E C O N O M I C S   B U L L E T I N

On dynamic Chamberlin−Heckscher−Ohlin trade patterns

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Abstract

Applying Atkeson and Kehoe's (2000) dynamic model to the dynamic Chamberlin−Heckscher−Ohlin approach, we examine the role of the timing of development (e.g., the removal of trade barriers) as a determinant of trade patterns.

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1 Introduction

Over the past several decades a vast literature has developed on the emergence of intra-industry trade (i.e., two-way trade of similar products). Helpman's (1981) seminal integration of the monopolistic competition trade model into the two-country by two-factor by two-good Heckscher-Ohlin (HO) framework has been extended and made popular by Helpman and Krugman (1985). This integration led to the widely-held belief that the HO and Chamberlinian monopolistic competition models are complementary in nature. Helpman (1981) called it the *'Chamberlin-Heckscher-Ohlin'* (CHO) approach.

Based on the CHO approach, Helpman and Krugman (1985, p. 173) advanced the empirical hypothesis that, on average, the more similar two countries' factor endowment ratios are, the larger the share of intra-industry trade within their bilateral trade volume will be.[1] However, since Helpman and Krugman's prediction depends on static analyses, differences in factor endowment ratios are exogenously given and not well explained. In this note, by examining the dynamic CHO model, we try to provide some theoretical background for this hypothesis on trade patterns.[2] Recently, Atkeson and Kehoe (2000) showed that in a dynamic HO model, *the timing of development* (e.g., the removal of trade barriers) affects the path of a country's development. Applying their dynamic model to the CHO approach, we examine the role of the timing of development as a determinant of trade patterns.

Section 2 provides the basic setup of the dynamic CHO model. Section 3 presents a trade-pattern proposition.

2 The Model

Consider a world economy consisting of a large number of small countries that differ only in the timing of their development. Some countries, the *early bloomers*, reach their steady states before other countries, the *late bloomers*, begin to develop. There are two types of commodities, differentiated products (Good 1) and consumable capital (Good 2), which are produced using reproducible capital, K, and a primary and time-invariant factor of production, L. The consumable capital is produced under constant returns to scale technology and we choose it as the numeraire.

The preferences of consumers in each country are given by

$$\int_0^\infty uX\,dt = \int_0^\infty f\left[U(V, C_2)\right] X\,dt, \tag{1}$$

$$\dot{X} = -\rho X, \tag{2}$$

where V is the quantity index for differentiated products, C_2 is the consumption of the consumable capital, and ρ is the rate of time preference. It will be assumed

[1] This hypothesis finds support in Helpman (1987) for the 1970s.

[2] Recently several studies have developed the dynamc HO model. See, Nishimura and Shimomura (2002, 2005), Shimomura (1992, 1993, 2004).

that U is homothetic and f is increasing in its arguments. Quantity index V takes the following Dixit-Stiglitz (1977) form:

$$V = \left[\int_{i=0}^{N} [x(i)]^{(\sigma-1)/\sigma} di \right]^{\sigma/(\sigma-1)}, \quad \sigma > 1, \tag{3}$$

where N is the *total* number of differentiated products, $x(i)$ is the consumption of the i-th variety of differentiated products, and σ is the elasticity of substitution between varieties.

Solving the static expenditure minimizing problem, we can define the expenditure function as

$$e(P)\psi(u) \equiv (\min PV + C_2, s.t., u = f[U(V, C_2)]), \tag{4}$$

where P is the price index for differentiated products and $\psi(u)$ is the inverse function of f. Given that the equilibrium is symmetric, we can derive the following conditions:

$$\begin{aligned} e'[N^{1/(1-\sigma)}p]\psi(u) &= N^{\sigma/(\sigma-1)}x, \\ N^{1/(1-\sigma)}e'[N^{1/(1-\sigma)}p]\psi(u) &= Nx. \end{aligned}$$

Assume that differentiated products are more capital-intensive than consumable capital. Differentiated products are produced by monopolistically competitive firms under increasing returns technology, while consumable capital is produced by competitive firms under constant returns technology. Assume that each firm in the differentiated products sector has the homothetic total cost function $c^1(w,r)\phi(y)$, where y is the output level of each firm. There are significant economies of scale: $\phi(y)/y$ is decreasing over the relevant range of output levels y. The marginal revenue will be equated to the marginal cost: $p[1-(1/\sigma)] = c^1(w,r)\phi'(y)$. Furthermore, free entry implies that price equals average cost: $p = \frac{c^1(w,r)\phi(y)}{y}$. By combining these conditions, one can easily see that all varieties will have the same output level $\bar{y}$, which is defined by[3]

$$1 - \frac{1}{\sigma} = \frac{\bar{y}\phi'(\bar{y})}{\phi(\bar{y})}. \tag{5}$$

The constraints on labor and capital within a country are

$$c_w^1(w,r)\phi(\bar{y})n + c_w^2(w,r)y_2 = L, \tag{6}$$

$$c_r^1(w,r)\phi(\bar{y})n + c_r^2(w,r)y_2 = K, \tag{7}$$

where n is the number of differentiated products produced in the country.

