E C O N O M I C S B U L L E T I N

Note on a generalized wage rigidity result

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Abstract

Considering Cournot competition, this note shows that, if the firms differ in labor productivities, the equilibrium wage rates under a centralized labor union are not independent of the number of firms and product differentiation if the labor union charges a uniform wage rate. However, if the centralized labor union can discriminate wage rate between the firms, the equilibrium wage rates do not depend on the number of firms and product differentiation. Hence, whether the wage rigidity result of Dhillon and Petrakis (2002) holds with asymmetric firms depends on the wage setting behavior of the labor union. The effects of the number of firms and product differentiation on the equilibrium wage rate are also shown.

I would like to thank an anonymous referee of this journal for helpful comments. The usual disclaimer applies.
Citation: Mukherjee, Arijit, (2007) "Note on a generalized wage rigidity result." *Economics Bulletin*, Vol. 10, No. 12 pp. 1-9
Submitted: August 7, 2007. **Accepted:** October 9, 2007.
URL: http://economicsbulletin.vanderbilt.edu/2007/volume10/EB-07J50004A.pdf

1. Introduction

The empirically observed wage rigidity has attracted attention of the researchers working on Industrial Organization and Labor Economics. Recently, Dhillon and Petrakis (2002) show that, under fairly general conditions, if there is a centralized labor union, the equilibrium wag rates do not depend on the number of firms and product differentiation.[1] Considering Cournot competition, this note extends this line of research with asymmetric firms.

We show that if the firms differ in labor productivities, the equilibrium wage rates under a centralized labor union are not independent of the number of firms and product differentiation if the labor union charges a uniform wage rate. However, if the centralized labor union can discriminate wage rate between the firms, the equilibrium wage rates do not depend of the number of firms and product differentiation. Hence, whether the wage rigidity result of Dhillon and Petrakis (2002) extends to the case of asymmetric firms depends on the wage setting behavior of the labor union.

Empirical evidence suggests that, in many situations, centralized labor unions charge uniform wage rates irrespective of the differences between the firms. As discussed in Haucap et al. (2000 and 2001), a common feature of many labor markets in continental Europe is "coverage extension rules", which implies that some or all employment terms are made generally binding for all industry participants and not only for the members of unions and employers' associations. "In Germany, for example, collective wage agreements between a union and an employers' association can be made compulsory even for independent employers through so-called *Allgemeinverbindlicherklärung* (AVE) … The Ministry of Labor can, on application of either unions or employers' associations, use an AVE to make some or all terms of a collectively negotiated employment contract generally binding for an entire industry, where otherwise only those unions, employers and employers' associations that have actually negotiated and signed the contract would be directly bound by it (§3 I TVG)" (Haucap et al., 2001). It is also noted in Haucap et al. (2001) that the number of AVEs almost continuously increased from 448 in 1975 to 588 in 1998.[2] Thus, it justifies our analysis with uniform wage setting by a centralized labor union.[3]

The remainder of the paper is organized as follows. The next section describes the model and shows the results. Section 3 concludes.

2. The model and the results

Since, in the presence of asymmetric firms, the calculations for showing the effects of the number of firms and product differentiation are cumbersome, we show these effects separately. In section 2.1, we consider the case of a homogeneous product and show the effects of the number of firms. In section 2.2, we consider a duopoly market structure and show the effects of product differentiation.

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[1] The irrelevance of the number of downstream firms on the upstream input price can also be found in the earlier works by Greenhut and Ohta (1976) and Tyagi (1999).

[2] Haucap et al. (2001) also show when the labor union may prefer uniform wage over discriminatory wage.

[3] The wage bargaining by the labor union for the UK Universities may also support the uniform wage setting by a centralized labor union in the presence of productivity differences. While the people working in different Universities may differ in productivities, the national labor union bargains for the similar wage rates for all the UK Universities.

