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Double Bertrand competition among intermediaries when consumers can default

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Abstract

This paper models a sequential double price competition among intermediaries when their expected revenue per sale is affected by consumers' default. If this revenue is non-monotonic with the asking price, the Walrasian outcome may not be an equilibrium and demand rationing may emerge instead.

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Double Bertrand Competition among Intermediaries when Consumers can Default.

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Key Words: Bertrand Competition, Double-sided Competition, Consumer Default, Demand Rationing.

JEL Codes: D43; D45; L13

1 Introduction

Because intermediaries buy goods from producers and resell them to consumers, they are necessarily involved in a double competition, competition on inputs as well as competition on outputs. Stahl (1988) made a significant contribution to the understanding of this complex double-sided competition by modelling a two-stage game of price competition among merchants. In particular, he develops an interesting sequential Bertrand competition game in which the competition for inputs takes place before the competition for output, and intermediaries are compelled to buy all the supply offered by producers. Stahl shows that such a sequential competition can generate two different outcomes. If the Walrasian asking price to consumers is higher than the sales-revenue-maximising price, the game displays the Walrasian equilibrium, confirming that trade organised through competing intermediaries is equivalent to a centralised trading system with a benevolent auctioneer. However, if the Walrasian asking price is lower than the sales-revenue-maximising price, the outcome of the game is different from the Walrasian equilibrium and depends on the rule that is applied when merchants post the same price for the inputs. If intermediaries share equally the market in the case of a tie, an equilibrium fails to exist. Alternatively, if one intermediary is randomly allocated the whole market, the equilibrium is characterised by an excess capacity, where some inputs are bought from producers but not resold to consumers.

Stahl's model of double Bertrand competition applies to any type of intermediation where intermediaries compete by prices in their input and output markets. It is not only a relevant framework to analyse competition among supermarkets/dealers reselling goods bought from the producers to the consumers, as Stahl does in his paper but Stahl's modelling has some interest for the financial markets as well. Indeed double Bertrand competition also applies to competition among intermediaries in the primary financial markets, in which the institutional investment firms buy the newly issued financial products (like Treasury and companies bonds/bills) from the issuers in need of funds and resell them to the final investors (like households). This double Bertrand competition is a relevant structure as well for the modelling of banking intermediation, where retail banks borrow funds from savers, mainly through deposits, and lend them to borrowers. This competitive structure in the banking area has been analysed in the literature by Yanelle (1989), Freixas and Rochet (1997), Toolsema (2001) and Bracoud (2002).

The original paper by Stahl assumes that the merchant's actual revenue per unit of sale equals the asking price to consumers, which requires that all customers entirely pay their bills to the intermediary. The present paper aims at generalising Stahl's analysis by capturing the possibility that some consumers may default on their payments to the intermediary. As a consequence, the average revenue per unit of good among the pool of customers could be less than the asking price, contrary to the assumption made in Stahl's original paper.

For banking intermediaries, possible default by borrowers is indeed a core feature of their lending business and has been a research field for many years. (See Freixas and Rochet (1997) for an overview). The interest of modelling default by consumers however goes much beyond the case of banking intermediation as default has become a relevant concept for many other markets that now allow deferred payments by consumers. It is indeed a widespread practice for many big retailers of furniture and white goods to provide their customers with the option to pay months after the delivery of the good, often without any interest being charged. This facility has become a significant feature of competition among providers of durable consumption goods, as reflected in retailers' advertisements in the media. The delay between the delivery and the payment does not need to be very long to generate a possible default. In primary and secondary financial markets, the delay between the delivery of a financial product and its financial settlement is enough for the intermediaries to bear the risk not to receive the totality of the posted price when reselling financial products.[1]

To capture such default risk on payments, the current paper assumes that the expected revenue per unit of sale depends not only on the price posted but also on the probability of default by consumers. An interesting case is that of an increasing default probability. The average probability of default may increase with the asking price for several reasons. Firstly, if a given unexpected shock in consumer's budget had to occur between the delivery and the payment, it would be more difficult to

[1] One can actually extend the argument to any intermediary that accepts cheques as a means of payment as the bank may have to reject the cheque if the client's account is not in credit at that time and this may translate into a default for the intermediary.

honour the payment if the good were more expensive. Secondly, a moral hazard effect can take place: dishonest consumers are more ready to bear the risk of judicial problems related to a break in their payments if the gain to be made (if they manage to get away with it) is higher. Thirdly, an adverse selection effect could also operate by keeping the best customers outside the market when the price increases, and therefore decreasing the average quality of the market (Akerlof (1970), Stiglitz and Weiss (1981)). These three mechanisms, operating individually or in combination, can generate an expected revenue per unit of sale that is a non-monotonic function of the posted price: the expected revenue per sale eventually decreases, because the decrease in the average probability of payment more than offsets the increase in the asking price.