Then, by defining $\xi \equiv \bar{y}/\phi(\bar{y})$, the zero-profit conditions can be written as

$$\xi p = c^1(w,r), \tag{8}$$

$$1 = c^2(w,r), \tag{9}$$

[3]This result depends crucially on homotheticity in production. See, Dixit and Norman (1980, pp. 284–5).

and we can obtain factor price functions $w(\xi p)$ and $r(\xi p)$. Utilizing these factor price functions, the national income is shown as

$$r(\xi p)K + w(\xi p)L. \tag{10}$$

The partial derivative of the national income is equal to the aggregate national output of differentiated products:

$$n\bar{y} = \xi r'(\xi p)K + \xi w'(\xi p)L. \tag{11}$$

From (10), we can obtain another condition for consumers:

$$\dot{K} = r(\xi p)K + w(\xi p)L - e[N^{1/(1-\sigma)}p]\psi(u). \tag{12}$$

Each consumer maximizes (1) subject to both (2) and (12). Associated with this problem is the Hamiltonian

$$H \equiv uX + \lambda[r(\xi p)K + w(\xi p)L - e(N^{1/(1-\sigma)}p)\psi(u)] + \delta\rho X, \tag{13}$$

where λ and δ are the shadow prices of K and X. The necessary conditions for optimality are

$$0 = u - \lambda e(N^{1/(1-\sigma)}p)\psi'(u), \tag{14}$$

$$\dot{\lambda} = -\lambda r, \tag{15}$$

$$\dot{\delta} = \rho\delta - u. \tag{16}$$

Letting $Z \equiv \lambda/X$ and combining (2) and (15), we can obtain

$$\dot{Z} = Z[\rho - r(\xi p)]. \tag{17}$$

3 Trade between Early and Late Bloomers

In comparing the development of early and late bloomers, we first solve for the steady-state output and prices for early bloomers. Imagine, for simplicity, that all but one of the countries start developing at the same time with the same initial capital endowment. Assume also that the late bloomer is a small country, so its behavior does not affect the time path for the world price.

Clearly, in equilibrium, all the identical early blooming countries make the same choices except in regard to the range of differentiated products that they produce; hence, the equilibrium for this world economy is the same as one for a single large country that does not trade.[4] Using this equivalence, the world resource constraints are the same as (6) and (7).

We denote the steady-state techniques of production for the early bloomers by $\bar{k}_1$ and $\bar{k}_2$, where $\bar{k}_i \equiv c_w^i/c_r^i$. Note that in a steady state, the world economy

[4]See Atkeson and Kehoe (2000).

will clearly produce some differentiated products. It will also produce consumable capital. Thus,

$$\bar{k}_2 < \bar{k} < \bar{k}_1, \tag{18}$$

where $\bar{k} \equiv K/L$ and (8) and (9) hold. These steady-state prices and quantities are found as the solutions to the following equilibrium conditions:[5]

$$\begin{aligned} \rho &= r(\xi p), & (19)\\ e[N^{1/(1-\sigma)}p]\psi(u) &= r(\xi \bar{p})K + w(\xi p)L, & (20)\\ \xi r'(\xi p)K + \xi w'(\xi p)L &= N\bar{y}, & (21)\\ N^{1/(1-\sigma)}e'[N^{1/(1-\sigma)}p]\psi(u) &= N\bar{y}. & (22) \end{aligned}$$

These four equations determine four variables: u, p, N, and K. Since the factor endowment ratio is identical among all early bloomers, there is no incentive for Heckscher-Ohlin trade (i.e., exchange of differentiated products for consumable capital). Still, since each country specializes in a different range of differentiated products, an incentive for intra-industry trade remains: all trade flows between early bloomers consist of two-way trade of differentiated products.

Now consider the small country starting the process of development late. In particular, assume that the rest of the countries in the world have already reached their steady states when this small country removes domestic distortions that have kept its capital stock relatively low and, hence, kept the country poor.