2.1. The effects of the number of firms

Let us consider an economy with $(n+m)$ firms producing a homogeneous product. Assume that production requires only labor. For notational convenience arrange the firms as $1,2,...n,n+1,n+2,...,n+m$. Without loss of generality, assume that each of the firms in $[1,n]$ requires one labor to produce one unit of output, while each of the firms in $[n+1,n+m]$ requires λ workers to produce one unit of output, where $\lambda \overset{\geq}{\underset{<}{-}} 1$. Hence, we consider asymmetry in labor productivities of the firms. Also, to make asymmetry meaningful in our analysis, assume that $n \geq 1$ and $m \geq 1$.

Assume that the inverse market demand function for the product is

$$P = a - q, \tag{1}$$

where the notations have usual meanings.

We assume that there is a centralized labor union that sets the wage rates for the firms. We will consider two possibilities: (i) where the labor union sets a uniform wage rate for all firms, and (ii) where the labor union can charge different wage rates to different firms. Following Haucap and Wey (2004), we can call the former wage setting behavior as "centralization" and the latter as "coordination". As a simplification, we assume that the reservation wage rates of the labors are zero.

We consider the following game. At stage 1, the labor union sets the wage rates. At stage 2, the firms produce like Cournot oligopolists and the profits are realized. We solve the game through backward induction.

Hence, we consider the case of a monopoly labor union as in Dunlop (1944) and Oswald (1982). Since, the purpose of this paper is to show that, under a centralized labor union, the equilibrium wage rates can depend on the number of firms and product differentiation in presence of asymmetric firms, it is enough for us to consider a monopoly labor union. However, note that our qualitative results holds even if there is bargaining between the labor union and the firms. Bargaining between the labor union and the firms will only complicate the calculations without adding much to the main purpose of the paper. Further, to make our point, we concentrate on the right-to-manage model of labor union, which is perhaps the most widely used model of labor union in the Industrial Organization literature.[4]

2.1.1. Uniform wage setting by the labor union

In this subsection we assume that the labor union charges a uniform wage rate to all firms.

Given the demand function and the uniform wage rate w, the equilibrium output of each of the firms in $[1,n]$ is $q_i = \frac{(a - w(m - \lambda m + 1))}{(n+m+1)}$, and the equilibrium output of each of the firms in $[n+1,n+m]$ is $q_j = \frac{(a - w(\lambda + \lambda n - n))}{(n+m+1)}$.

Therefore, the labor union maximizes the following expression to determine the wage rate:[5]

[4] See, Layard et al. (1991) for arguments in favor of right-to-manage models.

[5] It is important to note that, due to asymmetry among the firms, the labor union may not wish to serve all the firms. Hence, the labor union may charge a sufficiently high wage rate that encourages only the relatively efficient firms to hire labors and producing the product. However, it can be shown that the labor union prefers to serve all the firms if λ is not sufficiently different from 1. Since, wage determination in the presence of asymmetric firms is the main element of this paper, we do our analysis

$$\underset{w}{Max}\frac{wn(a-w(m-\lambda m+1))+\lambda wm(a-w(\lambda+\lambda n-n))}{(n+m+1)}. \quad (2)$$

The equilibrium wage rate is

$$w^{*}=\frac{a(n+\lambda m)}{2(n(1+m)+\lambda m(\lambda+\lambda n-2n))}, \quad (3)$$

which clearly shows that the equilibrium wage rate depends on the number of firms. Note that w^* is independent of the number of firms if either $\lambda=1$ or $n=0$ or $m=0$.

Hence, the following proposition is immediate.

Proposition 1: *If there is a centralized labor union that sets a uniform wage rate for all firms, the equilibrium wage rate depends on the number of firms if the firms differ in labor productivities.*

The reason for the above result follows easily from Dhillon and Petrakis (2002). They show that, if the firm's equilibrium output and profit are *log-linear in the wage rate and the market features such as the number of firms and product differentiation*, the equilibrium wage rate is independent of the market features. If the firms are asymmetric in labor productivities, it is immediate from the above analysis that the firm's equilibrium output and the profit[6] is not log-linear in the wage rate and the number of firms, and therefore, the equilibrium wage rate depends on the number of firms. It is important to note that, like Dhillon and Petrakis (2002), we have also considered that the production technologies of the firms are log-linear and the union utility is log-linear in the wage rate and aggregate employment. However, the asymmetry between the firms does not satisfy that the firm's equilibrium output and the profit are log-linear in the wage rate and the number of firms.

