E c o n o m i c s B u l l e t i n

Subgame-perfect market sharing agreements

Masaki Iimura
Graduate School of Humanities and Social Sciences, University of Tsukuba

Seiji Murakoshi
Graduate School of Humanities and Social Sciences, University of Tsukuba

Toru Hokari
Graduate School of Humanities and Social Sciences

Abstract

Jackson and Watts (2002, J Econ Theory) study a dynamic process of network formation assuming that each player is myopic. In this note, we study the same dynamic process but assume that each player is farsighted. In particular, we consider a finite-horizon version of such a dynamic process in a model of market sharing agreements introduced by Belleframme and Bloch (2004, Int Econ Review), and investigate which networks are likely to be realized when the number of the players is three.

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1 Introduction

Jackson and Watts (2002) study a dynamic process of network formation assuming that each player is myopic. In this note, we study the same dynamic process but assume that each player is farsighted. In particular, we consider a finite-horizon version of such a dynamic process in a model of market sharing agreements introduced by Belleframme and Bloch (2004), and investigate which networks are likely to be realized when the number of the players is three.

2 The model

There are three firms that produce a homogeneous good. Each firm has its *home market*. If two firms are linked by a *market sharing agreement*, each firm refrains from entering the other firm's home market. Otherwise, each firm enters other firms' home markets. Let $p = a - q$ be the inverse demand function in each market, where p is a price of the good and q is a market supply. We assume that the cost of production is zero. With this set-up, one can calculate the profits to the firms in each network of market sharing agreements (Figure 1). The model is a special case of the one studied by Belleframme and Bloch (2004).

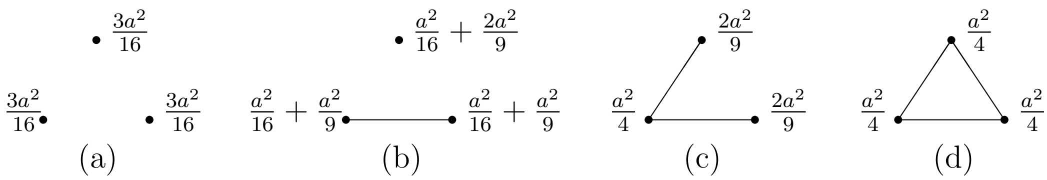


Figure 1: The profits to the firms in each network of market sharing agreements.

The formation of a link requires the consent of both parties involved, but severance can be done unilaterally. A network is **pairwise stable** (Jackson and Wolinsky, 1996) if (i) no pair of firms want to form a new link between them, and (ii) no one wants to sever any single direct link.

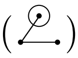
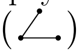
Among the four types of networks in Figure 1, the Pareto optimal networks are (b) and (d), and the pairwise stable networks are (a) and (d).

Let us consider the following discrete-time dynamic process. At each period $t \in \{1, 2, \ldots, T\}$, a pair of firms are chosen randomly. If they are already directly linked, they can decide whether to keep the link or sever it. If they are not linked, they can decide whether to form a new link between them.

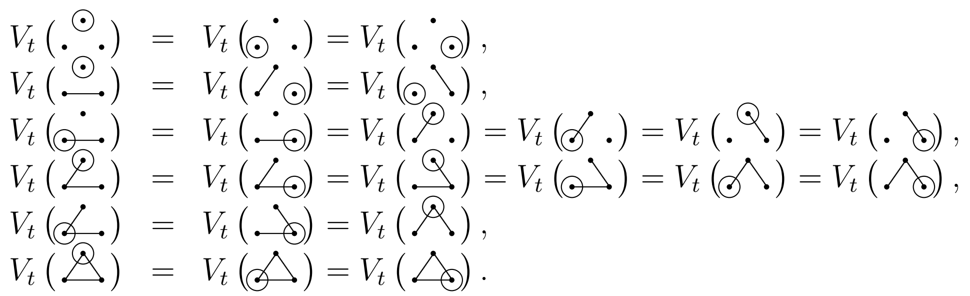
Jackson and Watts (2002) study this dynamic process for a more general model assuming that each player is myopic. Here, we assume that each firm is farsighted in the sense that it aims at maximizing the expected value of the sum of discounted profits, with a common discounting factor $\delta \in (0, 1]$. Then the above dynamic process defines an extensive form game. Assuming that T is finite, we use backward induction to find a subgame-perfect equilibrium of this game.

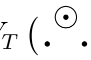
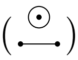
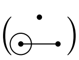
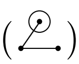
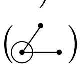
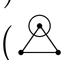
Dutta, Ghosal, and Ray (2005) study an infinite-horizon dynamic process similar to that of Jackson and Watts (2002) assuming that each player is farsighted. In their setting, when a pair of players is selected, each of them can sever the existing links with *other players* unilaterally. For simplicity, we do not incorporate such a feature into our model.

There is a trivial subgame-perfect equilibrium in which new links are never formed simply because each firm expects that other firms wouldn't agree to form a new link. In the rest of this note, we study a subgame-perfect equilibrium in which two firms form a new link whenever it is profitable for both to do so.

Let V_t () denote a subgame-perfect equilibrium payoff to the circled firm in the subgame starting period t with network (). Let $V_t(\cdot)$ be defined in a similar manner for each firm in each network.

