E C O N O M I C S   B U L L E T I N

On the coexistence of reciprocity and materialism

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Abstract

This paper studies the problem of the long-run preferences distribution in a large population using an evolutionary approach. Special attention is given to the investigation of the property of the mixed population equilibria, in which materialists and reciprocators coexist. Some of the players with reciprocal preference care about not only their own material payoffs but also about those of others, while the remaining ones are materialists who maximize their own material payoffs.

The author is grateful to Ikuo Ishibashi, Kazuhiro Yamamoto, Mami Matsuda, Noriyuki Yanagawa, Takashi Shimizu, and Yoichi Fukuo for their helpful comments and suggestions. Of course, the author is responsible for any remaining errors.
Citation: Maruyama, Akiko, (2007) "On the coexistence of reciprocity and materialism." *Economics Bulletin,* Vol. 3, No. 59 pp. 1-9
Submitted: May 30, 2007. **Accepted:** November 13, 2007.
URL: http://economicsbulletin.vanderbilt.edu/2007/volume3/EB-07C70011A.pdf

1. Introduction

Individuals frequently behave altruistically toward others despite the potential for their own material loss and other people's material gain. Many experimental studies, such as those by Fehr and Schmidt [5], and Bolton and Ockenfels [1], suggest that subjects appear not to care about their own monetary payoffs. Reciprocal behavior is a well-known human behavior that is similar to altruistic behavior. For example, Levine [8] leads the model of reciprocal behavior to explain the results of the ultimatum experiment and public goods games. Sethi and Somanathan [10] also provide a plausible specification of reciprocal preference in human society. Reciprocators are assumed to be altruistic toward each other and to be spiteful toward selfish individuals. Selfish individuals are interested exclusively in their own material payoffs. We refer to selfish individuals as materialists in the following. Spitefulness (altruism) is typically represented by negative (positive) weight on others' material payoffs.

The objective of this study is to analyze the state in which reciprocators and materialists coexist (throughout this study, this state is referred to as a *mixed population*). Sethi and Somanathan [10] investigate mainly the state in which all members are reciprocators/materialists (this state is referred to as a monomorphic population of reciprocators/materialists). In this paper, we explore the possibility that a mixed state is (locally) stable. Since reciprocators and materialists appear to coexist in real society, the study of a mixed population would also be worthwhile.

Reciprocal preferences are often explained by the evolutionary approach. In these evolutionary frameworks, it is typically assumed that the selection dynamics are *payoff monotonic*: a (heritable) preference that earns higher material payoff is typically replicated more rapidly over time. For the purpose of analysis of a mixed population, we assume replicator dynamics, which belongs to the class of payoff monotonic dynamics. On the other hand, in a strategic setting, evolution will favor the emergence of a preference that earns higher material payoff.

We consider a strategic setting that belongs to the class of aggregative games. The feature of this game is that an individual's material payoff depends on his own action and an aggregate of the actions of others. Such a payoff structure is employed by strategic market games, common pool resource extraction, and public goods games. In addition, when the problem of preference evolution is studied, an aggregative game is considered to be a plausible class as a strategic environment (see Sethi and Somanathan [10], Corchón [2]). We apply this aggregative game to a common pool resource game (Dasgupta and Heal [4], Koçkesen, Ok, and Sethi [6], Sethi and Somanathan [9]) in order to investigate the property of the static mixed population.

This paper is organized as follows. In Section 2, we describe our model. In Section 3, we explore under what condition either materialists or reciprocators are better off than the other in a static analysis. In Section 4, we consider the evolutionary dynamic model. The final section is the conclusion.

2. A Model

First, we consider a game in a strategic form in which no player has an a priori advantage. We focus on a particular class of strategic environments, i.e., aggregative games. Let Γ denote a symmetric n-person $(n \geq 2)$ aggregative game in normal form $\Gamma \equiv \{\mathbf{X}_i, \pi_i\}_{i\epsilon I}$, where $\mathbf{I} \in \{1, \cdots, n\}$ is the set of players and $\mathbf{X}_i$ and $\Pi_i : \times_{j=1}^{n} \mathbf{X}_j \rightarrow \mathbf{R}$ are respectively the action set and the material payoff function of player i , which is symmetric for all players:

$$\Pi_i(x_i, \mathbf{x}_{-i}) \equiv \pi_i = \pi_i\left(x_i, X\right).$$

Here, $X = \sum_{j=1}^{n} x_j$ is the aggregate action in the group, and π_i is assumed to be twice differentiable. Let the marginal payoff of player i be denoted as:

$$T\left(x_i, X\right) \equiv \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_i}{\partial X}\frac{dX}{dx_i}.$$

