1$.

The main motive for which spillovers among linked firms could be imperfect is related to the nature of knowledge accumulated through R&D investment. There is evidence that this knowledge is to some extent "tacit" and then not perfectly transmittable across firms. Even in the case of transmission of codified knowledge, receiving firms need (tacit) knowledge to interpret the piece of codified knowledge, and then "absorptive capacity" can be imperfect (Cowan and Foray 1997).

Finally, given the unit cost $c_i(g,e)$, firms compete in the market choosing quantities. $q_i(g,e) \in [0, A]$ denotes the action taken by firm *i* at this stage. The inverse demand function is linear: $p = A - \sum_{i \in N} q_i(g,e)$. In the Cournot-Nash equilibrium, quantities are given by:

$$q_i(g,e) = \frac{A - nc_i(g,e) + \sum_{j \neq i} c_j(g,e)}{n+1} \tag{2}$$

Net profits are given by:

$$\Pi_i(g,e) = (q_i(g,e))^2 - C(e_i) \tag{3}$$

In this paper, we will exclusively focus on symmetric networks. Networks are symmetric when all the firms have the same number of links, i.e. $n_i(g) = k \;\; \forall i$. Then, a symmetric network can be characterized by a number *k* identifying the number of links for each firm. GM define *k* as the degree of collaborative activity. When $k = n-1$, we obtain the complete network. We will call g^k a symmetric network of degree *k*.

Consider now a representative firm *i* in a symmetric network of degree *k*. Given the network g^k and other firms' investments, the representative firm *i* maximizes $\Pi_i(g,e)$ in e_i subject to $e_i \in [0, \bar{c}]$. We need to consider three types of firms: a) firm *i*; b) k firms linked to firm *i* (subscript *l*); c) $n-k-1$ firms not linked to *i* (subscript *m*). This results in a specific cost structure for each type of firm:

$$c_i(g^k) = \bar{c} - e_i - \beta k e_l \tag{4a}$$

$$c_l(g^k) = \bar{c} - e_l - \sum_{j \in N_l(g^k)} \beta e_j \tag{4b}$$

$$c_m(g^k) = \bar{c} - e_m - \sum_{j \in N_m(g^k)} \beta e_j \tag{4c}$$

Plugging (4a-4c) into firm *i*'s profit function and deriving with respect to e_i, we obtain the following first order condition[2]:

$$\frac{\partial \Pi_i}{\partial e_i} \equiv 2q_i(g,e)[n - k\beta] - 2e_i = 0 \tag{5}$$

Invoking symmetry across all firms, we impose $e_i = e_l = e_m = e(g^k)$. Rearranging the first order condition, we obtain the equilibrium effort:

$$e(g^k) = \frac{(A - \bar{c})(n - \beta k)}{(n+1)^2 - (n - \beta k)(1 + \beta k)} \tag{6}$$

Plugging (6) into (4a), one obtains the unit cost of production for the representative firm:

$$c(g^k) = \frac{\bar{c}(n - \beta k) - A(n - \beta k)(1 + \beta k)}{(n+1)^2 - (n - \beta k)(1 + k\beta)} \tag{7}$$

Plugging (7) into (2), and then (2) and (6) into (3), we finally obtain the level of profits for a symmetric network of degree *k*:

$$\Pi(g^k) = \frac{(A - \bar{c})^2((n+1)^2 - (n - \beta k)^2)}{((n+1)^2 - (n - \beta k)(\beta k + 1))^2} \tag{8}$$

Since every firm obtains the same level of profits in a symmetric network, the network which maximizes industry profits is the network which maximizes (8).

3. Results

We can now prove our result.

Proposition: *The complete network maximizes industry profits for* $\beta \le \beta^*$, *where* β^* *is the unique solution to* $\Pi(g^{n-1}) = \Pi(g^{n-2})$.

Proof: Suppose that *k* is a continuous variable in the range $[0, n-1]$. If we derive $\Pi(g^k)$ with respect *k*, we obtain that the first derivative has the same sign as $F = [(2n - 3\beta k - 1)(n+1)^2 - (n - \beta k)^3]$.

In $k = 0$, F is positive and independent of β ($F|_{k=0} = 2n^3 + 3n^2 - 1 > 0 \ \ \forall n \ge 4$). If $k = n-1$, we have $F|_{k=n-1} = [(2n - 3\beta(n-1) - 1)(n+1)^2 - (n - \beta(n-1))^3]$. This quantity is positive when $\beta \to 0$, negative in $\beta = 1$, and its first derivative with respect to β is negative

[2] It can be verified that second order conditions are satisfied.

(being equal to $\beta(n+1-\beta)^2-(n-1)(n+1)^2$, which is negative since $\beta<n-1$ and $n+1-\beta<n+1$). This implies that a value $\underline{\beta}$ exists for which $F=0$ at $k=n-1$. Finally:

$$\frac{\partial F}{\partial \beta}=-3\beta\left[(n+1)^2-(n-\beta k)^2\right]<0$$

These relations together imply that there are two relevant cases to consider: i) the first derivative of the profit function is always positive. This is true for $\beta \leq \underline{\beta}$. In this case the complete network maximizes profits; ii) the first derivative is positive and then negative. Then, treating k as continous, there will exist a value $k^*<n-1$ for which $F=0$, and the profit function is maximized at $k^*<n-1$.

