E C O N O M I C S   B U L L E T I N

Capital depreciation and the investment-uncertainty relationship: The role of symmetric adjustment costs

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Abstract

Some recent contributions (Femminis, 2007; Nakamura, 2002) demonstrate the important roles of capital depreciation in the investment-uncertainty relationship. This paper highlights the role of symmetric adjustment costs in the case that capital depreciates completely after finite periods. Various uncertainty-investment relationships emerge depending on the curvature of convex adjustment cost function. While investment decreases with uncertainty if the curvature is small, it increases with uncertainty if the curvature is large. If the curvature is in between, the investment-uncertainty relationship shows an inverted U-shaped curve: investment first increases and then decreases with uncertainty.

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1. Introduction

Recent contributions by Femminis (2007) and Nakamura (2002) demonstrate that the assumptions on capital depreciation often play a key role in determining the investment-uncertainty relationship. Saltari and Ticchi (2005) examine the investment decision of a risk-averse firm to show that the relationship is negative only when the coefficient of relative risk-aversion is lower than unity but larger than the labor share of income. As Femminis correctly points out, however, their result depends crucially on the assumption that capital depreciates fully after production, or its lifetime is just one period, which is essential to derive a closed form solution. Assuming that capital depreciates at a constant geometric rate, he shows that the relationship can be negative even when the risk-aversion coefficient is larger than unity.

On the contrary, Nakamura investigates the investment behavior of a risk-neutral firm under a different assumption of capital depreciation. As a result, it is shown that, if useful lifetime capital is shorter than the firm's horizon, or capital depreciates fully after some production periods, the relationship between current investment and future price uncertainties can be negative even without irreversibility of investment: the usual explanation of such a negative effect.

To derive clear the result, no costs are assumed to adjust the capital stock in the models of Saltari and Ticchi and of Femminis although the adjustment costs are commonly employed in the investment literature. Also, the adjustment costs play no other role than determining the level of investment in Nakamura's model. If the adjustment costs do not change the qualitative results, one can ignore them. However, if they play a key role in determining the sign of investment-uncertainty relationship, we should take them into account.

The purpose of this paper is to highlight the role of adjustment costs in determining the effect of uncertainty on investment. For this, we reexamine the results of Nakamura paying attention to the curvature of symmetric adjustment costs of investment. If marginal products of capital are constant regardless of capital input, the result of Hartman (1972) and of Abel (1983), i.e. the negative investment-uncertainty relationship appears. If they are decreasing with capital input, on the contrary, various relationships between investment and uncertainty emerge even without irreversibility or asymmetric adjustment costs. More precisely, with symmetric and convex adjustment costs, the relationship depends crucially on the curvature of adjustment cost function. It is negative if the curvature is small while the relationship is positive if it is large. If the curvature is in between, the relationship is inverted U-shaped: positive at low values of uncertainty whereas negative at high levels of uncertainty.[1]

2. The analytical framework

We utilize the simple setup in Caballero (1991), with two differences: (i) lifetimes of capital are finite, and (ii) adjustment costs associated with investment are symmetric. Consider a firm facing an isoelastic demand function, $P_t = Q_t^{(1-\psi)/\psi} Z_t$, where $\psi (\psi \geq 1)$ is a markup coefficient that takes unity under perfectly elastic demand, P_t and Q_t are respectively the price and output of the good, and Z_t is a stochastic term described by a lognormal random-walk process:

[1] Sarkar (2000) also shows an inverted U-shaped investment-uncertainty relationship using a real option model.

$$Z_t = Z_{t-1} \exp \varepsilon_t , \quad (1)$$

where $\varepsilon_t \sim N(-\sigma^2/2, \sigma^2)$ with $\sigma \geq 0$.