The present paper shows that the introduction of default leads to three mutually exclusive configurations, depending on the characteristics of the demand and supply functions for the good. The three possible outcomes of our framework are illustrated in Figures 1-3. As in Stahl's original paper, we get a case (illustrated in Figure 1) where there is no subgame perfect equilibrium in the game. However the introduction of default paradoxically makes this result less likely to occur than in the model by Stahl. Our framework could also generate a Walrasian equilibrium like in the original paper, which is represented in Figure 2. In this case the Walrasian asking price is higher than in the case with no default and the Walrasian bidding price is lower. More importantly, the non-monotonicity of the expected revenue per unit of sale, induced by consumers' default on payments, introduces a new form of inefficiency where consumers are rationed, as illustrated in Figure 3, a situation that could not arise in Stahl's framework. This result is in line with Stiglitz and Weiss' (1981) pioneer work on adverse selection and moral hazard in the loan market, where it is shown that credit rationing could be an outcome of asymmetric information. When applied to banking intermediation, the game-theoretical framework of double Bertrand competition that we develop in the current paper can therefore be seen as a formalization of the Stiglitz and Weiss rationing story. When applied to the markets of durable goods, the framework gives us some interesting insights on the possible consequences of introducing some facilities for the customers of delaying payments.

The paper is organised as follows: Section 2 presents the model, Section 3 characterises the Nash Equilibrium (NE) in the output subgames, which is the first step in solving our problem by backwards induction, and Section 4 defines the Subgame Perfect Nash Equilibrium (SPNE) of the whole game. Section 5 discusses the results and concludes.

2 The Model

Let $S(p)$ be the aggregate supply by producers when offered a price p, with $S(0) = 0$ and $S' > 0$. Let $D(p)$ denote the aggregate demand by consumers when facing an asking price p. Demand is characterised by $D' < 0$ and for prices higher or equal to p_{max}, demand is zero. The expected revenue per unit of sale is $\rho(p) = [1 - q(p)]\, p$ where q ($\in [0; 1]$) is the average probability of default in the pool of customers. Note that if $q = 0$, $\rho = p$ as in Stahl's original paper. We analyse the case where $q' > 0$, with $\lim_{p \to 0} q = 0$ and $\lim_{p \to p_{\max}} q = 1$. The expected revenue per unit of sale ρ is therefore concave, takes the value zero for $p = 0$ and $p = p_{max}$, and has a unique global maximum at $p = \bar{p_a}$.

The sequence of the game is as follows: at stage 1, two intermediaries simultaneously post a bidding price p_b^i with $i = 1, 2$; at stage 2, producers choose their intermediary, the latter having the obligation to accept their entire supply; at stage 3, intermediaries with the stocks formed at stage 2 post an asking price p_a^i; at stage 4, consumers choose their intermediary; at stage 5, intermediaries reject some consumers if their stocks are insufficient. Rationed consumers can then visit the other intermediary. In the case of tied bidding prices, producers are assumed to distribute equally among intermediaries, as it is a more realistic assumption than the rule, also used in Stahl's paper, by which a middleman would be chosen randomly to get the whole market. We assume a proportional rationing scheme for excess demand, although this type of rationing is not essential for our results to hold. [2] The residual demand facing the high-priced merchant 1 when the competitor has a capacity x is therefore $RD(p_a^1, p_a^2, x) = \max\{0; \alpha\} D(p_a^1)$ with $\alpha = \frac{D(p_a^2) - x}{D(p_a^2)}$.