We consider a symmetric equilibrium in the sense that for each $t \leq T$,



Note that V_T () $= \frac{3a^2}{16}$, V_T () $= \frac{a^2}{16} + \frac{2a^2}{9}$, V_T () $= \frac{a^2}{16} + \frac{a^2}{9}$, V_T () $= \frac{2a^2}{9}$, V_T () $= \frac{a^2}{4}$, and V_T () $= \frac{a^2}{4}$.

As mentioned earlier, we are interested in a subgame-perfect equilibrium in which two firms form a link whenever it is profitable for both to do so. Such an equilibrium can be found by solving the following Bellman equations:

- If $V_{t+1}\left(\odot\;\bullet\;\bullet\right) > V_{t+1}\left(\overset{\bullet}{\odot\!-\!\bullet}\right)$, then

$$V_t\left(\odot\;\bullet\;\bullet\right) = \frac{3a^2}{16} + \delta V_{t+1}\left(\odot\;\bullet\;\bullet\right).$$

- If $V_{t+1}\left(\odot\;\bullet\;\bullet\right) \leq V_{t+1}\left(\overset{\bullet}{\odot\!-\!\bullet}\right)$, then

$$V_t\left(\odot\;\bullet\;\bullet\right) = \frac{3a^2}{16} + \frac{2\delta}{3}V_{t+1}\left(\overset{\bullet}{\odot\!-\!\bullet}\right) + \frac{\delta}{3}V_{t+1}\left(\odot\;\bullet\!\setminus\!\bullet\right).$$

- If $\left[V_{t+1}\left(\overset{\odot}{\bullet\!-\!\bullet}\right) > V_{t+1}\left(\overset{\odot}{\angle}\!\!-\!\bullet\right) \text{ or } V_{t+1}\left(\overset{\bullet}{\odot\!-\!\bullet}\right) > V_{t+1}\left(\overset{\bullet}{\diagup}\!\!\odot\!-\!\bullet\right)\right]$ and $V_{t+1}\left(\overset{\bullet}{\odot\!-\!\bullet}\right) \geq V_{t+1}\left(\odot\;\bullet\;\bullet\right)$, then

$$V_t\left(\overset{\odot}{\bullet\!-\!\bullet}\right) = \frac{a^2}{16} + \frac{2a^2}{9} + \delta V_{t+1}\left(\overset{\odot}{\bullet\!-\!\bullet}\right).$$

- If $\left[V_{t+1}\left(\overset{\odot}{\bullet\!-\!\bullet}\right) > V_{t+1}\left(\overset{\odot}{\angle}\!\!-\!\bullet\right) \text{ or } V_{t+1}\left(\overset{\bullet}{\odot\!-\!\bullet}\right) > V_{t+1}\left(\overset{\bullet}{\diagup}\!\!\odot\!-\!\bullet\right)\right]$ and $V_{t+1}\left(\overset{\bullet}{\odot\!-\!\bullet}\right) < V_{t+1}\left(\odot\;\bullet\;\bullet\right)$, then

$$V_t\left(\overset{\odot}{\bullet\!-\!\bullet}\right) = \frac{a^2}{16} + \frac{2a^2}{9} + \frac{2\delta}{3}V_{t+1}\left(\overset{\odot}{\bullet\!-\!\bullet}\right) + \frac{\delta}{3}V_{t+1}\left(\overset{\odot}{\bullet\;\;\bullet}\right).$$

- If $V_{t+1}\left(\overset{\odot}{\bullet\!-\!\bullet}\right) \leq V_{t+1}\left(\overset{\odot}{\angle}\!\!-\!\bullet\right)$, $V_{t+1}\left(\overset{\bullet}{\odot\!-\!\bullet}\right) \leq V_{t+1}\left(\overset{\bullet}{\diagup}\!\!\odot\!-\!\bullet\right)$, and $V_{t+1}\left(\overset{\bullet}{\odot\!-\!\bullet}\right) \geq V_{t+1}\left(\odot\;\bullet\;\bullet\right)$, then

$$V_t\left(\overset{\odot}{\bullet\!-\!\bullet}\right) = \frac{a^2}{16} + \frac{2a^2}{9} + \frac{2\delta}{3}V_{t+1}\left(\overset{\odot}{\angle}\!\!-\!\bullet\right) + \frac{\delta}{3}V_{t+1}\left(\overset{\odot}{\bullet\!-\!\bullet}\right).$$

- If $V_{t+1}\left(\overset{\odot}{\bullet\!-\!\bullet}\right) \leq V_{t+1}\left(\overset{\odot}{\angle}\!\!-\!\bullet\right)$, $V_{t+1}\left(\overset{\bullet}{\odot\!-\!\bullet}\right) \leq V_{t+1}\left(\overset{\bullet}{\diagup}\!\!\odot\!-\!\bullet\right)$, and $V_{t+1}\left(\overset{\bullet}{\odot\!-\!\bullet}\right) < V_{t+1}\left(\odot\;\bullet\;\bullet\right)$, then

$$V_t\left(\overset{\odot}{\bullet\!-\!\bullet}\right) = \frac{a^2}{16} + \frac{2a^2}{9} + \frac{2\delta}{3}V_{t+1}\left(\overset{\odot}{\angle}\!\!-\!\bullet\right) + \frac{\delta}{3}V_{t+1}\left(\overset{\odot}{\bullet\;\;\bullet}\right).$$

