E C O N O M I C S   B U L L E T I N

Indeterminacy in a variety expansion model of endogenous growth

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Abstract

In this paper, I demonstrate that an expanding variety model of endogenous growth with temporary monopoly power exhibits the indeterminacy of equilibrium paths. This implies the existence of a global range of initial growth rates for any initial level of the state variable.

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1 Introduction

Many studies, beginning with Benhabib and Farmer (1994) and Boldrin and Rustichini (1994), have shown that a large class of endogenous growth models can exhibit the indeterminacy of equilibrium paths near a balanced growth path (BGP).

In this study, I construct a variety expansion model of endogenous growth in which the indeterminacy of equilibrium paths can emerge. The present model differs from that of Grossman and Helpman (1991, Ch. 3.2), who assume permanent monopoly power, only in that the innovators of new varieties enjoy a one-period monopoly. This paper shows that a minor extension to the Grossman–Helpman model implies a substantial change in the dynamic behavior of the economy.

In "standard" variety-based models with knowledge externalities, [1] one-dimensional dynamic systems have a unique and unstable steady state, which implies that the economy always follows the BGP. In the model presented below, the system is globally stable and there are many paths that are consistent with the transversality condition.

Related papers by Deneckere and Judd (1992), Gale (1996), and Matsuyama (1999, 2001) develop innovation cycle models with temporary monopoly power. However, these models do not exhibit the indeterminacy of equilibrium paths, and, thus, cannot explain sunspot phenomena, which are driven by changes in expectations.

The outline of the paper is as follows. The following section presents the basic model and the main result is described in Section 3. Section 4 compares the current model with existing ones, and Section 5 concludes the paper.

2 The Model

Time is discrete and extends to infinity. This model differs from that of Grossman and Helpman (1991, Ch. 3.2) only in the temporary nature of the innovator's monopoly. In the economy, households supply N units of labor inelastically and consume the final good, which is taken as the numeraire. The maximization problem can be written as

$$\max_{\{C_t\}_{t=0}^{\infty},\{A_t\}_{t=1}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \ln C_t \ ,$$
$$\text{subject to} \quad A_{t+1} + C_t = (1+r_t)A_t + w_t N \ ,$$

where $\beta \in (0,1)$ denotes the rate of time preference, A_t denotes the household's stock of assets from $t-1$ to t, which comprises shares of R&D firms, r_t denotes the interest factor from $t-1$ to t, and w_t denotes wage income. The Euler equation and

[1] See Romer (1990), Grossman and Helpman (1991), and Rivera-Batiz and Romer (1991).

the transversality condition characterize the solution to this maximization problem as follows:

$$\frac{C_{t+1}}{C_t} = \beta(1 + r_{t+1}), \forall t \geq 0 \ , \tag{1}$$

$$\lim_{T\to\infty} \beta^T \frac{A_{T+1}}{C_T} = 0 \ . \tag{2}$$

The production function in the final-goods sector is

$$C_t = \left[\int_0^{n_t} x_t(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}} \ ,$$

where $x_t(j)$ denotes the amount of variety i used in the final-goods sector, and the elasticity of substitution between each pair of inputs is equal to $\sigma > 1$. At any period, only a subset of differentiated inputs, $[0, n_t]$ is available to the economy. It follows that, in each sector, total factor productivity increases with the range, n. Cost minimization implies the following demand function:

$$x_t(j) = \frac{C_t}{p_t^{\sigma} \int_0^{n_t} p_t(j)^{1-\sigma} dj} \ ,$$

where $p_t(j)$ is the price of the intermediate good indexed by j. Note that the price elasticity is σ for all j. Profit maximization ensures that the marginal cost must equal unity, which implies the following condition:

$$w_t = [n_{t-1}(1 + \hat{\sigma} g_t)]^{\frac{1}{\sigma-1}} \ . \tag{3}$$

This equation is related to the determination of the initial growth rate, as explained in the next section.