We denote by $p_a^{mc}(p_b)$ the asking price that guarantees market clearing for a given bidding price p_b, i.e. $S(p_b) = D(p_a^{mc})$. The combinations (p_b, p_a^{mc}) are represented by $g(p_b)$ in Figures 1-3, which splits the space into prices generating excess demand on the left-hand side and prices creating an excess supply on the right-hand side. If producers are aware of the default problem the Walrasian prices are the combination (p_b^W, p_a^W) such that $S(p_b) = D(p_a)$ and $p_b = \rho(p_a)$. This special point is located at the intersection of g and the dotted curve $p_b = \rho(p_a)$ in Figures 1-3. [3]

Let us now represent in Figures 1-3 the combinations of prices such that $\rho(p_a)\, Q - p_b S(p_b) = 0$ where the actual sales Q are such that $Q = \min\{D(p_a); S(p_b)\}$. These

[2] 'Proportional rationing' is also called sometimes 'random rationing' as in Stahl's paper, referring to customers arriving in random order and being served on a first-come, first-served basis. In the remainder of the paper we will refer to proportional rationing, as it is the usual terminology used in the IO literature.

[3] Without default as in Stahl, the Walrasian prices are defined to be the combination of prices such that $S(p_b) = D(p_a)$ and $p_b = p_a$. This point would be therefore located at the intersection between g and the 45^0 line.

combinations correspond to either a zero monopoly rent when only one intermediary is active in the market, or an aggregate zero profit when the two active merchants post exactly the same prices. The zero-monopoly-rent curve, represented in bold in the figures, is built by adding two parts of two distinct curves: we take the part of the dotted curve $p_b = \rho(p_a)$ located on the left-hand side of g (excess demand) and the part of the solid curve $\rho(p_a)D(p_a) = p_b S(p_b)$ situated on the right-hand side of g (excess supply). We assume that $p_a D(p_a)$ is concave and consequently has a global maximum, denoted by $\widehat{p}_a$. This implies that the expected sales revenue with the non-monotonic revenue, $\rho(p_a)D(p_a)$, also has a unique global maximum (although associated with a lower revenue), located at $\widehat{\widehat{p}}_a$, which is necessarily lower than $\bar{p_a}$.

As already mentioned in the introduction, Figures 1-3 represent the three mutually exclusive configurations that may arise in our model, characterized by a different SPNE for the game. Configuration 1 refers to the case $p_a^W < \widehat{\widehat{p}}_a$. Because of the non-monotonicity of ρ the opposite case $p_a^W > \widehat{\widehat{p}}_a$, contrary to Stahl's analysis, gives rise to two sub-cases: $p_a^W < \overline{p}_a$ (configuration 2) and $p_a^W > \overline{p}_a$ (configuration 3).

We solve the sequential game by backwards induction in order to determine the SPNE of the model: Section 3 identifies the Nash equilibria asking prices in the second part of the game (the output subgames covering stages 3 to 5) for every possible configuration of bidding prices and capacities inherited from the first two stages of the game. Section 4 then solves the first stage of the game, and finds the Nash equilibrium bidding prices taking into account the consequences of intermediaries' decisions in terms of bidding prices for the rest of the game.

3 The NE in the output subgame

When analyzing the output subgames, we take the bidding prices of intermediaries and their capacities in terms of inputs as given. Two possible types of output subgames emerge: those coming from a situation where one player has posted a more attractive bidding price than its competitor and is therefore a monopoly in the subgame; and those emerging from intermediaries having posted the same bidding prices in the previous stages of the game with therefore equal capacities.

Before analyzing the two types of subgames, let $\widetilde{p}_b$ denote the highest possible bidding price compatible with a non-negative profit, therefore the highest p_b located on the bold zero-monopoly-rent curve.

Firstly, consider the subgames where one intermediary has posted a more attractive bidding price than its competitor in stage 1 (still lower than $\widetilde{p}_b$), thereby receiving the entire supply $x = S(p_b)$, and benefiting from a monopolist position in the output market. The NE is consequently the monopoly asking price subject to a capacity constraint x, i.e. the middle value among $\widehat{\widehat{p}}_a$, $\overline{p}_a$ and p_a^{mc}.The proof is given in Appendix

A.