Technology is described by a Cobb-Douglas production function, $Q_t = (AL_t^{\alpha} K_t^{(1-\alpha)})^{\gamma}$, with A a scale parameter, L_t labor, K_t capital, α the labor share, and $\gamma(\gamma \leq 1)$ a return-to-scale parameter. Assuming that labor is a variable input, the profit function, $\Pi(K_t, Z_t)$, becomes

$$\Pi(K_t, Z_t) = hZ_t^{\eta} K_t^{\mu} , \quad (2)$$

where $h \equiv (1-\alpha\gamma/\psi) A^{(\gamma/\psi)/(1-\alpha\gamma/\psi)} (\frac{\alpha\gamma}{\psi w})^{(\alpha\gamma/\psi)/(1-\alpha\gamma/\psi)}$, $\eta \equiv \frac{1}{1-\alpha\gamma/\psi} > 1$, and $\mu \equiv \frac{(1-\alpha)\gamma/\psi}{1-\alpha\gamma/\psi} \leq 1$.

Although most investment models assume that capital depreciates at a constant geometric rate, we assume that capital becomes fully obsolete after a finite lifetime. To simplify the analysis, we assume that the capital equipment can be in operation for only two periods, and hence the capital stock at period t is

$$K_t = I_t + I_{t-1} . \quad (3)$$

Evidently, I_{t-1} and I_{t+1} do not exist together at the same time, but have some linkage via I_t because $K_t = I_t + I_{t-1}$ and $K_{t+1} = I_{t+1} + I_t$. This overlapping nature of capital goods yields the relationship between current investment and future uncertainties.

We will use the minimum framework to demonstrate the main issue. The firm is in place for three periods and does not invest in the last period. Assuming that it does not discount the future profits, the firm's maximization problem becomes[2]

$$\max_{I_1, I_2} E[hZ_1^{\eta} K_1^{\mu} - I_1 - C(I_1) + hZ_t^{\eta} K_2^{\mu} - I_2 - C(I_2) + hZ_3^{\eta} K_3^{\mu}] ,$$

$$\text{subject to } K_1 = I_1 + I_0 ,\ K_2 = I_2 + I_1 ,\ K_3 = I_2 ,$$

where I_0 is given initial capital, $C(I)$ is an adjustment cost function of investment, and the price of capital is set equal to unity. We assume that the adjustment costs are not asymmetric but *symmetric*, which refers to the case in which the firm can adjust capital upward and downward at the same cost. Let us specify the adjustment cost function as follows:

$$C(I) = \gamma_1 I^{\beta} , \quad (4)$$

where $\beta \geq 1$ and γ_1 is a nonnegative parameter.

The first-order necessary conditions for a regular maximum are

$$\mu h Z_1^{\eta} K_1^{\mu-1} + \mu h E[Z_2^{\eta}] K_2^{\mu-1} = 1 + \gamma_1 \beta I_1^{\beta-1} , \quad (5a)$$

$$\mu h E[Z_2^{\eta}] K_2^{\mu-1} + \mu h E[Z_3^{\eta}] K_3^{\mu-1} = 1 + \gamma_1 \beta I_2^{\beta-1} . \quad (5b)$$

The interpretation of these conditions is straightforward. At the optimum, the sum of the expected marginal revenues of investment, the LHS, must be equal to its marginal cost, the RHS, for each equation.

[2] The main results will not change if we introduce constant discount rates.

3. Perfect Competition and Constant Returns

For now, it is assumed that the technology exhibits homogeneity of degree one with respect to capital and labor ($\gamma = 1$) and that demand is perfectly elastic ($\psi = 1$). Hence, the profit function is linear with respect to capital ($\mu = 1$). Taking the fact into account that, if $\ln x$ is normally distributed, then $E[x] = \exp\{E[\ln x] + \frac{1}{2}\mathrm{var}[\ln x]\}$, the first-order conditions become

$$hZ_1^{\eta}[1 + f_2(\sigma)] = 1 + \gamma_1 \beta I_1^{\beta-1}, \tag{6a}$$

$$hZ_1^{\eta}[f_2(\sigma) + f_3(\sigma)] = 1 + \gamma_1 \beta I_1^{\beta-1}, \tag{6b}$$

where $f_2(\sigma) = \exp\{\eta(\eta-1)\sigma^2/2\}$ and $f_3(\sigma) = \exp\{\eta(\eta-1)\sigma^2\}$. Here, we assume that $\beta > 1$ and $\gamma_1 > 0$ for the second-order conditions to be satisfied.