- If $\left[V_{t+1}(\;) > V_{t+1}(\;)\right.$ or $\left.V_{t+1}(\;) > V_{t+1}(\;)\right]$ and $V_{t+1}(\;) \geq V_{t+1}(\;)$, then

$$V_t(\;) = \frac{a^2}{16} + \frac{a^2}{9} + \delta V_{t+1}(\;).$$

- If $\left[V_{t+1}(\;) > V_{t+1}(\;)\right.$ or $\left.V_{t+1}(\;) > V_{t+1}(\;)\right]$ and $V_{t+1}(\;) < V_{t+1}(\;)$, then

$$V_t(\;) = \frac{a^2}{16} + \frac{a^2}{9} + \frac{2\delta}{3}V_{t+1}(\;) + \frac{\delta}{3}V_{t+1}(\;).$$

- If $V_{t+1}(\;) \leq V_{t+1}(\;)$, $V_{t+1}(\;) \leq V_{t+1}(\;)$, and $V_{t+1}(\;) \geq V_{t+1}(\;)$, then

$$V_t(\;) = \frac{a^2}{16} + \frac{a^2}{9} + \frac{\delta}{3}V_{t+1}(\;) + \frac{\delta}{3}V_{t+1}(\;) + \frac{\delta}{3}V_{t+1}(\;).$$

- If $V_{t+1}(\;) \leq V_{t+1}(\;)$, $V_{t+1}(\;) \leq V_{t+1}(\;)$, and $V_{t+1}(\;) < V_{t+1}(\;)$, then

$$V_t(\;) = \frac{a^2}{16} + \frac{a^2}{9} + \frac{\delta}{3}V_{t+1}(\;) + \frac{\delta}{3}V_{t+1}(\;) + \frac{\delta}{3}V_{t+1}(\;).$$

- If $V_{t+1}(\;) \geq V_{t+1}(\;)$, $V_{t+1}(\;) \geq V_{t+1}(\;)$, and $V_{t+1}(\;) > V_{t+1}(\;)$, then

$$V_t(\;) = \frac{a^2}{4} + \delta V_{t+1}(\;).$$

- If $V_{t+1}(\;) \geq V_{t+1}(\;)$, $V_{t+1}(\;) \geq V_{t+1}(\;)$, and $V_{t+1}(\;) \leq V_{t+1}(\;)$, then

$$V_t(\;) = \frac{a^2}{4} + \frac{2\delta}{3}V_{t+1}(\;) + \frac{\delta}{3}V_{t+1}(\;).$$

- If $\left[V_{t+1}(\cdot) < V_{t+1}(\cdot) \text{ or } V_{t+1}(\cdot) < V_{t+1}(\cdot)\right]$ and $V_{t+1}(\cdot) > V_{t+1}(\cdot)$, then

$$V_t(\cdot) = \frac{a^2}{4} + \frac{2\delta}{3} V_{t+1}(\cdot) + \frac{\delta}{3} V_{t+1}(\cdot).$$

- If $\left[V_{t+1}(\cdot) < V_{t+1}(\cdot) \text{ or } V_{t+1}(\cdot) < V_{t+1}(\cdot)\right]$ and $V_{t+1}(\cdot) \leq V_{t+1}(\cdot)$, then

$$V_t(\cdot) = \frac{a^2}{4} + \frac{2\delta}{3} V_{t+1}(\cdot) + \frac{\delta}{3} V_{t+1}(\cdot).$$

- If $V_{t+1}(\cdot) \geq V_{t+1}(\cdot)$, $V_{t+1}(\cdot) \geq V_{t+1}(\cdot)$, and $V_{t+1}(\cdot) > V_{t+1}(\cdot)$, then

$$V_t(\cdot) = \frac{2a^2}{9} + \delta V_{t+1}(\cdot).$$

- If $V_{t+1}(\cdot) \geq V_{t+1}(\cdot)$, $V_{t+1}(\cdot) \geq V_{t+1}(\cdot)$, and $V_{t+1}(\cdot) \leq V_{t+1}(\cdot)$, then

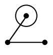
$$V_t(\cdot) = \frac{2a^2}{9} + \frac{2\delta}{3} V_{t+1}(\cdot) + \frac{\delta}{3} V_{t+1}(\cdot).$$

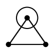
- If $\left[V_{t+1}(\cdot) < V_{t+1}(\cdot) \text{ or } V_{t+1}(\cdot) < V_{t+1}(\cdot)\right]$ and $V_{t+1}(\cdot) > V_{t+1}(\cdot)$, then

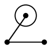
$$V_t(\cdot) = \frac{2a^2}{9} + \frac{\delta}{3} V_{t+1}(\cdot) + \frac{\delta}{3} V_{t+1}(\cdot) + \frac{\delta}{3} V_{t+1}(\cdot).$$

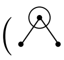
- If $\left[V_{t+1}(\cdot) < V_{t+1}(\cdot) \text{ or } V_{t+1}(\cdot) < V_{t+1}(\cdot)\right]$ and $V_{t+1}(\cdot) \leq V_{t+1}(\cdot)$, then

$$V_t(\cdot) = \frac{2a^2}{9} + \frac{\delta}{3} V_{t+1}(\cdot) + \frac{\delta}{3} V_{t+1}(\cdot) + \frac{\delta}{3} V_{t+1}(\cdot).$$

- If V_{t+1} () $\geq V_{t+1}$ (), then

V_t () $= \frac{a^2}{4} + \delta V_{t+1}$ ().