Secondly, let us consider subgames where both intermediaries have posted the same bidding price $p_b \leq \widetilde{p}_b$ in stage 1. When competing for consumers in stage 3, each merchant has the same capacity constraint equal to $x = \frac{S(p_b)}{2}$ and a fixed total cost of $p_b \frac{S(p_b)}{2}$. The market-clearing asking price p_a^{mc} is obviously dependent on p_b, for a higher bidding price is associated with a lower p_a^{mc}. Depending on the level of p_b in the output subgame considered, p_a^{mc}'s relative position with respect to $\widehat{\widehat{p}}_a$ and $\overline{p}_a$ (the latter parameters being independent of the subgame considered) may be different. A priori three rankings could emerge in the output subgames. As we will see later, depending on which configuration prevails (figures 1-3), not all three rankings we analyse below may be relevant. The proofs of NE in the output subgames defined below are detailed in Appendix B.

Low p_b necessarily generates the ranking $\widehat{\widehat{p}}_a < \overline{p}_a < p_a^{mc}$. This configuration leads to a NE in the output subgame at $\overline{p}_a$, characterized by demand rationing. This case is a direct consequence of the non-monotonicity of ρ.[4] For higher bidding prices, we could get $\widehat{\widehat{p}}_a < p_a^{mc} < \overline{p}_a$. In this scenario the NE in the output subgame is the market-clearing asking price p_a^{mc}. Finally, as soon as the bidding price is large enough to drive p_a^{mc} below $\widehat{\widehat{p}}_a$, there is no NE in the subgame any longer.

Let now relate the three subgames outcomes described above to the configurations in Figures 1-3. As already mentioned, the above rankings may not all be relevant for the three figures: indeed it may depend upon the position of the lowest market-clearing price compatible with nonnegative profit, $p_a^{mc}(\widetilde{p}_b)$ relative to $\widehat{\widehat{p}}_a$ and $\overline{p}_a$. In configuration 3, $p_a^{mc}(\widetilde{p}_b) = p_a^{mc}(\rho(\overline{p}_a))$, which is necessarily higher than $\widehat{\widehat{p}}_a$ and $\overline{p}_a$. Therefore in all subgames where intermediaries post the same bidding price, only the first ranking ever applies, generating demand rationing at $\overline{p}_a$. In configuration 2, $p_a^{mc}(\widetilde{p}_b) = p_a^W$, which is between $\widehat{\widehat{p}}_a$ and $\overline{p}_a$. The first two rankings therefore occur and subgames with the same bidding prices can therefore be characterized either by demand rationing or market clearing. Finally Configuration 1 has a market-clearing asking price that could decrease to below the Walrasian level, and therefore below $\overline{p}_a$ and $\widehat{\widehat{p}}_a$. As a result, configuration 1 covers the three rankings and therefore displays the following outcome across subgames: demand rationing; market clearing; and inexistence of equilibrium.

Let us denote by p_b^E the maximum bidding price posted by both intermediaries for which a NE in the related subgame exists, i.e. $p_a^{mc}(p_b^E) = \widehat{\widehat{p}}_a$. It is depicted for each configuration in Figures 1-3. In configuration 3, no problem of existence ever arises. As a result, in this configuration, $p_b^E = \widetilde{p}_b = \rho(\overline{p}_a)$. In configuration 2, no problem of

[4] Without default, as in Stahl's framework, the market-clearing price of the subgame p_a^{mc} is always lower than $\overline{p}_a (= p_{max})$.

existence arises either, thereby $p_b^E = \widetilde{p}_b = p_b^W$. In configuration 1, the range of highest bidding prices generates an absence of NE and consequently $p_b^E (< p_b^W) < \widetilde{p}_b$.

4 The SPNE of the game

Knowing the payoffs for all subgames starting at stage 3, we can now solve the first stage of the game, where competition for inputs takes place. Competition in the input market necessarily drives the bidding price to the highest value compatible with equilibrium in the subgame, i.e. p_b^E. For any other lower price indeed, one intermediary would have an incentive to deviate by offering a slightly higher bidding price in order to achieve a monopolist position in the output market. The discrete jump in quantities that this deviation represents will always more than offset the negative effect in terms of inputs costs. To put this formally: $\rho(p_a)\min\{D(p_a); S(p_b+\epsilon)\} - (p_b+\epsilon)S(p_b+\epsilon) > \rho(p_a)\min\{\frac{D(p_a)}{2}; \frac{S(p_b)}{2}\} - p_b\frac{S(p_b)}{2}$.