In this case, as Caballero emphasizes, investment at each period does not depend on either past or future capital stocks, and this lack of "intertemporal linkage" does not depend on the firm's horizon. From (6a), we have the positive investment-uncertainty relationship, or

$$\frac{dI_1}{d\sigma} = \frac{hZ_1^{\eta} f_2'(\sigma)}{\gamma_1 \beta(\beta-1) I_1^{\beta-2}} > 0. \tag{7}$$

This is consistent with Hartman (1972) and Abel's (1983) conclusion for infinitely durable capital. Complete depreciation of capital has nothing to do with the sign of investment-uncertainty relationship under the assumption of perfect competition and constant returns to scale.

4. Imperfect Competition or Decreasing Returns

When competition is imperfect ($\psi > 1$), or when technology exhibits decreasing returns to scale ($\gamma < 1$), the profit function is a decreasing function of capital ($\mu < 1$). Therefore, investment at each period depends on both past and future capital stocks:

$$\mu hZ_1^{\eta}(I_0 + I_1)^{\mu-1} + \mu hZ_1^{\eta} f_2(\sigma)(I_1 + I_2)^{\mu-1} = 1 + \gamma_1 \beta I_1^{\beta-1}, \tag{8a}$$

$$\mu hZ_1^{\eta} f_2(\sigma)(I_1 + I_2)^{\mu-1} + \mu hZ_1^{\eta} f_3(\sigma) I_2^{\mu-1} = 1 + \gamma_1 \beta I_2^{\beta-1}. \tag{8b}$$

From the above two equation, we have[3]

$$sign[dI_t/d\sigma] = sign[HA(I_1, I_2, \sigma) + SUB(I_1, I_2, \sigma) + AC(I_1, I_2, \sigma)], \tag{9}$$

where $HA(I_1, I_2, \sigma) = \bar{Z} f_2'(\sigma) f_3(\sigma)(I_1 + I_2) I_2^{\mu-2} > 0$,

$SUB(I_1, I_2, \sigma) = -\bar{Z} f_2(\sigma) f_3'(\sigma) I_2^{\mu-1} < 0$,

$AC(I_1, I_2, \sigma) = \bar{Z} f_2'(\sigma)(I_1 + I_2)\gamma_1 \beta(\beta-1) I_2^{\mu-2} > 0$, and $\bar{Z} = -\mu(\mu-1)hZ_1^{\eta} > 0$.

The above equation shows that the total effect of uncertainty on investment is the sum of the Hartman-Abel effect, $HA(I_1, I_2, \sigma)$, the substitution effect, $SUB(I_1, I_2, \sigma)$, and the adjustment cost effect, $AC(I_1, I_2, \sigma)$. Figure 1 also demonstrates this decomposition. An increased uncertainty increases the expected marginal revenue product of K_2 (Jensen's inequality argument) and hence increases the desired capital stock, thereby augmenting I_1. This is the Hartman-Abel effect.

[Insert Figure 1 around here]

[3] See Appendix-A for the derivation.

At the same time, an increased uncertainty increases I_2. Since the shock to demand follows a random walk (in logs) in our specification of stochastic process, which is quite common in the literature, uncertainty at the third period is higher than at the second. A rise in σ increases the marginal revenue product of K_3 more than that of K_2, thereby increasing K_3 more than K_2. Since $K_2 = I_2 + I_1$ and $K_3 = I_2$, the firm increases I_2 more than I_1. Under decreasing marginal returns to capital, an increase in I_2 decreases the marginal product of I_1 at the second period, $\mu(I_1 + I_2)^{\mu-1}$, since I_1 and I_2 are *substitute* ($\mu < 1$). Therefore, an increase in σ has a negative impact on I_1 through increasing I_2. The substitution effect captures this channel.