- If V_{t+1} () $< V_{t+1}$ (), then

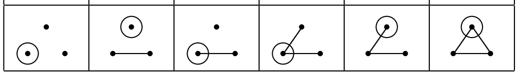
V_t () $= \frac{a^2}{4} + \frac{2\delta}{3} V_{t+1}$ () $+ \frac{\delta}{3} V_{t+1}$ ().

One can use a spreadsheet program such as Excel to solve these equations. A sample Excel file is available at the following web page:

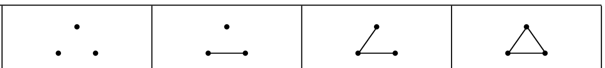
`http://member.social.tsukuba.ac.jp/hokari/`

The results are shown in Table 2, where B, C, D, E, F, and G represent the following six "states" in each period:

B	C	D	E	F	G



Assuming that the process starts with the empty network, the corresponding subgame-perfect equilibria are described in Figures 2, 3, 4, and 5. By using these figures, one can calculate the probability distribution of the networks in the final period for each case as follows:



$T = 11$	1.000000	0.000000	0.000000	0.000000
$T = 12$	0.760436	0.017342	0.000000	0.222222
$T = 16$	0.249625	0.009634	0.000000	0.740741
$T = 23$	0.194153	0.007493	0.000000	0.798354

For $T = 11$, since $V_2(\mathrm{B}) > \mathrm{V}_2(\mathrm{D})$, no one wants to form a link at $t = 1$. Similarly, since $V_3(\mathrm{B}) > \mathrm{V}_3(\mathrm{D})$, $V_4(\mathrm{B}) > \mathrm{V}_4(\mathrm{D}), \ldots$, $V_{11}(\mathrm{B}) > \mathrm{V}_{11}(\mathrm{D})$, the probability that the complete network is realized in the final period is zero. For $T \geq 12$, this probability becomes positive, and it increases as T does.

The analysis of this note can be extended to the cases of more than three players. In the three-person case, there are only six "states" in each period. The number of the states in each period is twenty for the four-person case, and eighty nine for the five-person case (Figures 6 and 7). So, in order to study the five-person case, all one has to do is writing down the Bellman equations for eighteen nine states and creating a spreadsheet to solve them. It might sound as a tedious job, but we think that it can be done.

References

[1] Belleflamme, P. and F. Bloch (2004) "Market sharing agreements and collusive networks" *International Economic Review* **45**, 387–411.

[2] Dutta, B., S. Ghosal, and D. Ray (2005) "Farsighted network formation" *Journal of Economic Theory* **122**, 143–164.

[3] Jackson, M.O., and A. Watts (2002) "The evolution of social and economic networks" *Journal of Economic Theory* **106**, 265–295.

[4] Jackson, M.O., and A. Wolinsky (1996) "A strategic model of social and economic networks" *Journal of Economic Theory* **71**, 44–74.