We now check if there is any possible deviation from p_b^E, our unique candidate as a SPNE. There is none for configurations 2 and 3. Since p_b^E is also equal to $\widetilde{p}_b$ in these configurations as shown at the end of the previous section, it is indeed never profitable for an intermediary to deviate from p_b^E through an increase in the bidding price, for there is no asking price able to generate a positive profit at all. Price p_b^E will effectively be a SPNE in configurations 2 and 3. In configuration 1, however, p_b^E is lower than $\widetilde{p}_b$, and therefore there is some profitable deviation from p_b^E by increasing the bidding price. This configuration actually does not display any SPNE at all, as the only way to avoid a deviation would be to bid $\widetilde{p}_b$, for which however there is no NE in the related output market subgame.
We summarise our results below:

Proposition *If $p_a^W < \widehat{\widehat{p}}_a < \overline{p}_a$ (configuration 1), the sequential game does not admit any SPNE. If $\widehat{\widehat{p}}_a < p_a^W < \overline{p}_a$ (configuration 2), the SPNE of the game is the Walrasian outcome $(\rho(p_a^W), p_a^W)$. If $\widehat{\widehat{p}}_a < \overline{p}_a < p_a^W$ (configuration 3), the SPNE is $(\rho(\overline{p}_a), \overline{p}_a)$ and displays demand rationing.*

When an equilibrium exists, it is characterized by zero profit in line with standard results in a Bertrand environment. Note that in the case of a tie, if the goods were allocated to a single intermediary chosen randomly rather than distributed equally among intermediaries, the inexistence result for configuration 1 would be replaced by a SPNE at $(p_b^E, \widehat{\widehat{p}}_a)$, with some of the stocks bought by intermediaries being not resold to consumers. As already mentioned, we do not think that this is a reasonable assumption.

5 Discussion and Conclusion

The outcome of nonexistence of equilibrium (or excess capacity if a single intermediary is randomly allocated the whole market) corresponding to configuration 1 will arise for fewer configurations of demand and supply functions than in the no-default case like Stahl. Indeed, some demand and supply functions could lead to $p_a^W < \widehat{p}_a$ in Stahl's framework whereas the introduction of default in the same environment could generate the opposite ranking with $p_a^W > \widehat{\widehat{p}}_a$. This reduction in inefficiency stems from the loss of revenue per unit of good generated by the default of some consumers - $\rho(p_a) < p_a$ - which pushes the Walrasian asking price up compared to Stahl's framework.[5]

A Walrasian equilibrium is also a possible outcome of our framework (configuration 2). However due to the default on payment the Walrasian asking price is higher than in Stahl's framework whereas the Walrasian bidding price is lower. There is undeniably a loss of efficiency: Consumers and producers end up worse off, and the level of transactions at equilibrium is less than that in the absence of default.

More importantly, we have demonstrated that another form of inefficiency arises in our framework (configuration 3), taking the form of a SPNE with demand rationing in situations where the Walrasian outcome would have emerged in the no-default case like Stahl. Default itself is not enough to generate such an outcome. Our result of demand rationing also requires the non-monotonicity of the unit receipt. The monotonicity of ρ (even with $\rho < p_a$ due to default) would not be sufficient to create demand rationing because the asking price generating the highest expected revenue per unit of sale would be p_{max}, and therefore the Walrasian asking price would be always lower that this maximum, ruling out configuration 3.

When applied to banking intermediation, where banks compete for deposits and loan applicants, our framework recasts the seminal analysis of credit rationing by Stiglitz and Weiss (1981) in a game-theoretical framework where double competition is explicitly modelled.

Our framework also provides some interesting insights on the consequences of delayed payments in the white goods and other durable goods markets. It seems that the market is currently in configuration 2, since no demand rationing is observed. However consumers should be aware that they pay a higher price as a result of this new facility. Our framework also predicts that, should the practice of delaying payment intensify, the industry of durable goods for households might well experience some demand rationing in the future, as shown in configuration 3.

[5] It may also occur (but not necessarily) that the sales-revenue-maximising asking price in presence of default is lower than the one with full repayment, i.e., $\widehat{\widehat{p}}_a < \widehat{p}_a$. This second effect, should it happen, would complement the effect due to a higher Walrasian and possibly reverse the ranking.