Let us now see the effects of the number of firms on the equilibrium wage rate.

Proposition 2: *(i) Assume* $\lambda>0$. *If* $\lambda<1$, *we get* $\frac{\partial w^*}{\partial n}<0$. *However, if* $1<\lambda<2$, *we get* $\frac{\partial w^*}{\partial n}\underset{<}{\geq}0$ *for* $\frac{1}{(\lambda-1)}\underset{>}{\leq}m$.[7]

(ii) If $\lambda>1$, *we get* $\frac{\partial w^*}{\partial m}<0$. *However, if* $\frac{1}{2}<\lambda<1$, *we get* $\frac{\partial w^*}{\partial m}\underset{<}{\geq}0$ *for* $\frac{\lambda}{(1-\lambda)}\underset{<}{\geq}n$.[8]

under the assumption that λ is sufficiently close to 1 so that the labor union serves all the firms and solves the maximization problem (2).

[6] It is easy to check that the profit of the i th firm is q_i^2, $i=1,2,...,n+m$.

[7] Though it will be immediate from the proof of this proposition that $\frac{\partial w^*}{\partial n}<0$ if $\lambda>2$, we will not focus on the higher values of λ, since as mentioned in footnote 5, the labor union would serve only the relatively efficient firm for higher values of λ. For example, it is easy to show that if $\lambda>1$, $n=m=1$ and the centralized labor union charges a uniform wage rate to all firms, then the labor union prefers to serve both firms instead of serving only the firm with higher labor productivity provided $\lambda<2$. When $n,m>1$, the relevant values of λ for which the labor union serves all the firms are less than $\lambda^*(n)$ that is lower than 2. So, for $n,m>1$, we need to restrict λ between 1 and $\lambda^*(n)$.

Proof: (i) We find from (3) that if $\lambda > 0$ and $\lambda \neq 1$, then $\frac{\partial w^*}{\partial n} \overset{\geq}{<} 0$ provided

$$\frac{1}{(\lambda - 1)} \overset{\geq}{<} m. \qquad (4)$$

Hence, if $\lambda < 1$, we get $\frac{\partial w^*}{\partial n} < 0$, since $m \geq 1$. However, if $\lambda > 1$, we get $\frac{1}{(\lambda - 1)} > 1$ provided $\lambda < 2$. Hence, for $1 < \lambda < 2$, we get $\frac{\partial w^*}{\partial n} \overset{\geq}{<} 0$ for $\frac{1}{(\lambda - 1)} \overset{\leq}{>} m$.

(ii) We find from (3) that if $\lambda \neq 1$, then $\frac{\partial w^*}{\partial m} \overset{\geq}{<} 0$ provided

$$-\frac{\lambda}{(\lambda - 1)} \overset{\geq}{<} n. \qquad (5)$$

Hence, if $\lambda > 1$, we get $\frac{\partial w^*}{\partial m} < 0$. However, if $\lambda < 1$, we get that $\frac{\lambda}{(1-\lambda)} > 1$ provided $\lambda > \frac{1}{2}$. Hence, for $\frac{1}{2} < \lambda < 1$, we get $\frac{\partial w^*}{\partial m} \overset{\geq}{<} 0$ for $\frac{\lambda}{(1-\lambda)} \overset{\geq}{<} n$. Q.E.D.

The reason for the above result is easy to understand. The presence of asymmetric firms makes the total labor demand curve as a kinked function, and the kink occurs at that wage rate where the firms with relatively lower labor productivities do not find it profitable to produce.

Let us now consider Proposition 2(i). If $\lambda < 1$, then, in our analysis, the firms in $[1, n]$ are relatively inefficient, and none of them demand labor if $w > \frac{a}{(m - \lambda m + 1)} \equiv \bar{w}$. An increase in n implies that the number of inefficient firms increases, which makes the segment of the labor demand curve, where all firms find production profitable, more elastic, but does not affect $\bar{w}$. This is shown in Figure 1.