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2	a^2=	1										
3	delta=	0.98										
4												
5												
6	t	V(B)	V(C)	V(D)	V(E)	V(F)	V(G)	t	V(B)-V(D)	V(C)-v(F)	V(D)-V(E)	V(F)-V(G)
7	11	0.1875	0.2847222	0.173611111	0.25	0.222222	0.25	11	0.013889	0.0625	-0.07639	-0.02778
8	10	0.37125	0.5319907	0.348287037	0.445093	0.453611	0.495	10	0.022963	0.07838	-0.09681	-0.04139
9	9	0.551325	0.7535645	0.522433642	0.639248	0.67148	0.7351	9	0.028891	0.082085	-0.11681	-0.06362
10	8	0.7277985	0.9571505	0.695033924	0.831456	0.879181	0.970398	8	0.032765	0.07797	-0.13642	-0.09122
11	7	0.90074253	1.1478081	0.865447451	1.021086	1.078932	1.20099	7	0.035295	0.068876	-0.15564	-0.12206
12	6	1.070227679	1.3288661	1.033279339	1.207749	1.272209	1.42697	6	0.036948	0.056657	-0.17447	-0.15476
13	5	1.236323126	1.5025224	1.198294655	1.391219	1.46	1.648431	5	0.038028	0.042522	-0.19292	-0.18843
14	4	1.399096663	1.6702358	1.360362507	1.571373	1.642977	1.865462	4	0.038734	0.027259	-0.21101	-0.22249
15	3	1.55861473	1.8329812	1.519419525	1.748154	1.821602	2.078153	3	0.039195	0.011379	-0.22873	-0.25655
16	2	1.714942435	1.9914174	1.675446013	1.921551	1.996203	2.28659	2	0.039496	-0.00479	-0.2461	-0.29039
17	1	1.868143587	2.1491228	2.013625219	2.252366	2.273361	2.490858					
18												
19												
20	t	V(B)	V(C)	V(D)	V(E)	V(F)	V(G)	t	V(B)-V(D)	V(C)-v(F)	V(D)-V(E)	V(F)-V(G)
21	12	0.1875	0.2847222	0.173611111	0.25	0.222222	0.25	12	0.013889	0.0625	-0.07639	-0.02778
22	11	0.37125	0.5319907	0.348287037	0.445093	0.453611	0.495	11	0.022963	0.07838	-0.09681	-0.04139
23	10	0.551325	0.7535645	0.522433642	0.639248	0.67148	0.7351	10	0.028891	0.082085	-0.11681	-0.06362
24	9	0.7277985	0.9571505	0.695033924	0.831456	0.879181	0.970398	9	0.032765	0.07797	-0.13642	-0.09122
25	8	0.90074253	1.1478081	0.865447451	1.021086	1.078932	1.20099	8	0.035295	0.068876	-0.15564	-0.12206
26	7	1.070227679	1.3288661	1.033279339	1.207749	1.272209	1.42697	7	0.036948	0.056657	-0.17447	-0.15476
27	6	1.236323126	1.5025224	1.198294655	1.391219	1.46	1.648431	6	0.038028	0.042522	-0.19292	-0.18843
28	5	1.399096663	1.6702358	1.360362507	1.571373	1.642977	1.865462	5	0.038734	0.027259	-0.21101	-0.22249
29	4	1.55861473	1.8329812	1.519419525	1.748154	1.821602	2.078153	4	0.039195	0.011379	-0.22873	-0.25655
30	3	1.714942435	1.9914174	1.675446013	1.921551	1.996203	2.28659	3	0.039496	-0.00479	-0.2461	-0.29039
31	2	1.868143587	2.1491228	2.013625219	2.252366	2.273361	2.490858	2	-0.14548	-0.12424	-0.23874	-0.2175
32	1	2.205115247	2.4720315	2.309799434	2.535226	2.521165	2.691041					
33												
34												
35	t	V(B)	V(C)	V(D)	V(E)	V(F)	V(G)	t	V(B)-V(D)	V(C)-v(F)	V(D)-V(E)	V(F)-V(G)
36	16	0.1875	0.2847222	0.173611111	0.25	0.222222	0.25	16	0.013889	0.0625	-0.07639	-0.02778
37	15	0.37125	0.5319907	0.348287037	0.445093	0.453611	0.495	15	0.022963	0.07838	-0.09681	-0.04139
38	14	0.551325	0.7535645	0.522433642	0.639248	0.67148	0.7351	14	0.028891	0.082085	-0.11681	-0.06362
39	13	0.7277985	0.9571505	0.695033924	0.831456	0.879181	0.970398	13	0.032765	0.07797	-0.13642	-0.09122
40	12	0.90074253	1.1478081	0.865447451	1.021086	1.078932	1.20099	12	0.035295	0.068876	-0.15564	-0.12206
41	11	1.070227679	1.3288661	1.033279339	1.207749	1.272209	1.42697	11	0.036948	0.056657	-0.17447	-0.15476
42	10	1.236323126	1.5025224	1.198294655	1.391219	1.46	1.648431	10	0.038028	0.042522	-0.19292	-0.18843