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7 Appendix

7.1 Appendix A: The monopolist equilibrium in the output subgames

In subgames with an intermediary with all goods from the producers ($x = S(p_b)$), the monopolist chooses p_a in order to maximise the expected sales $\rho(p_a)\min\{D(p_a); S(p_b)\}$. Recall that p_a^{mc} is such that $D(p_a^{mc}) = S(p_b)$, $\widehat{\widehat{p}}_a$ maximises $\rho(p_a)D(p_a)$ whereas $\bar{p_a}$ is the maximum of $\rho(p_a)S(p_b)$ with $\widehat{\widehat{p}}_a < \bar{p_a}$.

For $p_a \leq p_a^{mc}$ the supply is binding and therefore maximizing the sales consists in maximizing $\rho(p_a)$, $S(p_b)$being constant. The argmax is therefore $\min\{p_a^{mc}; \bar{p_a}\}$.

For $p_a \geq p_a^{mc}$ the demand is binding and therefore maximizing the sales involves maximizing $\rho(p_a) D(p_a)$. The argmax is therefore $\max\left\{p_a^{mc}; \widehat{\widehat{p}}_a\right\}$.

The final stage is to find which of the two asking prices defined above generates the highest sales over the whole range of p_a.

- If $p_a^{mc} \leq \widehat{\widehat{p}}_a \leq \overline{p}_a$ then $\min\{p_a^{mc}; \bar{p_a}\} = p_a^{mc}$ and $\max\{p_a^{mc}; \widehat{\widehat{p}}_a\} = \widehat{\widehat{p}}_a$. Given that $[\rho(p_a^{mc})D(p_a^{mc})]' \geq 0$ therefore $\widehat{\widehat{p}}_a$ maximises the sales.

- If $\widehat{\widehat{p}}_a < p_a^{mc} \leq \overline{p}_a$ then $\min\{p_a^{mc}; \bar{p_a}\} = \max\left\{p_a^{mc}; \widehat{\widehat{p}}_a\right\} = p_a^{mc}$. Therefore p_a^{mc} maximises the sales.

- Finally if $\widehat{\widehat{p}}_a \leq \overline{p}_a < p_a^{mc}$ then $\min\{p_a^{mc}; \bar{p_a}\} = \bar{p_a}$and $\max\{p_a^{mc}; \widehat{\widehat{p}}_a\} = p_a^{mc}$. Given that $[\rho(p_a^{mc})S(p_b)]' < 0$ therefore $\overline{p}_a$is the maximum.

7.2 Appendix B: The Nash equilibria in the output subgames

Let us consider any bidding price lower than or equal to $\widetilde{p}_b$.

- We firstly show that the bidding price generating the ranking $\widehat{\widehat{p}}_a < \overline{p}_a < p_a^{mc}$ leads to a NE at $\overline{p}_a$ with demand rationing.

We rule out as equilibria all situations with an excess of inputs ($p_a > p_a^{mc}$), since an intermediary could profitably decrease the asking price: the deviating intermediary would capture all consumers and get sale revenue for some otherwise unused inputs. There is both an increase in ρ (as the asking price is on the negative slope of ρ) and in the quantities sold, for $\min\left\{\frac{S(p_b)}{2}; D(p_a - \varepsilon)\right\} > \frac{D(p_a)}{2}$.

Nor can we accept as equilibria situations in which $\overline{p}_a < p_a \leq p_a^{mc}$. An intermediary could deviate by offering a lower asking price to increase its unit sale revenue ρ while keeping the inputs supply $\frac{S(p_b)}{2}$ as the binding side.

Lastly, we reject as equilibria situations in which $p_a < \overline{p}_a < p_a^{mc}$. An intermediary could deviate by offering a higher asking price to increase its unit sale revenue ρ while keeping the inputs supply $\frac{S(p_b)}{2}$ as the binding side.

From $\overline{p}_a$ characterized by demand rationing, there exists no deviation as ρ is at its maximum.

- Secondly, we show that if p_a^{mc} becomes lower than $\overline{p}_a$ generating the ranking $\widehat{\widehat{p}}_a < p_a^{mc} < \overline{p}_a$ the NE is p_a^{mc}.