Without adjustment costs, the cost for increasing I_1 is equal to that for increasing I_2 regardless of the sizes of I_1 and I_2. With convex adjustment costs, however, the marginal cost of I_2 is larger than that of I_1 if $I_2 > I_1$. In general, with convex adjustment costs, firms try to distribute investments equally over the planning period to minimize the associated costs. This intertemporal smoothing effect of convex adjustment costs narrows the difference between an increase in I_2 and an increase in I_1.[4] The smaller is an increase in I_2, the smaller is the aforementioned substitution effect. This partial offset of the substitution effect via suppressing an increase in I_2 is the adjustment cost effect. Importantly, without the adjustment cost effect, the substitution effect surely dominates the Hartman-Abel effect, and hence we have a negative relationship between investment and uncertainty for I_1.[5]

PROPOSITION: ***If there is no increasing and convex adjustment cost associated with investment, an increased uncertainty reduces investment at the initial period***.

PROOF: See Appendix-B.

With large convex adjustment costs, however, the sum of the adjustment cost and Hartman-Abel effects can dominate the substitution effect so that the sign of the investment-uncertainty relationship becomes positive. Even though it is the fact with a small σ, the sign can be negative with a large σ. Since I_2 increases more rapidly than I_1 with σ, the substitution effect also increases sharply with σ. Hence, with a large σ, the substitution effect may dominate the sum of the other two effects so that the total effect becomes negative. Since the three effects depend crucially on σ, the sign of the investment-uncertainty relationship cannot be determined unambiguously even in our simple setup. We therefore have to use numerical results to illustrate the investment-uncertainty relationship.

[4] Although we do not consider it explicitly in this paper, discount factors also have an “anti-smoothing” effect.

[5] Nakamura (2002) derives the same result under the assumption of identically and independently distributed price shocks.

5. Numerical Analysis

Throughout the numerical analysis, we assume that $\eta = 2$ and $\mu = 0.5$,[6] and that, without loss of generality, $\mu h Z_1^{\eta} = 1$. To confirm the aforementioned proposition, we start with the case without adjustment costs: $\gamma_1 = 0$. As Figure 2(A) shows, investment decreases with uncertainty. When adjustment costs increase rapidly with investment, on the contrary, investment is expected to increases with uncertainty. From Figure 2(B), where $\gamma_1 = 0.05$ and $\beta = 2$, we can verify the finding.

[Insert Figures 2(A), (B), and (C) around here]

Figure 2(C) shows the investment-uncertainty relationship when adjustment costs increase with investment but not rapidly: $\gamma_1 = 0.005$ and $\beta = 2$. In this case, investment increases with σ initially. However, after a certain point (around $\sigma = 0.25$) it decreases with σ. These observations are consistent with our analysis.

6. Conclusion

This paper has investigated the role of symmetric adjustment costs of investment under the assumption that capital depreciates completely after finite periods. Suppose that marginal returns to capital are constant. Then the conclusion of Hartman (1972) and of Abel (1983) holds that greater future uncertainty increases current investment regardless of the curvature of adjustment cost function.

In contrast, when marginal returns to capital are decreasing, the investment-uncertainty relationship depends crucially on the curvature of adjustment costs. When there are no convex adjustment costs, higher future uncertainty lowers current investment, even without irreversibility of investment. With a high curvature, on the contrary, the conclusion of Hartman and of Abel holds. In addition, if the curvature is not high, current investment is initially increasing and then decreasing with uncertainty.

In order to demonstrate the role of adjustment costs in determining the investment-uncertainty relationship, the simple assumptions are employed that capital lives two periods, and that the firm's horizon is three periods. Needless to say, it is very important to investigate what happens to the investment-uncertainty relationship under more realistic assumptions: longer life of capital and a more distant horizon of the firm. Also, the lifetime of capital should be endogenously determined by the firm. Mauer and Ott (1995) have taken an important step along this line.

[6] For example, $\alpha = 2/3$, $\gamma = 1$ and $\psi = 4/3$ in the imperfect competition case, or $\alpha = 2/3$, $\gamma = 3/4$ and $\psi = 1$ in the decreasing returns to scale case.