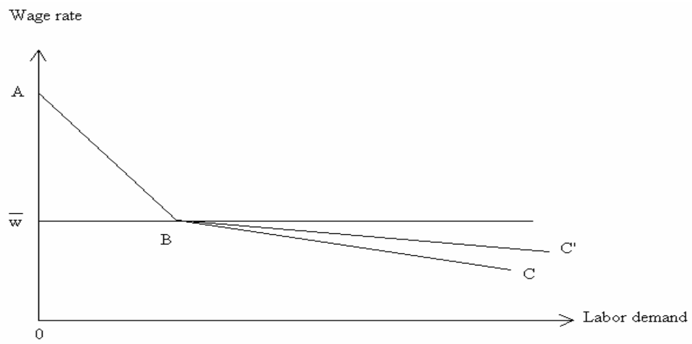

Figure 1: The effect of a higher n when $\lambda < 1$.

[8] Though it will be immediate that $\frac{\partial w^*}{\partial m} < 0$ for $\lambda < \frac{1}{2}$, following footnotes 5 and 7, we restrict our attention to $\lambda > \frac{1}{2}$.

Assume that the total labor demand curve for a given $\lambda < 1$, n and m is given by the kinked curve ABC. The wage rate $\overline{w}$ is the wage rate at which the labor demand by the lower productive firms is zero. The labor demand is coming from all the firms on the segment BC, while, on the segment AB, the labor demand is coming only from the firms with higher labor productivities, i.e., from the firms in $[n+1, n+m]$. Now, if n increases, it rotates BC to BC', and makes this segment more elastic. As a result, the equilibrium wage rate reduces with n.

However, if $\lambda > 1$, then, the firms in $[1, n]$ are relatively efficient, and these firms stop producing if $w > \frac{a}{(\lambda + \lambda n - n)} \equiv w'$. If n increases, it shifts the segment of the labor demand curve above w' outward. However, higher n also implies that w' falls, since, now the inefficient firms are facing competition from more efficient firms. Hence, in this situation, higher n not only shifts the labor demand curve outward, it also reduces the critical level of the wage rate at which kink occurs. This is shown in Figure 2.

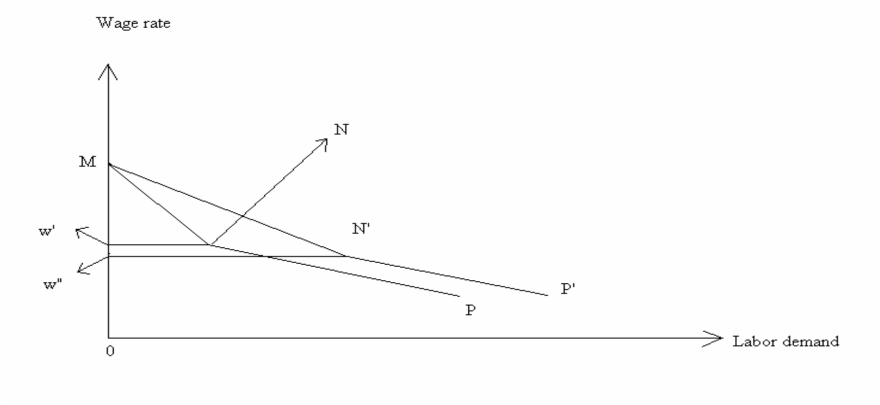

Figure 2: The effect of a higher n when $\lambda > 1$.

Assume that the labor demand curve for a given $\lambda > 1$, n and m is given by the kinked curve MNP. The wage rate w' is the wage rate at which the labor demand by the lower productive firms is zero. Now, if n increases, it shifts the labor demand curve to $MN'P'$, and the kink occurs at w''. So, while the outward shift of the labor demand curve tends to increase the equilibrium wage rate, the fall of w' to w'' tends to reduce the equilibrium wage rate. Hence, the net effect of a change in n depends on the number of inefficient firms and the labor productivities of the inefficient firms.