43	9	1.399096663	1.6702358	1.360362507	1.571373	1.642977	1.865462	9	0.038734	0.027259	-0.21101	-0.22249
44	8	1.55861473	1.8329812	1.519419525	1.748154	1.821602	2.078153	8	0.039195	0.011379	-0.22873	-0.25655
45	7	1.714942435	1.9914174	1.675446013	1.921551	1.996203	2.28659	7	0.039496	-0.00479	-0.2461	-0.29039
46	6	1.868143587	2.1491228	2.013625219	2.252366	2.273361	2.490858	6	-0.14548	-0.12424	-0.23874	-0.2175
47	5	2.205115247	2.4720315	2.309799434	2.535226	2.521165	2.691041	5	-0.10468	-0.04913	-0.22543	-0.16988
48	4	2.504099246	2.7394137	2.579899995	2.785421	2.748457	2.88722	4	-0.0758	-0.00904	-0.20552	-0.13876
49	3	2.76790979	2.9752558	2.824111852	3.012967	2.961039	3.079476	3	-0.0562	0.014217	-0.18886	-0.11844
50	2	3.004503291	3.2004729	2.941240726	3.101048	3.122644	3.267886	2	0.063263	0.077828	-0.15981	-0.14524
51	1	3.131913225	3.3571689	3.076692794	3.23912	3.296025	3.452528					
52												
53												
54	t	V(B)	V(C)	V(D)	V(E)	V(F)	V(G)	t	V(B)-V(D)	V(C)-v(F)	V(D)-V(E)	V(F)-V(G)
55	23	0.1875	0.2847222	0.173611111	0.25	0.222222	0.25	23	0.013889	0.0625	-0.07639	-0.02778
56	22	0.37125	0.5319907	0.348287037	0.445093	0.453611	0.495	22	0.022963	0.07838	-0.09681	-0.04139
57	21	0.551325	0.7535645	0.522433642	0.639248	0.67148	0.7351	21	0.028891	0.082085	-0.11681	-0.06362
58	20	0.7277985	0.9571505	0.695033924	0.831456	0.879181	0.970398	20	0.032765	0.07797	-0.13642	-0.09122
59	19	0.90074253	1.1478081	0.865447451	1.021086	1.078932	1.20099	19	0.035295	0.068876	-0.15564	-0.12206
60	18	1.070227679	1.3288661	1.033279339	1.207749	1.272209	1.42697	18	0.036948	0.056657	-0.17447	-0.15476
61	17	1.236323126	1.5025224	1.198294655	1.391219	1.46	1.648431	17	0.038028	0.042522	-0.19292	-0.18843
62	16	1.399096663	1.6702358	1.360362507	1.571373	1.642977	1.865462	16	0.038734	0.027259	-0.21101	-0.22249
63	15	1.55861473	1.8329812	1.519419525	1.748154	1.821602	2.078153	15	0.039195	0.011379	-0.22873	-0.25655
64	14	1.714942435	1.9914174	1.675446013	1.921551	1.996203	2.28659	14	0.039496	-0.00479	-0.2461	-0.29039
65	13	1.868143587	2.1491228	2.013625219	2.252366	2.273361	2.490858	13	-0.14548	-0.12424	-0.23874	-0.2175
66	12	2.205115247	2.4720315	2.309799434	2.535226	2.521165	2.691041	12	-0.10468	-0.04913	-0.22543	-0.16988
67	11	2.504099246	2.7394137	2.579899995	2.785421	2.748457	2.88722	11	-0.0758	-0.00904	-0.20552	-0.13876
68	10	2.76790979	2.9752558	2.824111852	3.012967	2.961039	3.079476	10	-0.0562	0.014217	-0.18886	-0.11844
69	9	3.004503291	3.2004729	2.941240726	3.101048	3.122644	3.267886	9	0.063263	0.077828	-0.15981	-0.14524
70	8	3.131913225	3.3571689	3.076692794	3.23912	3.296025	3.452528	8	0.05522	0.061144	-0.16243	-0.1565
71	7	3.25677496	3.5011642	3.206808723	3.387932	3.451776	3.633478	7	0.049966	0.049388	-0.18112	-0.1817
72	6	3.379139461	3.6360293	3.332605964	3.532051	3.60043	3.810808	6	0.046533	0.0356	-0.19945	-0.21038
73	5	3.499056672	3.7641136	3.454765898	3.672167	3.743507	3.984592	5	0.044291	0.020606	-0.2174	-0.24109
74	4	3.616575539	3.8869683	3.573750011	3.808747	3.882023	4.1549	4	0.042826	0.004945	-0.235	-0.27288
75	3	3.731744028	4.0056229	3.689875794	3.942117	4.016658	4.321802	3	0.041868	-0.01103	-0.25224	-0.30514
76	2	3.844609147	4.1279749	3.992514028	4.237306	4.258227	4.485366	2	-0.1479	-0.13025	-0.24479	-0.22714
77	1	4.144414311	4.4152359	4.25303975	4.483593	4.469484	4.645659					

