Capacity Choice in the Mixed duopoly with Product Differentiation

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Abstract

This note shows that when products are complements in the mixed duopoly market, both public and private firms choose excess capacity. This contrasts with substitute case, where public firm strategically chooses under-capacity while private firm keeps holding excess capacity.

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1. Introduction

The capacity choice problem in the mixed oligopolies has been analyzed in the recent papers of Nishimori and Ogawa (2004) and Lu and Poddar (2005)[1]. The main result obtained in these studies is that, in the mixed duopoly market, the public firm strategically chooses under-capacity while the private firm chooses excess capacity. This result is sharp contrast with the conventional wisdom that holding excess capacity plays an essential role as a strategic device in the pure oligopoly market. While two papers cited above employ the model of deterministic demand, Lu and Poddar (2006) extends the analysis to an uncertain demand environment. They use a simple two-stage mixed duopoly model where firms choose capacity in the first stage without knowing which state of nature is going to be realized and show that public firm may choose excess capacity depending on the realized demand.

This paper is also concerned with capacity choice of firms in the mixed market. However, its focus is somewhat different from that of Lu and Poddar (2006). In this paper, we focus on the effect of product differentiation on the capacity choice behavior, while the previous literature use a model in which products in a mixed duopoly are perfect substitutes. Our result shows that while the production differentiation under substitute products does not alter the result that public firm chooses under-capacity and private firm chooses excess capacity, the result does not carry over when products are complements; Both public and private firms choose excess capacity in the mixed market.

2. Model

There are two firms ($i = 1, 2$) operating in a differentiated good market with inverse demand given by

$$p_i = a - q_i - bq_j \qquad a > 0, b \in (-1, 1), b \neq 0, \tag{1}$$

where p_i is a price for firm i and q_i denotes the output of firm i. (1) is derived by assuming that the surplus of the representative consumer is given by[2].

$$CS = a(q_1 + q_2) - 0.5(q_1^2 + 2bq_1q_2 + q_2^2) - p_1q_1 - p_2q_2.$$

The substitutability of the products will be measured by $b \in (-1, 1)$, where positive b is associated with substitutes, negative values with complements. While firm 1 is a

[1] Nett (1994), Matsumura and Matsushima (2003), and Ishibashi and Matsumura (2005) analyze the endogenous determination of cost structure in a mixed market under different settings.

[2] In this paper, we employ a version of the consumer surplus function used by Vives (1984), Furth and Kovenock (1993), and Bárcena-Ruiz and Garzón (2003) etc.. Following these literature, the consumer surplus can be obtained by inserting (1) into CS function. From this substitution, we have $CS = a(q_1 + q_2) - (q_1^2 + 2bq_1q_2 + q_2^2)/2 - [(a - q_1 - bq_2)q_1 + (a - q_2 - bq_1)q_2]$, which can be rearranged as $CS = 0.5(q_1^2 + q_2^2) + bq_1q_2$. Notice that if the two products are perfectly substitutes ($b = 1$), CS reduces to a standard expression of $Q^2/2$, where $Q \equiv q_1 + q_2$.

profit maximizing private firm, firm 2 is a firm maximizing the social surplus which is the summation of consumer surplus (CS) and firms' profits ($\pi_1 + \pi_2$).

The firms have same technology, represented by the cost function, $C(q_i, x_i)$, where q_i and x_i are production quantity and capacity of firm i, respectively. We assume that firms choose their capacity in the first stage. After observing the capacity choice, firms choose their quantities. Following Vives (1986), Nishimori and Ogawa (2004) and Lu and Poddar (2005), we simply assume that the cost function is given by

$$C(q_i, x_i) = mq_i + (q_i - x_i)^2.$$

Under this U-shaped cost function, the long-run average cost is minimized when quantity equals to production capacity, $q_i = x_i$.

The objective function of firm 1 is its profit, given by

$$\pi_1 = p_1 q_1 - mq_1 - (q_1 - x_1)^2. \tag{2}$$

The firm 2 maximizes the social surplus (SS), given by

$$SS = CS + \pi_1 + \pi_2, \tag{3}$$

where $CS = 0.5(q_1^2 + q_2^2) + bq_1 q_2$ and $\pi_i = (p_i - m)q_i - (q_i - x_i)^2$.

3. Equilibrium

Following the standard equilibrium concept, we solve the model from the second stage.

