E C O N O M I C S   B U L L E T I N

Straightforwardness of Game Forms with Infinite Sets of Outcomes

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Abstract

We show that no game form with an infinite set of outcomes can bestraightforward and that no voting scheme with an infinite range can be non-manipulable.

I would like to thank Nick Baigent and Francis Bloch for their comments and suggestions.
Citation: Codognato, Giulio, (2007) "Straightforwardness of Game Forms with Infinite Sets of Outcomes." *Economics Bulletin*, Vol. 3, No. 5 pp. 1-4
Submitted: December 4, 2006. **Accepted:** January 23, 2007.
URL: http://economicsbulletin.vanderbilt.edu/2007/volume3/EB-06C70017A.pdf

1 Introduction

Gibbard (1973) showed that straightforward game forms with at least three outcomes must be dictatorial and, as a corollary, that also non-manipulable voting schemes with at least three outcomes must be dictatorial. Gibbard's proof of these theorems does not require that the set of outcomes is finite.

We analyze straightforwardness of game forms with infinite sets of outcomes. We show that, as a consequence of Gibbard's theorem, when the set of outcomes is infinite, no game form can be straightforward. This, in turn, implies that no voting scheme with an infinite range can be non-manipulable.

2 Notation and definitions

According to Gibbard, a game form is characterized by a set $I = \{1, \ldots, n\}$ of players with $n > 1$, a nonempty set X of outcomes, a nonempty set S_i of strategies, for each player $i \in I$, and a function $g : S_1 \times \ldots \times S_n \to X$ whose range is X. An ordering of a set Z is a complete and transitive binary relation on Z. Given an ordering R of X, a strategy $s_i^\star \in S_i$ is R-dominant for player $i \in I$ if $g(s_1, \ldots, s_{i-1}, s_i^\star, s_{i+1}, \ldots, s_n) R g(s_1, \ldots, s_n)$, for all $(s_1, \ldots, s_n) \in S_1 \times \ldots \times S_n$. A game form is straightforward if, for every ordering R of X and for every player $i \in I$, there is a strategy which is R-dominant for i. A player $k \in I$ is a dictator for the game form g if, for every outcome $x \in X$, there is a strategy $s_k(x) \in S_k$ such that $g(s_1, \ldots, s_{k-1}, s_k(x), s_{k+1}, \ldots, s_n) = x$, for all $(s_1, \ldots, s_n) \in S_1 \times \ldots \times S_n$. A game form g is dictatorial if there is a dictator for it.

Gibbard introduced the notion of a game form to generalize the notion of a voting scheme, which he defined as follows. A voting scheme is a game form v such that, for some set Z with $X \subset Z$, S_i is the set of orderings of Z, for each $i \in I$. A voting scheme is manipulable if for, some $i \in I$, for some n-tuple $(R_1, \ldots, R_n)$ of orderings of Z, and for some ordering $R^\star$ of Z, $v(R_1, \ldots, R_{i-1}, R^\star, R_{i+1}, \ldots, R_n) \bar{R}_i v(R_1, \ldots, R_n)$, where $\bar{R}_i$ denotes the asymmetric part of R_i.

3 The theorem

Gibbard proved the following theorem.

Gibbard's Theorem. *Every straightforward game form with at least three outcomes is dictatorial.*

He then showed a corollary concerning voting schemes on the basis of the following result, which we state as a lemma.

Lemma. *Every non-manipulable voting scheme is a straightforward game form.*

This lemma provides a meaningful interpretation of the notion of straightforwardness of game forms. Gibbard's corollary follows immediately.

Corollary. *Every non-manipulable voting scheme with at least three outcomes is dictatorial.*

Our main result, which we state as a theorem, is actually a further corollary of Gibbard's theorem.

Theorem. *No game form with an infinite set of outcomes is straightforward.*

Proof. Let X be infinite and suppose that g is a straightforward game form. Since X contains more than two outcomes, by Gibbard's theorem, g is dictatorial. Let k be the dictator for g. Consider now an ordering P of X such that, for each $x \in X$, there is a $y \in X$ such that yPx and not xPy. As k is the dictator for g, for every outcome $x \in X$, there is a strategy $s_k(x) \in S_k$ such that $g(s_1, \ldots, s_{k-1}, s_k(x), s_{k+1}, \ldots, s_n) = x$, for all $(s_1, \ldots, s_n) \in S_1 \times \ldots \times S_n$. But then, there is no strategy which is P-dominant for k, contradicting the assumption that g is straightforward. ■

From this theorem and the lemma, we can derive the following corollary, which provides an immediate interpretation of our result in terms of manipulation of voting schemes.

Corollary. *No voting scheme with an infinite range is non-manipulable.*

4 Conclusion

We have shown that, on unrestricted domains of orderings, there exists no straightforward game form - and hence no non-manipulable voting scheme - with an infinite set of outcomes. We leave as an open problem for further research the characterization of domain restrictions which allow for the existence of straightforward game forms and non-manipulable voting schemes with an infinite range.

References

[1] Gibbard A. (1973),"Manipulation of voting schemes: a general result," *Econometrica* **41**, 587-601.