Table 2

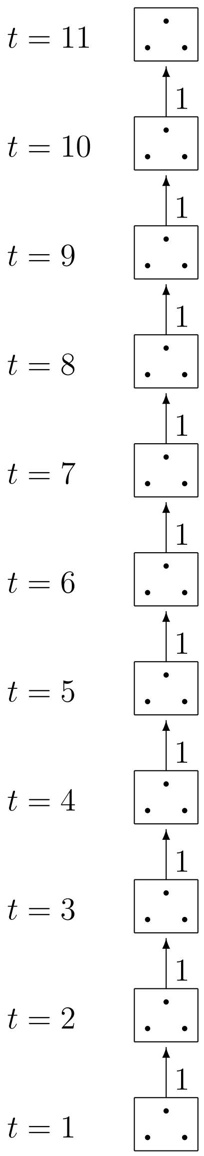


Figure 2: The subgame-perfect equilibrium path when $T = 11$ and $\delta = 0.98$.

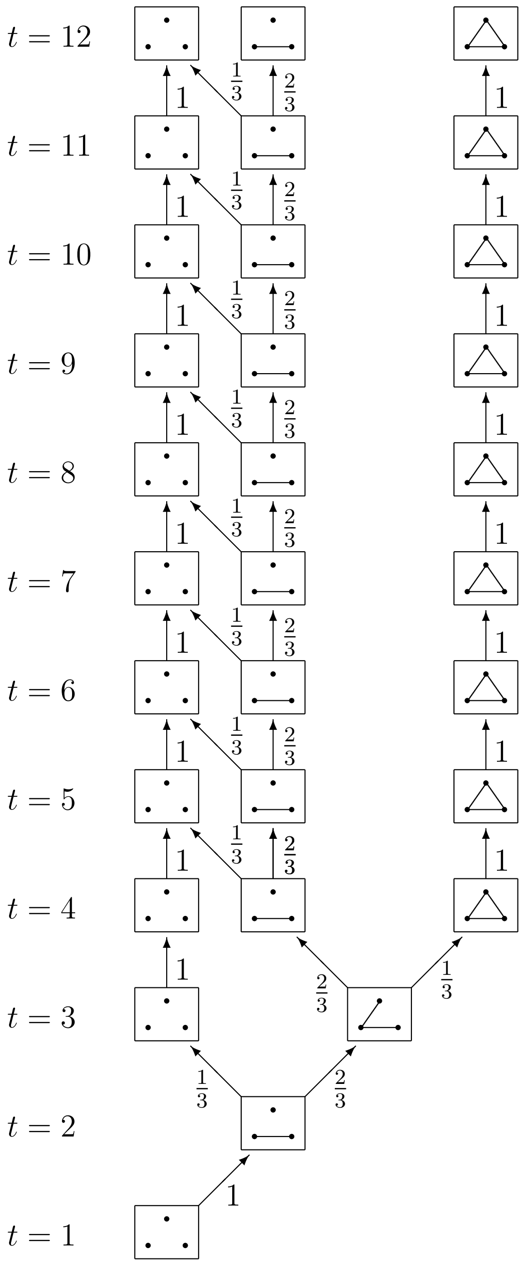


Figure 3: The subgame-perfect equilibrium path when $T = 12$ and $\delta = 0.98$.

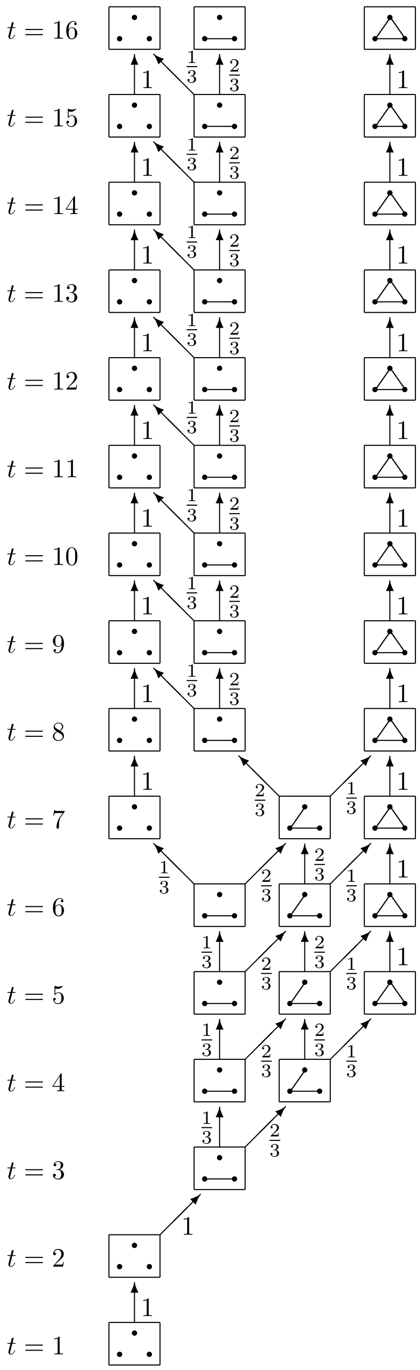


Figure 4: The subgame-perfect equilibrium path when $T = 16$ and $\delta = 0.98$.

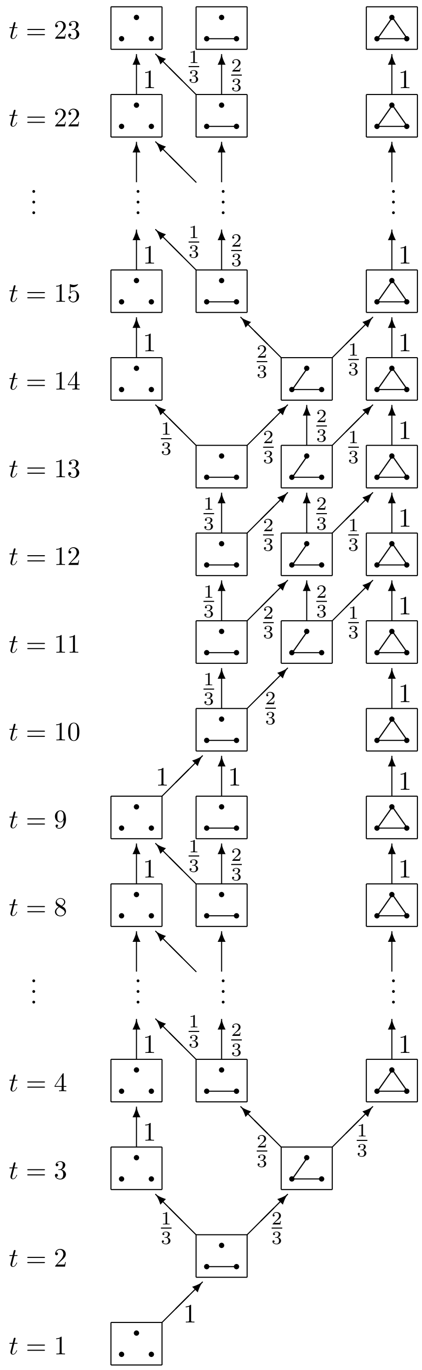


Figure 5: The subgame-perfect equilibrium path when $T = 23$ and $\delta = 0.98$.