Next, we introduce the notion of reciprocity. Let us consider a heterogeneous population, in which players belong to two different types, namely, materialists, who have materialistic preferences, and reciprocators, who have reciprocal preferences. Materialists are material-payoff maximizers, i.e., they are concerned with only their own material payoffs. On the other hand, reciprocators are concerned with not only their own material payoffs but also those of others. There are n-persons in the group, where $k \in \{0, \cdots, n\}$-persons are materialists and $(n-k)$-persons are reciprocators. Assuming that all players know the distribution of preferences, it is not necessary that all players know which of two preferences, reciprocal or materialistic, each player has.

Formally, player i's objective function is defined as follows:

$$\begin{cases} u_i\left(x_i\right) = \pi_i(x_i, X), & for\ i \in M, \\ u_i(x_i) = \pi_i(x_i, X) + \beta_r \sum\limits_{j \in R\backslash\{i\}} \pi_j(x_j, X) + \beta_m \sum\limits_{j \in M} \pi_j(x_j, X), & for\ i \in R \end{cases}$$

where M is the set of all materialists and R is the set of all reciprocators. It is assumed that the material payoff functions and the action space are symmetric for all players. Furthermore,

$$\beta_r = \frac{\alpha}{1+\lambda}, \text{ and } \beta_m = \frac{\alpha(1-\lambda)}{1+\lambda}.$$

$\alpha \in (0, 1)$ and $\lambda > 0$ are parameters. Since $\lambda > 0$, $\beta_r \in (0, \alpha)$ and $\beta_m \in (-\alpha, \alpha)$. These specifications are led by the following definition from Sethi and Somanathan [10],

$$\beta_{ij} = \frac{\alpha_i + \lambda\left(\alpha_j - \alpha_i\right)}{1+\lambda}.$$

That is, β_{ij} is the weight placed by player i on player j's payoff, which is the weighted average of α_i and $(\alpha_j - \alpha_i)$. A parameter α_i is player i's certain amount of pure altruism toward player j. Thus, λ implies the weight placed by player i on the deviation of j's altruism from i's own, where the weight placed by i on his own α_i is 1. If player j is a reciprocator, reciprocator i places the weight $\beta_r > 0$ on j's payoff (where $\alpha_j = \alpha_i = \alpha$). Conversely, if player j is a materialist $(\alpha_j = 0)$, reciprocator i places the weight β_m on j's payoff. If player i is more concerned with the difference in the altruism of others from i's own $(\lambda > 1)$, then reciprocators are spiteful toward materialists. On the other hand, if $0 < \lambda < 1$, reciprocators are altruistic towards all players. We define this game as a n-person game $\Gamma(k)$, $k \in \{0, \cdots, n\}$. Since player's objective functions are heterogenous, the equilibria of $\Gamma(k)$ will typically be asymmetric.

Furthermore, for the static analysis in the next section, we apply Γ to a symmetric common pool resource game (which we refer to as the CPR game throughout). Let us suppose that there are n players in a group and each player has access to a common pool resource. Player i chooses the extraction effort $x_i \geq 0$, and then $X = \sum_{j=1}^{n} x_j$ is the aggregate extraction effort. Let $F(X, K)$ denote the aggregate production function, where K is the resource stock. For simplicity, we assume that the resource stock is exogenously given and constant, i.e., $K = K_0$. Then, $f(X) = F(X, K_0)$, which is assumed to be a differentiable real function, such that $f'(X) > 0$, $f''(X) < 0$ and $f(0) = 0$. Each member of the group receives a share of the total product that corresponds to his share of the aggregate extractive effort. The cost of effort w is constant and exogenously given.

The payoff to player i is given by

$$\pi_i(x_i, X) = \frac{x_i}{X} f(X) - x_i w = x_i \left(A(X) - w\right),$$

where $A(X) \equiv f(X)/X$ is the average value of the effort. It is assumed throughout that $f(X)$ has an upper bound, i.e., $\lim_{X \to \infty} f'(X) = 0$, and that $f'(0) > w$ to guarantee an interior solution. As is well known, if all members are materialists under open access, we will have an equilibrium of effort that is unique, interior, symmetric, and inefficient.