Denote with $\lfloor k^* \rfloor$ the floor of k^* (i.e. the greatest integer less than or equal to k^*) and with $\lceil k \rceil$ the ceiling of k^* (i.e. the least integer greater than or equal to k^*). If we let k take only integer values, and given the behavior of $\Pi(g^k)$, the actual maximum is $\max\left\{\Pi(g^{\lfloor k^* \rfloor}),\Pi(g^{\lceil k^* \rceil})\right\}$. Define as $\overline{\beta}$ the value for which $F=0$ at $k=n-2$. For any $\beta \geq \overline{\beta}$, the maximum will be in some $k<n-2$. Consider finally $\Pi(g^{n-1})-\Pi(g^{n-2})$. Since it is positive in $\underline{\beta}$, negative in $\overline{\beta}$ and continous, from Weierstrass intermediate value theorem there is at least one β where $\Pi(g^{n-1})-\Pi(g^{n-2})=0$ (Weierstrass intermediate value theorem states that if f is a function which is continuous at any point of the interval [a,b] and $f(a)f(b)<0$ then $f(x)=0$ at some $x\in(a,b)$). Notice that if this value is unique (call this value β^*), then for all $\beta \leq \beta^*$ the complete network maximizes industry profit. Proving analytically that $\Pi(g^{n-1})-\Pi(g^{n-2})$ has a unique solution turns out to be difficult, since it is a polynomial of high order both in β and n. However, numerical solutions, reported above, show that it is actually unique. Then, the proposition follows. ■

Table 1 reports the exact value of β^* as a function of n. This value is independent from A and $\bar{c}$, since these parameters enter the profit function only as a factor $A-\bar{c}$. From the table we see that β^* is high in concentrated markets: it is about 0.91 with $n=4$, 0.79 with $n=6$ and 0.75 for $n=8$. β^* is decreasing with n, but it is above 0.65 even for a very large number of firms.

Table 1: β^* as a function of n

n	4	6	8	10	12	14	16	18	20
β^*	**0.9097**	**0.7944**	**0.7505**	**0.7274**	**0.7131**	**0.7034**	**0.6964**	**0.6911**	**0.6869**
n	*30*	*40*	*50*	*60*	*70*	*80*	*90*	*100*	*200*
β^*	**0.6749**	**0.6691**	**0.6657**	**0.6635**	**0.6619**	**0.6607**	**0.6598**	**0.6591**	**0.6559**

The intuition for our result is straightforward. The reason why GM obtain an intermediate degree of collaboration as optimal relies on the fact that equilibrium effort is declining in k. Indeed, if $\beta=1$, from equation (6) we obtain:

$$\frac{\partial e(g^k)}{\partial k} = -\frac{(A-\bar{c})[(n+1)^2-(n-k)^2]}{[(n+1)^2-(n-k)(1+k)]^2} < 0$$

The negative relation between R&D efforts and degree of cooperation is due to two effects. If a firm increases its number of collaborators, by increasing its R&D effort it reduces the cost of more firms, making them tougher competitors. Furthermore, since each firm has more links, it operates *ceteris paribus* at lower costs, and this reduces the incentive to invest in R&D for non-linked firms.

This negative relation leads to a unit cost that is non-monotonic in *k*: when *k* increases, firms have access to more firms' R&D efforts, but these are lower. When *k* is large, the unit cost can actually increase with *k*. Finally, effort enters profits in two ways: through the quantity produced (which is non-monotonic in *k*) and through investement costs in R&D (which are declining in *k*). GM shows that the net effect is such that the complete network is never profit maximizing.

When $\beta < 1$, the negative effect of an increase in *k* is smaller, since the cost reduction obtained by linked firms is lower, and non linked firms operate at a higher cost, *ceteris paribus*. From equation (6), we obtain:

$$\frac{\partial e(g^k)}{\partial \beta} = -\frac{(A-\bar{c})\beta[(n+1)^2-(n-\beta k)^2]}{[(n+1)^2-(n-\beta k)(1+\beta k)]^2} < 0$$

When β is sufficiently low, the net effect of the forces discussed above is such that the complete network maximizes profits.

4. Conclusion

In this paper we showed how the result obtained in the GM model of R&D networks concerning the symmetric network structure which maximizes industry profits depends on their assumption of perfect spillovers between linked firms. When spillovers are imperfect, the complete network is industry profit-maximizing if spillovers are sufficiently low.

We did not discuss, in this paper, the notions of stability of the R&D network. However, the result we want to stress is that, when $\beta \leq \beta^*$, firms' private incentives in forming links (captured for instance by a notion of pairwise stability) cannot be excessive compared to firms' collective incentives (captured by industry profit-maximizing networks). This means that, in this case, the misalignment of private and collective incentives cannot be used to explain the stylized fact of the instability of R&D collaborative agreements.

Whether β^* is "high" or "low" in absolute terms may be a matter of empirical scrutiny. We argued that, in principle, there are reasons why spillovers can be imperfect among linked firms, and this should be taken in account when considering GM results. More generally, our result suggests that, when discussing the properties of R&D networks, one should consider the interplay of technological effects of cooperation, here captured by the spillover parameter β, and the nature of market competition.

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