As already explained, we can rule out all situations in which there is an excess of supply ($p_a > p_a^{mc}$), since an intermediary could profitably deviate by decreasing the asking price. Although there may be a decrease in the unit sale revenue, the deviating intermediary registers a discrete jump in the quantity sold, which more than offsets the possible infinitesimal decrease in ρ. Indeed, starting from a situation with excess of supply, $\left(\rho\left(p_a - \varepsilon\right)\min\left\{\frac{S(p_b)}{2}; D\left(p_a - \varepsilon\right)\right\} > \rho\left(p_a\right)\frac{D(p_a)}{2}\right.$.

Nor can we accept any situation with demand rationing ($p_a < p_a^{mc}$). An intermediary can offer a higher asking price, which increases its unit sale revenue ρ, and still provides enough residual demand to guarantee that the input quantity is binding: $RD\left(p_a + \varepsilon\right) = D\left(p_a + \varepsilon\right)\left[\frac{D(p_a) - \frac{1}{2}S(p_b)}{D(p_a)}\right]$. The rationing ratio being a positive constant depending only on the initial situation, RD is strictly decreasing in ε. As at the initial p_a, RD is equal to $D\left(p_a\right) - \frac{1}{2}S\left(p_b\right) > \frac{1}{2}S\left(p_b\right)$, there exists an ε such that the input side is still binding for the deviating intermediary.

From p_a^{mc} it is impossible to deviate through a decrease in the asking price: ρ would decrease and the quantities could not increase since the amount of inputs is binding.

We now analyze whether p_a^{mc} is robust to a deviation through an increase in the asking price. Let us define the residual demand that an intermediary would face through an increase in the asking price. Given that the other intermediary posts the market-clearing asking price, the residual demand that the deviating intermediary (with the less attractive asking price) faces is exactly the aggregate demand for the good (at this less attractive price) divided by 2. Indeed, $RD = D\left(p_a^{mc} + \varepsilon\right)\frac{D(p_a^{mc}) - \frac{1}{2}S(p_b)}{D(p_a^{mc})}$. As $D\left(p_a^{mc}\right)$ is equal to $S\left(p_b\right)$, this residual demand is exactly equal to $\frac{1}{2}D\left(p_a^{mc} + \varepsilon\right)$ and remains binding. It is important to ask at this point if $\rho\left(p_a^{mc} + \varepsilon\right)\frac{1}{2}D\left(p_a^{mc} + \varepsilon\right)$ is higher or lower than $\rho\left(p_a^{mc}\right)\frac{1}{2}D\left(p_a^{mc}\right)$.[6] As p_a^{mc} is higher than $\widehat{\widehat{p}}_a$, the deviating intermediary's sales revenue is unambiguously decreased, so

[6] We can use the configuration of sale revenue (computed with total demand) which has the same shape as the sales revenue of an individual intermediary (after sharing the total demand among 2 intermediaries) in order to conclude as to whether the sales receipts of the deviating intermediary are increased or decreased.

that the deviation is not profitable. The market-clearing price is then the NE of the subgame.

- Thirdly for p_a^{mc} lower than $\widehat{\widehat{p}}_a$, we get the ranking $p_a^{mc} < \widehat{\widehat{p}}_a < \bar{p_a}$. There is no NE in the subgame.

Any excess of supply or demand can be ruled out for the reasons explained previously in the second point. From p_a^{mc}, sales revenue increases through an increase in the asking price: indeed as $p_a^{mc} < \widehat{\widehat{p}}_a$, $\rho\left(p_a^{mc} + \varepsilon\right) \frac{1}{2} D\left(p_a^{mc} + \varepsilon\right) > \rho\left(p_a^{mc}\right) \frac{1}{2} D\left(p_a^{mc}\right)$. See previous point for the equivalence between residual demand and half the demand at the new price.