From these, we obtain,

$$\frac{\partial \pi_i}{\partial x_i} = A(X) - w, \tag{1}$$

$$\frac{\partial \pi_i}{\partial X} = x_i A'(X) < 0, \tag{2}$$

$$\frac{\partial \pi_i}{\partial x_i} = T(x_i, X) = A(X) - w + x_i A'(X),$$

$$\frac{\partial T(x_i, X)}{\partial x_i} = A'(X) < 0. \tag{3}$$

It is noteworthy that, since $f(X)$ is strictly concave and $f(0) = 0$ from the assumptions, we have $A'(X) < 0$ for all $X \geq 0$. Furthermore, we assume that

$$\frac{\partial T(x_i, X)}{\partial X} = A'(X) + x_i A''(X) < 0. \tag{4}$$

The sign of (4) means the assumption of strategic substitutability. This assumption is general in the analysis of the symmetric aggregative game (see, Corchón [2], Sethi and Somanathan [10]). From (3) and (4), the marginal payoff function is strictly decreasing in x_i and X, and

$$\frac{\partial^2 \pi_i}{\partial x_i^2} = \frac{\partial T(x_i, X)}{\partial x_i} + \frac{\partial T(x_i, X)}{\partial X} < 0.$$

Therefore, the payoff function π_i is strictly concave.

When there are reciprocators or materialists in this CPR game, the necessary condition for the Nash equilibrium is given by

$$\frac{\partial u_i}{\partial x_i} = T(x_i, X) \leq 0, \qquad \forall i \in M, \tag{5}$$

$$\frac{\partial u_j}{\partial x_j} = T(x_j, X) + \beta_r \sum_{i \in R \setminus \{j\}} x_i A'(X) + \beta_m \sum_{i \in M} x_i A'(X) \leq 0, \qquad \forall j \in R. \tag{6}$$

We define this game as a n-person CPR game $\Gamma_C(k)$, $k \in \{0, \cdots, n\}$.

3. Static Analysis

Consider a CPR game $\Gamma_C(k)$, $k \in \{0, \cdots, n\}$. Our main questions are under what condition either reciprocators or materialists have a higher material payoff in a given equilibrium of $\Gamma_C(k)$, where there are both reciprocators and materialists, i.e., for any $k \in \{1, \cdots, n-1\}$.

It is useful to introduce the following concept before we introduce the main subject.

Definition 1 *If players with the same preference take the same action, an equilibrium* $\mathbf{x}$ *of* $\Gamma_C(k)$ *is referred to as* intragroup symmetric*.*

Lemma 1 *For any $k \in \{0, \cdots, n\}$, every equilibrium of $\Gamma_C(k)$ is an intragroup symmetric.*

The following proposition shows the sufficient condition for outperformance of reciprocators (materialists).

Proposition 1 *In any equilibrium of $\Gamma_C(k)$, $k \in \{1, \cdots, n-1\}$, if $\lambda > (<)\ \bar{\lambda} = (n-1)/k$ and $\lambda > 1$, $\pi_j(x_j, X) > (<)\ \pi_i(x_i, X)$ for any $i \in M$, $j \in R$.*

Proposition 1 implies that, when $\lambda > (<)\ \bar{\lambda}$, for all $k \in \{1, \cdots, n-1\}$, reciprocators (materialists) in a group will outperform materialists (reciprocators) in that group. If $1 < \lambda$ holds, reciprocators are always spiteful toward materialists. Then, reciprocators have an advantage over materialists. That is, if $\bar{\lambda} < \lambda$, reciprocators have the urge to extract more of the common resource than the extraction level chosen by materialists. If $\bar{\lambda} > \lambda > 1$, reciprocators are willing to strengthen altruistic links with each other rather than respond spitefully to the presence of materialists. Since reciprocators do not raise their extractive effort much, materialists are able to be better off than reciprocators.