The demand side of the model and the structure of final production, as described above, are identical to those of Grossman and Helpman (1991, Ch. 3.2). There is only one departure from the Grossman and Helpman model. This is related to the structure of the innovation sector. I introduce temporary monopoly power into the model, whereas Grossman and Helpman assume permanent monopoly power. Due to the temporary nature of the monopoly, the "old" intermediates, $[0, n_{t-1}]$ are supplied *competitively*, whereas the "new" intermediates, $[n_{t-1}, n_t]$, are supplied *monopolistically*. Assuming that producing a unit of intermediate goods requires one unit of labor, the price of old intermediates is $p_t = w_t$, $\forall j \in [0, n_{t-1}]$ and the new intermediates are priced at $\frac{\sigma w_t}{\sigma - 1}$. Thus, denoting the growth rate of varieties by $g_t \equiv (n_t - n_{t-1})/n_{t-1}$, the demand function for machine j is

$$x_t(j) = \begin{cases} x_t^c = \frac{C_t}{w_t n_{t-1}(1+\hat{\sigma} g_t)} & , \text{if } \ j \in [0, n_{t-1}] \\ x_t^m = \frac{\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} C_t}{w_t n_{t-1}(1+\hat{\sigma} g_t)} & , \text{if } \ j \in [n_{t-1}, n_t] \end{cases} \ , \tag{4}$$

where $\hat{\sigma} \equiv \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}$. This equation implies the profit function

$$\pi_t = \frac{w_t x_t^m}{\sigma - 1} = \frac{(\sigma-1)^{\sigma-1} C_t}{\sigma^\sigma n_{t-1}(1+\hat{\sigma} g_t)} \,. \tag{5}$$

In summary, the present model is almost identical to existing models in the literature. However, unlike in Grossman and Helpman (1991, Ch. 3.2), the innovator has only temporary monopoly power. Unlike in Matsuyama (1999), the savings rate is endogenously determined by the intertemporal optimization of an infinitely lived agent. Again, unlike in Matsuyama (2001), there is no capital accumulation, and the constant elasticity of substitution (CES) specification is assumed for final production.

3 Equilibrium

The previous section described the basic framework. In this section, I close the model by using two equilibrium conditions.

Suppose that innovating new varieties at period t incurs start-up costs of a^{RD}/n_{t-1} units of labor per variety *in the previous period*.[2] Using (1) and (5), the free-entry condition, $\pi_t/(1+r_t) \leq a^{RD} w_{t-1}/n_{t-1}$, and the labor-market clearing condition ensures that

$$\frac{C_t}{w_t} \begin{cases} = \frac{a^{RD}\sigma^\sigma}{\beta(\sigma-1)^{\sigma-1}}(1+\hat{\sigma} g_{t+1}) & \text{if } g_{t+1} > 0 \\ \leq \frac{a^{RD}\sigma^\sigma}{\beta(\sigma-1)^{\sigma-1}} & \text{if } g_{t+1} = 0 \end{cases} , \tag{6}$$

$$N = (n_t - n_{t-1}) x_t^m + n_{t-1} x_t^c + a^{RD} g_{t+1} \,. \tag{7}$$

Then, from (4), (5), (6), and (7), the following law of motion can be derived:

$$g_{t+1} = \begin{cases} \frac{N - \eta f(g_t)}{a^{RD} + \hat{\sigma}\eta f(g_t)} & \text{if } N \geq \eta \\ 0 & \text{if } N < \eta \end{cases} , \tag{8}$$

where

$$f(g) = \frac{1 + \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} g_t}{1 + \hat{\sigma} g_t} \,,$$

$$\eta = \frac{a^{RD}\sigma^\sigma}{\beta(\sigma-1)^{\sigma-1}} \,,$$

and $f' < 0$ and $f'' > 0$ hold. Thus, the following equation is derived:

$$g'_{t+1} = \max\left[\frac{-\eta(a^{RD} + \hat{\sigma}N) f'(g_t)}{(a^{RD} + \hat{\sigma}\eta f(g_t))^2},\, 0\right] \geq 0 \,,$$

$$g''_{t+1} \leq 0 \,.$$

[2] Note that I adopt the *knowledge-driven specification* of Rivera-Batiz and Romer (1991).

As shown in figure 1, the one-dimensional system has a unique and globally stable steady state.[3] In (3), g_0 is not given; only n_{-1} is given. Hence, for any n_{-1}, many equilibrium paths can satisfy both the law of motion (8) and the transversality condition (2).[4] Hence, the initial growth rates are globally indeterminate. The following proposition summarizes the preceding argument.

Proposition 1 *A variety expansion model of endogenous growth that incorporates temporary monopoly profits for innovators exhibits the indeterminacy of initial growth rates. This implies that, for any initial values of the state variable, n_{-1}, a global range of initial growth rates are consistent with both the law of motion and the transversality condition.*

Along with figure 1, this proposition states that the indeterminacy of initial growth rates also arises in the local dynamics. It is worth pointing out that local indeterminacy implies that sunspot phenomena can emerge in the neighborhood of a BGP. Thus, the economy can fluctuate in the neighborhood of the steady state.[5]

The model described in this paper is almost identical to those of Grossman and Helpman (1991, Ch. 3.2) and Matsuyama (1999, 2001). A minor change made to these models substantially changes the dynamic behavior of the economy. In Grossman and Helpman (1991), as is well known, the economy follows a unique BGP after starting from any initial number of varieties, and there are no transitional dynamics. In Matsuyama (1999, 2001), the equilibrium path can be locally determined.[6] Unlike existing growth models with a similar structure, the model of this paper exhibits local indeterminacy.