8 Figures

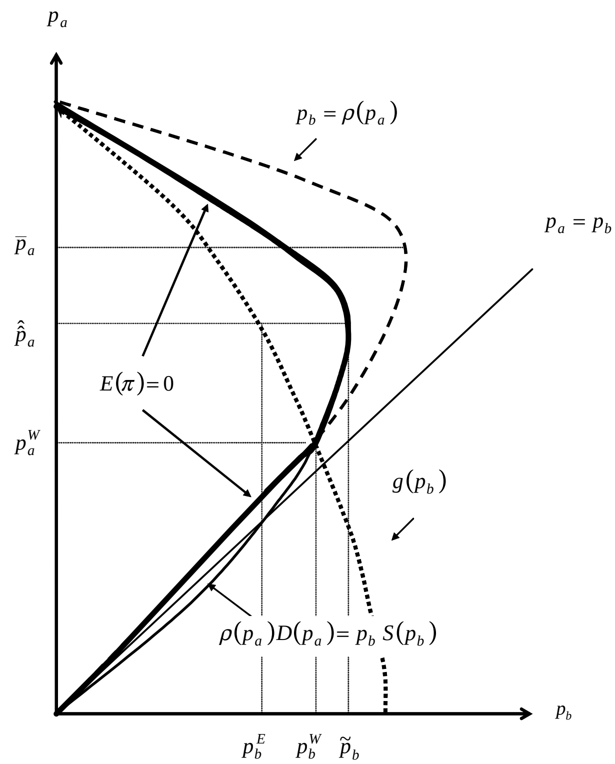

Figure 1: Configuration 1 where $p_a^W < \widehat{\widehat{p}}_a < \overline{p}_a$ displaying absence of equilibrium.

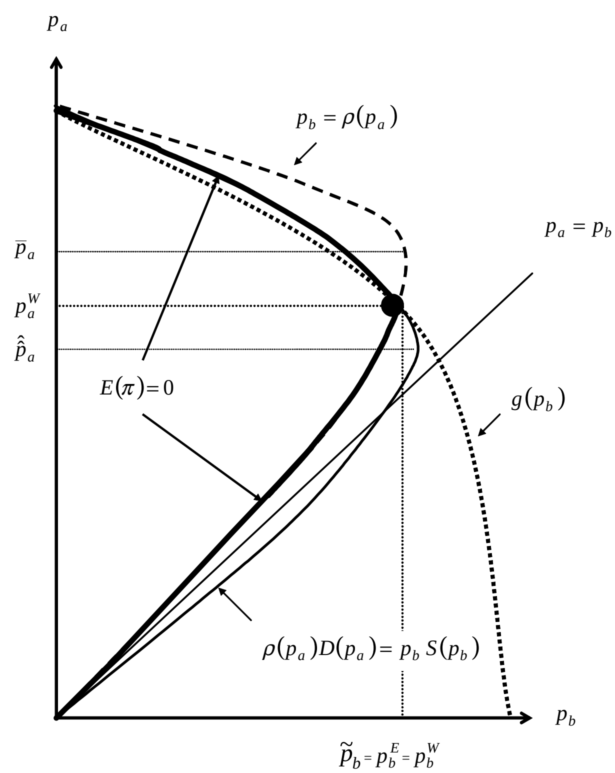

Figure 2: Configuration 2 where $\widehat{\widehat{p}}_a < p_a^W < \overline{p}_a$ displaying Walrasian equilibrium.

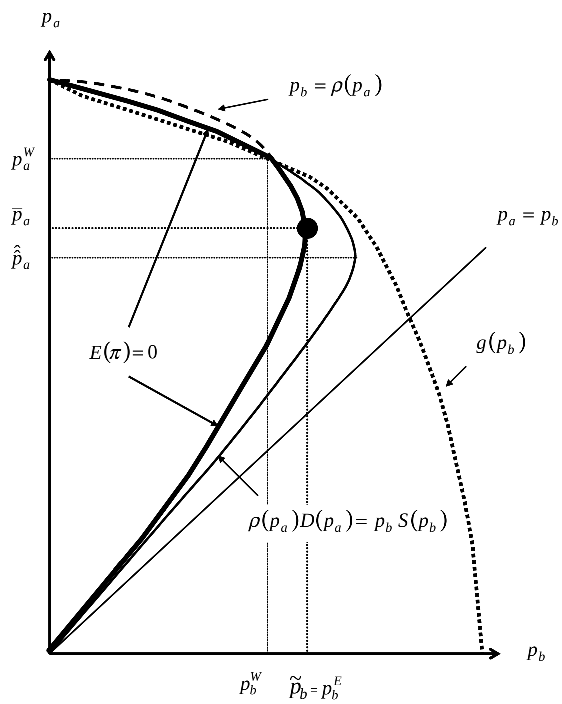

Figure 3: Configuration 3 where $\widehat{\widehat{p}}_a < \overline{p}_a < p_a^W$ displaying demand rationing.