Moreover, our result implies that the threshold weight $\bar{\lambda}$ depends on the number of materialists (reciprocators) and the number of persons n in a group. Given the group size n, $\bar{\lambda}$ is a decreasing function of the number of materialists. For instance, if there are a large number of materialists in a group of constant size n, $\bar{\lambda}$ becomes small. That is to say, in a case in which there are a large number of materialists, smaller values of $\lambda \left(> \bar{\lambda}\right)$ are sufficient for reciprocators to outperform materialists in $\Gamma_C(k)$ than in the case in which there are few materialists. When $\lambda \left(> \bar{\lambda}\right)$ is small, β_m is not very large, and β_r is large. Conversely, if there are few materialists in a group of (constant) size n, the threshold weight becomes large. In this case, a relatively large λ is necessary for the outperformance of reciprocators, compared to the case of large k. If $\lambda \left(> \bar{\lambda}\right)$ is large, reciprocators place large negative weights on the payoffs of materialists, and the weight on the payoffs of reciprocators is small.[1] These results have evolutionarily implications as well, in the sense that evolution will favor the emergence of the preference that earns a higher material payoff.

Lemma 1 and Proposition 1 are an extension of those of Koçkesen, Ok, and Sethi [6] and Sethi and Somanathan [10] to the analysis of the state with a mixed (group) population that contains heterogenous preferences.[2]

4. Evolutionary Dynamics

Now, we analyze the long-run preference distribution in a large population. It is assumed that the population is infinite.[3] Let p_m and p_r denote the share of materialists and the share of reciprocators in the global population, respectively, where $p_m + p_r = 1$. Furthermore, n-persons are randomly pulled out from the global population, and they then match randomly with each other in a group of size n that they formed. Therefore, the probability $\gamma_k(p_m)$ that k materialists are contained in the group formed is

$$\gamma_k(p_m) = \binom{n}{k} (p_m)^k (p_r)^{n-k}.$$

It is assumed that all players of a group know the distribution of preferences in the game. Let $\mu_m(k)$ (> 0) and $\mu_r(k)$ (> 0) denote the expected equilibrium payoff to materialists and reciprocators respectively in a group with population composition $k \in \{0, \cdots, n\}$.[4] Generally, each game $\Gamma(k)$ may have multiple equilibria, and then $\mu_m(k)$ and $\mu_r(k)$ originally depend on the probabilities with which a variety of equilibria are realized in $\Gamma(k)$. Our assumptions, along the lines of those by Sethi and Somanathan [10], are as follows: the probability that any certain equilibrium is realized is exogenously given for any given group composition k. As a result, $\mu_m(k)$ and $\mu_r(k)$ are well defined. In the global population, the expected payoffs to materialists and reciprocators are $\bar{\mu}_m(p_m) = \sum_{k=1}^{n} [\gamma_k(p_m) / \sum_{l=1}^{n} \gamma_l(p_m)] \mu_m(k)$ and $\bar{\mu}_r(p_m) = \sum_{k=0}^{n-1} [\gamma_k(p_m) / \sum_{l=0}^{n-1} \gamma_l(p_m)] \mu_r(k)$, respectively. Now, in order to focus on the stability of the interior states $p_m \in (0,1)$, we consider replicator dynamics, which is included in the class of payoff monotonic dynamics. If the population is very large and the generations fuse continuously, we may assume that the state $p_m(t)$ evolves as a differentiable function of t. The rate of increase in $\dot{p}_m/p_m$ is a measure of the materialists' evolutionary success. We may express this success as the difference between the payoff

[1] If the number of materialists k is constant, then $\bar{\lambda}$ is an increasing function of n. In this case, the same logic to the above discussion can be applied here as well, since a decline in n with constant k has a similar effect to an increase in k with constant n.

[2] Koçkesen, Ok, and Sethi [6] show that, in any CPR game, materialistic preferences obtain strictly lower (absolute) payoffs than *interdependent preferences*, as materialists are concerned with their own (absolute) payoff and their payoff relative to the average payoff in the population.

[3] The results obtained also hold for a sufficiently large population.

[4] We can observe the assumption that the payoff to players is positive in many papers. In this paper, in $\Gamma_C(k)$, $k \in \{1, \cdots, n-1\}$, if $\lambda > 1$, all players' payoffs are positive (see the proof of proposition1). Further, in $\Gamma_C(0)$ and $\Gamma_C(n)$, the payoff to players is also positive (this can be proved by the same method as the proof of Proposition 1). In this section, with the assumptions of positive $\mu_m(k)$ and $\mu_r(k)$, we can obtain the following result in a more general aggregative game that includes the common pool resource game.