Second Stage. Given the production capacities, the firm 1 maximizes (2) with respect to q_1 and firm 2 maximizes (3) with respect to q_2. The maximization problem of each firm yields

$$\begin{aligned} q_1 &= (a - m + 2x_1 - bq_2)/4, \quad &(4)\\ q_2 &= (a - m + 2x_2 - bq_1)/3. \quad &(5)\end{aligned}$$

From (4) and (5), we obtain the output levels as

$$\begin{aligned} q_1 &= \frac{(3-b)(a-m) - 2bx_2 + 6x_1}{12 - b^2}, \quad &(6)\\ q_2 &= \frac{(4-b)(a-m) + 8x_2 - 2bx_1}{12 - b^2}. \quad &(7)\end{aligned}$$

First Stage. In the first stage, firms know that the decision on the capacity level has effects on the firms' output decision in the second stage. Hence, we can formulate the maximization problem of private firm as follows.

$$\begin{array}{ll} \max_{x_1} & \pi_1 = (a - q_1 - bq_2)q_1 - mq_1 - (q_1 - x_1)^2, \\ \text{s.t.} & (6) \text{ and } (7). \end{array}$$

Solving the problem, we have

$$x_1 = \frac{12(3-b)(a-m) - 24bx_2}{72 - 24b^2 + b^4}. \tag{8}$$

Similarly, the maximization problem for the public firm can be formulated as

$$\begin{aligned} \max_{x_2} SS &= [0.5(q_1^2 + q_2^2) + bq_1q_2] + [(a - q_1 - bq_2)q_1 - mq_1 - (q_1 - x_1)^2] \\ &\quad +[(a - q_2 - bq_1)q_2 - mq_2 - (q_2 - x_2)^2], \\ \text{s.t.} &\quad (6) \text{ and } (7). \end{aligned}$$

Solving the problem, we have

$$x_2 = \frac{(48 - 15b - 3b^2 + b^3)(a-m) - 2b(15 - b^2)x_1}{48 - 18b^2 + b^4}. \tag{9}$$

From (6)-(9), production quantity and capacity levels are given by

$$\begin{aligned} x_1 &= \frac{12(1-b)(a-m)}{24 - 18b^2 + b^4}, \\ x_2 &= \frac{(a-m)(b^3 - 3b^2 - 15b + 24)}{24 - 18b^2 + b^4}, \\ q_1 &= \frac{(1-b)(12 - b^2)(a-m)}{24 - 18b^2 + b^4}, \\ q_2 &= \frac{(288 - 168b - 72b^2 + 26b^3 + 4b^4 - b^5)(a-m)}{(24 - 18b^2 + b^4)(12 - b^2)}. \end{aligned}$$

Hence, we have

$$x_1 - q_1 = \frac{b^2(1-b)(a-m)}{24 - 18b^2 + b^4}, \tag{10}$$

$$x_2 - q_2 = -\frac{b(1-b)(a-m)}{24 - 18b^2 + b^4}. \tag{11}$$

From (10) and (11), we obtain the following result.

Proposition. Assume $a > m$. For any $b \in (-1, 1)$, private firm 1 chooses over-capacity, $x_1 > q_1$. When the products are substitutes, $b \in (0, 1)$, public firm 2 chooses under-capacity, $x_2 < q_2$, and it chooses excess capacity when the products are complements, $b \in (-1, 0)$.

There is a simple mechanism that justifies the behavior of public firm. The public firm tries to make private firm produce much in the duopoly market. When the products are substitute $b \in (0, 1)$, from (6), we see that there is a negative relationship between the capacity level of public firm and the output level of private firm. In this case, the public firm can improve the social surplus by reducing it's own capacity. Enlarging the production share in the market is desirable for the private firm. Hence, the private firm chooses over capacity and the public firm chooses under capacity as a strategic device. On the other hand, when the products are complements ($b \in (-1, 0)$), an increase in capacity level of public firm increases the output level of private firm. Hence, the under-capacity strategy for public firm does not carry over in the case of product complements. In fact, for every $b \in (-1, 0)$, exactly the opposite is true.

4. Conclusion

This paper introduces the product differentiation into a mixed duopoly model to get new result concerning the capacity choice behavior of public firm. The main result obtained in the previous literature is that, in the mixed duopoly market with perfect substitute products, the public firm strategically chooses under-capacity and private firm keeps holding excess capacity. This paper shows this result does not hold in the market where the products are complements. In fact, for every complementarity's parameter, $b \in (-1, 0)$, both public and private firms choose excess capacity.

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