Similar argument follows for the case of Proposition 2(ii).

2.1.2. Wage discrimination

We have shown in the previous section that the wage rigidity result of Dhillon and Petrakis (2002) does not hold if the centralized labor union sets a uniform wage rate and the firms differ in labor productivities.

In this section we show that if the centralized union keeps the flexibility of charging different wage rates to different firms, the wage rigidity result of Dhillon and Petrakis (2002) holds in the presence of the asymmetric firms. Let us now consider wage discrimination between the firms. Given the wage rates, the equilibrium output of each of the firms in $[1,n]$ is $q_i = \dfrac{(a-(n+m)w_i + \sum_{\substack{k=1\\ i\neq k}}^{n} w_k + \lambda \sum_{j=n+1}^{n+m} w_j)}{(n+m+1)}$ and the equilibrium output of each of the firms in $[n+1, n+m]$ is $q_j = \dfrac{(a-\lambda(n+m)w_j + \sum_{i=1}^{n} w_i + \lambda \sum_{\substack{s=n+1\\ j\neq s}}^{n+m} w_s)}{(n+m+1)}$.

Therefore, the labor union maximizes the following expression to determine the wage rates for each firm:

$$\underset{w_1,\ldots,w_n,w_{n+1},\ldots,w_{n+m}}{Max} \frac{\sum_{i=1}^{n} w_i q_i + \lambda \sum_{j=n+1}^{n+m} w_j q_j}{(n+m+1)}. \quad (6)$$

The equilibrium wage rates are $w_i^* = \dfrac{a}{2}$, $i = 1,2...,n$, and $w_j^* = \dfrac{a}{2\lambda}$, $j = n+1,...,n+m$.

Hence, the following proposition is immediate.

Proposition 3: *If a centralized labor union can discriminate wage rate between the firms, the equilibrium wage rates are independent of the number of firms even if the firms differ in labor productivities.*

If the union discriminates wage rate between the firms, it considers the labor demand of different firms separately. Hence, it is important to see whether, in the firm's output and the profit, the firm's own wage rate and market features such as the number of firms behave like a log-linear function. It follows from the above analysis that, in the firm's equilibrium output and profit, the firm's own wage rate and the number of firms behave like a log-linear function. As a result, the equilibrium wage rate of each firm is independent of the number of firms.

2.2. The effects of product differentiation

Let us now consider a horizontally differentiated duopoly market structure. Assume that firm 1 requires one labor to produce one unit of output, and firm 2 requires λ labors to produce one unit of output. Like section 2.1, we assume that the reservation wage rates for the labors are zero.

Assume that the inverse market demand function for firm i is

$$P_i = a - q_i - \gamma q_j, \qquad i \neq j, \quad (7)$$

where $\gamma \in [0,1]$ shows the degree of product differentiation. $\gamma = 0$ implies that the products are isolated, while $\gamma = 1$ implies that the products are homogeneous.

2.2.1. Uniform wage setting

Let us first consider the situation where the centralized labor union sets a uniform wage rate for both firms. Given the demand specification and the uniform wage rate w, the equilibrium outputs of firms 1 and 2 are respectively $q_1 = \dfrac{(a(2-\gamma) - 2w + \gamma\lambda w)}{(4-\gamma^2)}$ and $q_2 = \dfrac{(a(2-\gamma) - 2\lambda w + \gamma w)}{(4-\gamma^2)}$.

The centralized labor union determines the uniform wage rate by maximizing the following expression:[9]

$$\underset{w}{Max}\frac{w(a(2-\gamma)-2w+\gamma\lambda w)+\lambda w(a(2-\gamma)-2\lambda w+\gamma w)}{(4-\gamma^2)}. \quad (8)$$

The equilibrium wage rate is $w^* = \frac{a(2-\gamma)(1+\lambda)}{4(1+\lambda^2-\lambda\gamma)}$. We also find that $\frac{\partial w^*}{\partial \gamma} < 0$.