$\bar{\mu}_m(p_m)$ or $\bar{\mu}_r(p_m)$ and the average payoff $\bar{\mu}(p_m) = p_m\bar{\mu}_m(p_m) + p_r\bar{\mu}_r(p_m)$ of the global population. Thus, we obtain two replicator equations: $\dot{p}_m(t) = [\bar{\mu}_m(p_m) - \bar{\mu}(p_m)]\, p_m$ and $\dot{p}_r(t) = [\bar{\mu}_r(p_m) - \bar{\mu}(p_m)]\, p_r$. From the definition of $\bar{\mu}(p)$, these two equations and take the same form:

$$\dot{p} = \Psi(p) \cdot p(1-p) \equiv g(p),$$

where $p = p_m$, and $\Psi(p) \equiv [\bar{\mu}_m(p) - \bar{\mu}_r(p)]$. We can rewrite $\Psi(p)$ as

$$\Psi(p) = \sum_{k=1}^{n} \tfrac{\gamma_k(p)}{1-\gamma_0(p)}\mu_m(k) - \sum_{k=0}^{n-1} \tfrac{\gamma_k(p)}{1-\gamma_n(p)}\mu_r(k).$$

If interior solutions $\hat{p} \in (0,1)$ exist, $\hat{p}$ satisfies $\Psi(p) = 0$. Suppose that $g(p)$ is differentiable in $(0,1)$ and continuous in $[0,1]$. Then, Proposition 2 shows that there is, at least, one interior equilibrium $\hat{p}$.

Proposition 2 *In $\Gamma(k)$, two equilibria, $p = 1$ and $p = 0$, are locally unstable, and there exists at least one interior equilibrium $\hat{p}$ satisfying $\bar{\mu}_m(p) = \bar{\mu}_r(p)$, in $(0,1)$.*

Proof. It is noteworthy that $g(p)$ is a differentiable function on the open interval $(0,1)$.

$$g'(p) = \Psi'(p) \cdot (1-p)\,p + \Psi(p) \cdot (1-2p).$$

When $p \to 1$, $\gamma_k(p) \to 0$, $\gamma_0(p) \to 0$ and $\gamma_n(p) \to 0$. Moreover, we have $\Psi'(p) \to 0$, if $p \to 1$. Accordingly,

$$g'(p) \to -\Psi(1).$$

where $\Psi(1) = -\sum_{k=0}^{n-1} \{\gamma_k(1) / [1-\gamma_n(1)]\}\, \mu_r(k)$. From l'Hospital's rule,

$$\lim_{p\to 1}\left(\tfrac{\gamma_k(p)}{1-\gamma_n(p)}\right) = \begin{cases} 0, & \text{for} \quad k \in \{1, 2, \cdots, n-2, n\} \\ 1, & \text{for} \quad k = n-1 \end{cases}$$

Therefore, when $p \to 1$, then $g'(p) \to -\Psi(1) = \mu_r(n-1) > 0$ holds. Similarly, if $p \to 0$, we obtain $g'(p) \to \Psi(0) = \mu_m(1) > 0$. These signs of $g'(p)$ together imply that there exists at least one interior equilibrium $\hat{p} \in (0,1)$ where $g(\hat{p}) = 0$. ■

Proposition 2 shows that there exists at least one equilibrium in which reciprocators and materialists coexist. Next, we consider the stability of an interior solution. Let us now assume that, in Γ_C, $\Psi(p)$ is a monotone decreasing function in $[0, 1/2]$ when $\lambda > n-1$.[5] Proposition 3 shows the sufficient condition for the local stability of an interior solution in Γ_C.

Proposition 3 *Suppose that, in Γ_C, when $\lambda > n-1$, $\Psi(p)$ is a monotone decreasing function in $[0, 1/2]$. If $\lambda > \bar{\lambda} = n-1$, there exists one locally stable equilibrium $\hat{p}$ in $(0, 1/2)$.*

Proof. It is noteworthy that $\lim_{p\to 0} \Psi(p) = \mu_m(1) > 0$ from Proposition 2. We also have

$$\Psi\left(\tfrac{1}{2}\right) = \frac{\left(\frac{1}{2}\right)^n}{1-\left(\frac{1}{2}\right)^n}\left[\mu_m(n) - \mu_r(0)\right] + \sum_{k=1}^{n-1} \frac{\left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{n-k}}{1-\left(\frac{1}{2}\right)^n}\left[\mu_m(k) - \mu_r(k)\right].$$

[5]It is noteworthy that when $\lambda > n-1$, $\Psi(p)$ becomes a monotone decreasing function in $(0, 1/2]$ with some combinations of the value of $\mu_m(k)$, $\mu_r(k)$, $k \in \{0, \cdots, n\}$.