4 Temporary versus Permanent Monopoly

In this section, I compare my model with that of Grossman and Helpman (1991, Ch. 3.2) to explain why incorporating temporary, rather than permanent, monopoly has such a pronounced effect on the dynamic behavior of the economy.

Innovators finance the start-up costs in the stock market and their profits are paid out to shareholders as dividends. I introduce a new variable, V_t^{GH}, which denotes the stock market price of the innovator who invents a new variety in period t, in the

[3] In this context, I assume that $N > \eta$. This ensures that the economy inherits a sufficiently large labor supply (or sufficiently small start-up costs of R&D) to enable it to grow: that is, $g_{+1} > 0$ for any $g_t \geq 0$.

[4] It is easy to verify, by using $A_{T+1} = (n_{T+1} - n_T)\pi_T$, that the transversality condition (2) is always satisfied.

[5] See Benhabib and Farmer (1999) for details.

[6] Matsuyama (2001) does not rule out the possibility of global indeterminacy.

context of the model of Grossman and Helpman (1991, Ch. 3.2). Because the patent is protected forever in the Grossman and Helpman model, the price is

$$V_t^{GH} = \frac{\pi_t}{1+r_t} + \frac{\pi_{t+1}}{(1+r_t)(1+r_{t+1})} + \cdots \;, \tag{9}$$

which is the present discounted value of its profit stream from period t onwards. Denoting by V_t the market value of the innovator in the model yields the following expression:

$$V_t = \frac{\pi_t}{1+r_t} \;. \tag{10}$$

Equation (9) and (10) can be rewritten as

$$r_t V_t^{GH} = \pi_t + (V_{t+1}^{GH} - V_t^{GH}) \;,$$
$$r_t V_t = \pi_t - V_t \;.$$

Clearly, while shareholders in the Grossman and Helpman model enjoy not only dividends, but also experience capital gains (or losses), $V_{t+1}^{GH} - V_t^{GH}$, those in the current model enjoy only one-period dividends. In other words, innovators' stock market values are positive for only one period. Thus, the Grossman and Helpman model predicts potential price bubbles. This implies that the dynamic system can be unstable: the price of stocks continues to change when the initial price level is off the BGP. Therefore, in the Grossman and Helpman model, the transversality condition uniquely determines the dynamic paths that are on the BGP by excluding bubble solutions.

However, the dynamic system of this model is not determined by the stock market condition, but by the labor market condition (7). In addition, because positive stock market values for innovators are eliminated after one period, shareholders do not experience capital gains (or losses). Hence, it may seem that this model does not allow the possibility of price bubbles. This implies the global stability of the dynamic equation (8), which generates the indeterminacy of equilibrium dynamic paths.

5 Conclusion

In this paper, I have shown that a minor extension to the variety expansion model of Grossman and Helpman dramatically changes the predicted dynamic behavior of the economy. More precisely, I made the additional assumption that innovators' profits are temporary. That is, an innovator who invents a new intermediate good enjoys monopoly profits only temporarily. The one-dimensional dynamic system derived from the model of this paper has a unique steady state, which is globally stable. This contrasts with similar existing models. For any initial value of a state variable, many initial growth rates are consistent with both the law of motion and the transversality condition, Hence, there is indeterminacy in this sense.

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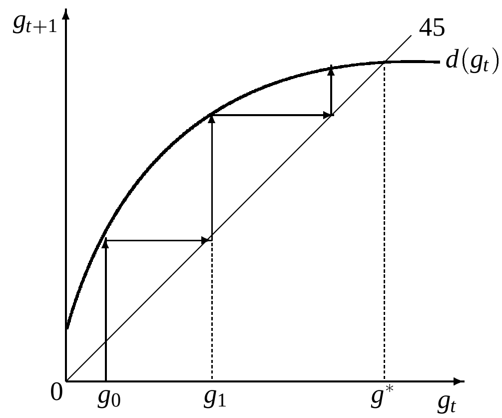

Figure 1: A unique and globally stable steady state