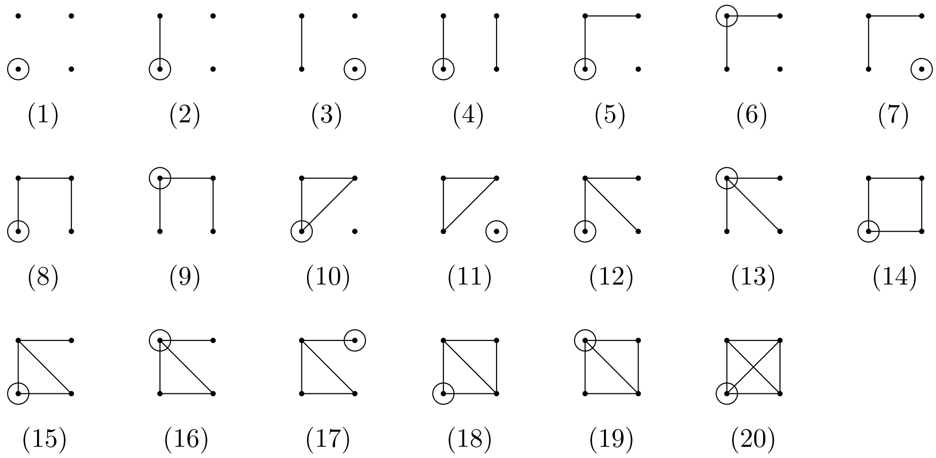


Figure 6: The states in each period for the four-person case.

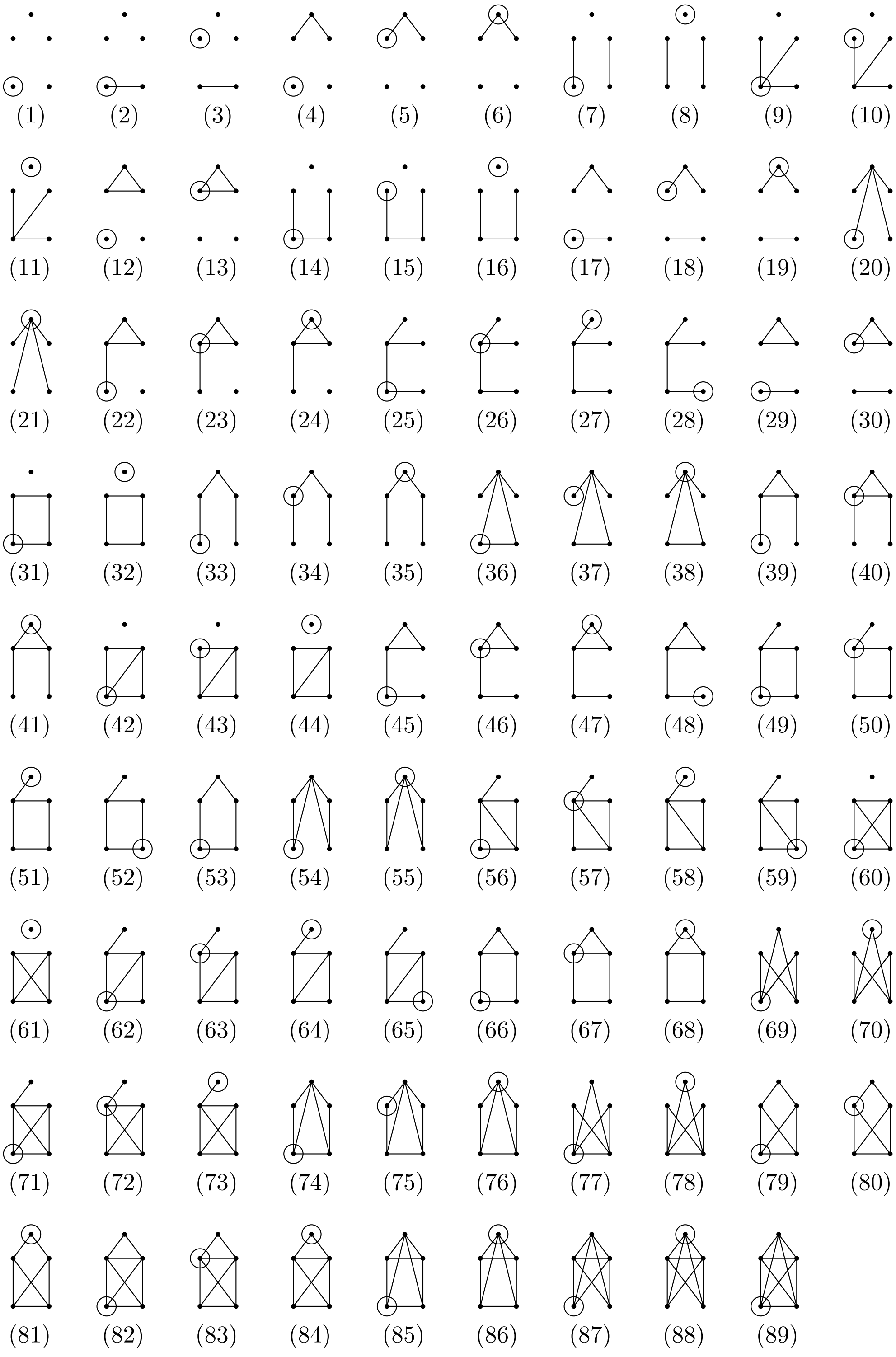


Figure 7: The states in each period for the five-person case.