As $\lambda > n-1$, $\mu_m(k) < \mu_r(k)$ for $k \in \{1, \cdots, n-1\}$, from Proposition 1. From this and $\mu_m(n) < \mu_r(0)$, $\Psi(1/2) < 0$ is hold .[6] Since $\Psi(p)$ monotonically decreases in $[0, 1/2]$, there exists one locally stable equilibrium $\hat{p} \in (0, 1/2)$. ■

Proposition 3 implies that, in Γ_C, when reciprocators are spiteful toward materialists, the interior equilibrium $\hat{p} \in (0, 1/2)$ is locally stable. Then, if there is an initial point in a $\hat{p}$-neighborhood, a global population will converge to the mixed population in time. Furthermore, from Proposition 2, both the monomorphic population of materialists ($p = 1$) and the monomorphic population of reciprocators ($p = 0$) are locally unstable. Therefore, even if the distribution of preferences changes with time, there are both materialists and reciprocators in the global population at any time. Our result is obtained without the assumption of the separability in material payoffs that is assumed in Sethi and Somanathan [10].

5. Concluding Remarks

In this study, we explored the states of mixed populations in which materialists and reciprocators coexist. First, our static analysis in a CPR game proves that reciprocators earn strictly greater (lower) material payoffs than materialists if reciprocators are much (less) concerned with the difference between the altruism of others and their own in an equilibrium. Moreover, we find that the threshold weight for reciprocators (materialists) to outperform depends on the distribution of preferences in a CPR game.

Next, under replicator dynamics, we show that there exists, at least, one equilibrium in which materialists and reciprocators coexist in an aggregative game. Moreover, this equilibrium is locally stable when reciprocators are spiteful toward materialists in a CPR game. Therefore, the global population will then converge to a mixed (global) population over time if there is an initial point in a neighborhood of that interior equilibrium. In addition, we find that the monomorphic (global) population of each preference is locally unstable.

Appendix

Proof of Lemma 1 Let us suppose that there exists $i,j \in R$ such that $x_i > x_j$ at some equilibrium $\mathbf{x}$ of $\Gamma_C(k)$. From the necessary condition for equilibrium (6), we must have $\partial u_i/\partial x_i = 0 \geq \partial u_j/\partial x_j$, or

$$T(x_i, X) + \beta_r x_j A'(X) + \beta_r \sum_{j \in R \setminus \{i,j\}} x_j A'(X) + \beta_m \sum_{j \in M} x_j A'(X)$$
$$\geq T(x_j, X) + \beta_r x_i A'(X) + \beta_r \sum_{i \in R \setminus \{i,j\}} x_j A'(X) + \beta_m \sum_{j \in M} x_j A'(X).$$

[6] The fact $\mu_r(0) > \mu_m(n)$ can be proved along the same lines of Sethi and Somanathan (2001 [proof of Proposition 5]). However, in Γ_C, we can obtain $\mu_r(0) > \mu_m(n)$ without the separability condition $\partial \pi_i / \partial x_i \partial X = 0$ in their proof. Moreover, we can show that $\Gamma_C(0)$ and $\Gamma_C(n)$, respectively, have a unique equilibrium following the same procedure as Sethi and Somanathan (2001 [claim 1 of Proposition 4]).

Therefore we obtain

$$A(X) - w + (1-\beta_r)x_i A'(X) \geq A(X) - w + (1-\beta_r)x_j A'(X)$$
$$\therefore \ (x_i - x_j)A'(X) \geq 0.$$

where $(1-\beta_r) > 0$ from $\alpha \in (0,1)$ and $\lambda > 0$. Since $A'(X) < 0$, $x_i \leq x_j$. However, this contradicts $x_i > x_j$. Hence, for all i,$j \in R$, $x_i = x_j$. The result for any i,$j \in M$, $x_i = x_j$ follows by applying the above reasoning to (5). ■

Proof of Proposition 1 Let $\mathbf{x} \in \mathbf{R}_+^N$ be an equilibrium of $\Gamma_C(k)$, $k \in \{1, \cdots, n-1\}$ and the aggregate equilibrium effort be $X = \sum_{i=1}^n x_i$. Let y and z denote the equilibrium extraction effort of $i \in M$ and $j \in R$ respectively, from Lemma 1.