Hence the following proposition is immediate.

Proposition 4: *If the firms differ in labor productivities and a centralized labor union charges uniform wage rate to the firms, the equilibrium wage rate depends on the degree of product differentiation. As the degree of product differentiation increases (i.e., γ falls), the equilibrium wage rate increases.*

The intuition of this result is similar to the intuition provided in subsection 2.1.1.

2.2.2. Wage discrimination

Let us now consider the situation where the labor union discriminates wage between the firms. Given that the labor union charges w_1 and w_2 to firms 1 and 2 respectively, the equilibrium outputs of firms 1 and 2 are respectively $q_1 = \frac{(a(2-\gamma)-2w_1+\gamma\lambda w_2)}{(4-\gamma^2)}$ and $q_2 = \frac{(a(2-\gamma)-2\lambda w_2+\gamma w_1)}{(4-\gamma^2)}$.

The labor union determines the wage rates by maximizing the following expression:

$$\underset{w_1,w_2}{Max}\frac{w_1(a(2-\gamma)-2w_1+\gamma\lambda w_2)+\lambda w_2(a(2-\gamma)-2\lambda w_2+\gamma w_1)}{(4-\gamma^2)}. \quad (9)$$

The equilibrium wage rates are $w_1^* = \frac{a}{2}$ and $w_2^* = \frac{a}{2\lambda}$.

Hence, the proposition is immediate.

Proposition 5: *If a centralized labor union can discriminate wage between the firms, the equilibrium wage rates are independent of the degree of product differentiation even if the firms differ in labor productivities.*

The intuition of this result is similar to the intuition provided in subsection 2.1.1.

3. Conclusion

Considering Cournot competition, this note shows that if the firms differ in labor productivities and the centralized labor union charges a uniform wage rate to the firms, the wage rigidity result of Dhillon and Petrakis (2002) does not hold. The asymmetry between the firms does not satisfy that the firm's output and profit is log-linear in the wage rate and the market features such as the number of firms and

[9] The qualification made in footnote 5 also holds for the subsection 2.2.

product differentiation. As a result, the equilibrium wage rate depends on the number of firms and product differentiation.

However, if the centralized labor union can discriminate wage between the firms, the wage rigidity result of Dhillon and Petrakis (2002) holds even if the firms differ in labor productivities. In this situation, the firm's own wage rate and the market features such as the number of firms and product differentiation behave like a log-linear function in the firm's equilibrium output and profit. Hence, whether the wage rigidity result of Dhillon and Petrakis (2002) extends to the case of asymmetric firms depends on the wage setting behavior of the labor union.

References

Dhillon, A. and E. Petrakis, 2002, 'A generalized wage rigidity result', *International Journal of Industrial Organization*, 20: 285 – 311.

Dunlop, J. T., 1944, *Wage determination under trade union*, New York, Macmillan.

Greenhut, M. L. and H. Ohta, 1976, 'Related market conditions and interindustrial mergers', *The American Economic Review*, 66: 267-77.

Haucap, J., U. Pauly and C. Wey, 2000, 'The incentives of employers' associations to raise rivals' costs in the presence of collective bargaining', in de Gijsel, Olthoff and Zwick (Eds.), *The unemployment debate: current issues*, 185 – 226, Marburg, Metropolis Verlag.

Haucap, J., U. Pauly and C. Wey, 2001, 'Collective wage setting when wages generally are generally binding: an antitrust perspective', *International Review of Law and Economics*, 21: 287 – 307.

Haucap, J. and C. Wey, 2004, 'Unionisation structures and innovation incentives', *Economic Journal*, 114: C149 – 65.

Layard, R., S. Nickell and R. Jackman, 1991, *Unemployment, macroeconomic performance and the labour market*, Oxford University Press, Oxford.

Oswald, A. J., 1982, 'The microeconomic theory of the trade union', *Economic Journal*, 92: 269 – 83.

Tyagi, R. K., 1999, 'On the effects of downstream entry', *Management Science*, 45: 59-73.