The path of development for the late bloomer depends on the late bloomer's capital-labor ratio relative to the ratios in the cone of diversification. This cone can be described by means of a Lerner diagram as in Figure 1: each isoquant describes the combinations of factor inputs that produce an output level $1/p$ given the production function. The cone is defined as the set of capital and labor endowments with which the late bloomer produces both differentiated products and consumable capital. The cone consists of the pairs of capital and labor (K, L) such that the ratio $k = K/L$ is in the interval $[\bar{k}_2, \bar{k}_1]$. When the late bloomer's factor endowments lie in this cone, the late bloomer's factor prices are equal to those of the early bloomers.

Notice from Figure 1 that if the late bloomer starts the process of development with some $k_0 \in (0, \bar{k}_2)$, then over time the country's capital-labor ratio converges to the point A at the edge of the steady-state cone of diversification. This process can be explained as follows: The late bloomer starts outside the cone of diversification and specializes in producing consumable capital. While the late bloomer is outside this cone, it has a lower capital-labor ratio than the early bloomers use in the production of consumable capital and, hence, a higher rental rate on capital. Thus, the late bloomer accumulates capital until its capital-labor ratio equals that used in the rest of the world to produce consumable capital. With this ratio, the late bloomer's rental rate on capital

[5] Note that (19) and (20) are derived from (17) and (12).

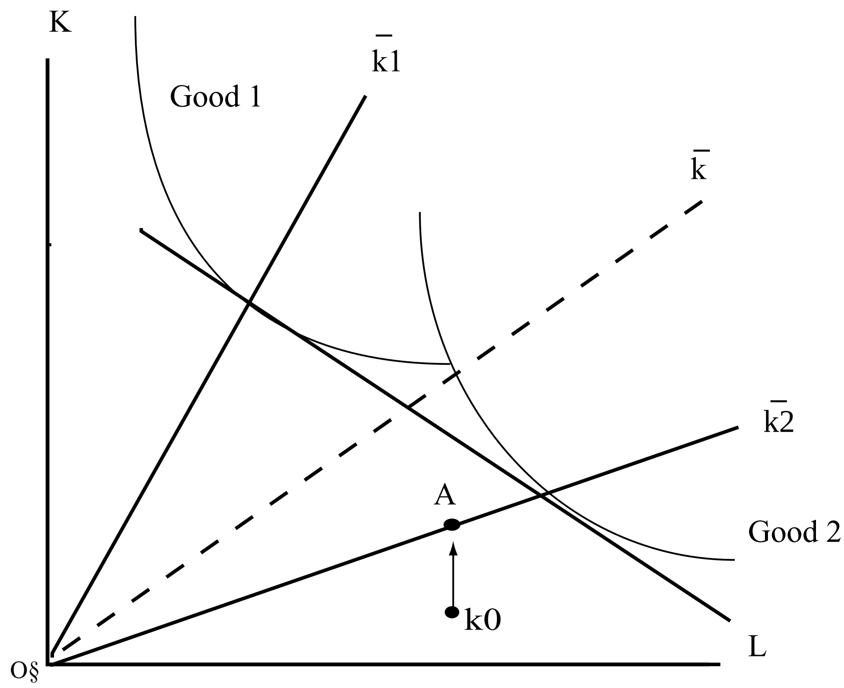

FIGURE 1§

fall to the steady-state rate prevailing in the rest of the world, consumers in the late blooming country have no further incentive to accumulate capital, and the country's growth stops at the lower edge of the cone of diversification.

The capital-labor ratio $\bar{k}_2$ is strictly less than that of the early bloomers, $\bar{k}$. Although factor prices are equalized between early and late bloomers, differences in factor endowment ratios remain in the steady state: the late bloomer never produces differentiated products (i.e., the capital-intensive good) and, instead, imports them in exchange for consumable capital.[6]

Proposition 1: *While trade between early and late bloomers is inter-industry trade, trade within early bloomers is intra-industry trade.*

Applying Atkeson and Kehoe's (2000) dynamic multi-country model to the CHO framework, we emphasize the role of *the timing of development* (e.g., the removal of trade barriers) as a determinant of trade patterns. Although our result depends critically on several restrictive assumptions, it establishes a link between the workhorse model of monopolistic competition and the timing of development. Hopefully this analysis provides a useful paradigm for considering how the timing of developent works as a determinant of intra-industry trade.

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[6] Also notice that the steady-state output for the late bloomer $(r\bar{k}_2 + w)$ is lower than that for the early bloomers.

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