From the assumptions that $A'(X) < 0$, $f'(0) > w$, and f is bounded above, there exists a unique open access aggregate effort $X^O > 0$ such that $A(X) \lesseqqgtr w$ whenever $X \gtreqqless X^O$.

First, we claim that $X < X^O$ whenever $y > 0$ in $\Gamma_C(k)$. To examine this, let us suppose $X \geq X^O$, which implies that $A(X) - w \leq 0$. However, from (5), $T(y, X) = A(X) - w + yA'(X) = 0$ must hold at equilibrium. This and (3) improve $A(X) - w > 0$. Hence, $X < X^O$.

Next, we show that $\Gamma_C(k)$ has $y > 0$. If $y = 0$ and $z = 0$, then $X = ky + (n-k)z = 0 < X^O$. Hence, $A(X) - w > 0$ holds. On the other hand, from (5) and (6), we have $T(y, X) = T(z, X) = A(X) - w \leq 0$. Hence, $y > 0$ or $z > 0$.

Now, let us suppose that $y = 0$ and $z > 0$. If $X \geq X^O$, then $A(X) - w \leq 0$ so that, by $A'(X) < 0$,

$$\frac{\partial u_j}{\partial x_j} = A(X) - w + zA'(X) + \beta_r \sum_{j \in R} zA'(X) < 0.$$

This improves $z = 0$, contradicting the idea that $z > 0$ in a Nash equilibrium. Accordingly, we must have $X < X^O$ whenever $z > 0$. This implies that $A(X) - w > 0$. However, from (5) and $y = 0$, $A(X) - w \leq 0$ must hold. Hence, $y > 0$ or $z = 0$. Therefore, $y > 0$ and $X < X^O$ hold in $\Gamma_C(k)$.

Furthermore, we show that, if $\lambda > 1$, $\Gamma_C(k)$ has $y > 0$ and $z > 0$. We assume that $z = 0$. From the necessary condition for equilibrium (6),

$$\frac{\partial u_i}{\partial x_i} = A(X) - w + \beta_m \sum_{j \in M} yA'(X) \leq 0.$$

From the assumption that $A'(X) < 0$ and $\lambda > 1$, we obtain $A(X) - w < 0$. However, since $y > 0$, we must have $A(X) - w > 0$ from $T(y, X) = 0$ and $A'(X) < 0$. Hence, $y > 0$ and $z > 0$ must hold when $\lambda > 1$. This improves $X < X^O$.

Finally, we compare the material payoff of a materialist and a reciprocator when $\lambda > 1$ holds at equilibrium of $\Gamma_C(k)$. Take any $i \in M$ and $j \in R$. When $\lambda > 1$, (6) can be rewritten as follows:

$$\frac{\partial u_j}{\partial x_j} = T(z, X) + \frac{\alpha A'(X)}{1+\lambda} \left[(n-k-1)\, z - (\lambda - 1)\, ky\right] = 0. \tag{7}$$

(5) and (7) imply that

$$A'(X)(z-y) + \frac{\alpha A'(X)}{1+\lambda}\left[(n-k-1)\,z - (\lambda-1)\,ky\right] = 0;$$

$$\therefore,\ z = \frac{(1+\lambda)+\alpha k\,(\lambda-1)}{(1+\lambda)+\alpha\,(n-k-1)}\ y. \tag{8}$$

If $(n-1)/k < \lambda$, then

$$1 = \frac{(1+\lambda)+\alpha\,(n-k-1)}{(1+\lambda)+\alpha\,(n-k-1)} < \frac{(1+\lambda)+\alpha k\,(\lambda-1)}{(1+\lambda)+\alpha\,(n-k-1)} = \frac{z}{y}.$$

We obtain $z > y$. By (1) and $A(X) - w > 0$, we have $\pi_j(z, X) > \pi_i(y, X)$. If $(n-1)/k > \lambda > 1$, then $z < y$ and $\pi_j(z, X) < \pi_i(y, X)$. ■

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