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Appendices

Appendix-A: Derivation of (9):

Let us totally differentiate (8a) and (8b), and express them in a matrix form:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} dI_1 \\ dI_2 \end{bmatrix} = -\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} d\sigma ,$$

where $a_{11} = -\overline{Z}\{(I_0 + I_1)^{\mu-2} + f_2(\sigma)(I_1 + I_2)^{\mu-2}\} - \gamma_1 \beta(\beta - 1) I_1^{\beta-2}$,

$a_{12} = a_{21} = -\overline{Z} f_2(\sigma)(I_1 + I_2)^{\mu-2}$,

$a_{11} = -\overline{Z}\{ f_2(\sigma)(I_1 + I_2)^{\mu-2} + f_3(\sigma) I_2^{\mu-2}\} - \gamma_1 \beta(\beta - 1) I_2^{\beta-2}$,

$b_1 = \mu h Z_1^{\eta} f_1'(\sigma)(I_1 + I_2)^{\mu-1}$, $b_2 = \mu h Z_1^{\eta}\{ f_2'(\sigma)(I_1 + I_2)^{\mu-1} + f_3'(\sigma) I_2^{\mu-1}\}$,

and $\overline{Z} = -\mu(\mu - 1) h Z_1^{\eta} > 0$.

The second-order conditions are $a_{11} < 0$, $a_{22} < 0$ and $\Delta = a_{11}a_{22} - a_{12}{}^2 > 0$.

Using Cramer's rule, we obtain:

$$\frac{dI_1}{d\sigma} = \frac{\mu h Z_1^{\eta}(I_1 + I_2)^{\mu-2}}{\Delta}\{HA(I_1, I_2, \sigma) + SUB(I_1, I_2, \sigma) + AC(I_1, I_2, \sigma)\} .$$

Since $\mu h Z_1^{\eta}(I_1 + I_2)^{\mu-2} / \Delta$ is positive, we have (9).

Appendix-B: Proof of Proposition

By definition,

$$HA(I_0, I_1, \sigma) + SUB(I_0, I_1, \sigma) = \bar{Z} I_2^{\mu-2}\{ f_2'(\sigma) f_3(\sigma)(I_1 + I_2) - f_2(\sigma) f_3'(\sigma) I_2 \} .$$

Since $f_3(\sigma) = f_2(\sigma)^2$, $f_2'(\sigma) = \eta(\eta-1)\sigma f_2(\sigma)$, and $f_3'(\sigma) = 2\eta(\eta-1)\sigma f_2(\sigma)^2$,

$$HA(I_0, I_1, \sigma) + SUB(I_0, I_1, \sigma) = \bar{Z} I_2^{\mu-2} \eta(\eta-1)\sigma f_2(\sigma)^3 (I_1 - I_2) .$$

Therefore,

$$sign[HA(I_0, I_1, \sigma) + SUB(I_0, I_1, \sigma)] = sign[I_1 - I_2] .$$

Subtracting (8b) from (8a), we have

$$I_1^{\beta-1} - I_2^{\beta-1} = (\mu h Z_1^{\eta} / \gamma_1 \beta)\{(I_0 + I_1)^{\mu-1} - f_3(\sigma) I_2^{\mu-1}\} ,$$

where I_0 is a nonnegative constant and $\mu h Z_1^{\eta} / \gamma\beta > 0$. Therefore,

$$sign[I_1^{\beta-1} - I_2^{\beta-1}] = sign[(I_0 + I_1)^{\mu-1} - f_3(\sigma) I_2^{\mu-1}] .$$

Suppose that $I_1 \geq I_2$. Then, $I_1^{\beta-1} - I_2^{\beta-1} \geq 0$, and $(I_0 + I_1)^{\mu-1} - f_3(\sigma) I_2^{\mu-1} < 0$ since $f_3(\sigma) > 1$ when $\sigma > 0$. These two inequalities contradicts the above equation. Instead, suppose that $I_1 < I_2$ and therefore $I_1^{\beta-1} - I_2^{\beta-1} < 0$. Then $(I_0 + I_1)^{\mu-1} - f_3(\sigma) I_2^{\mu-1}$ can be either positive or negative. Hence, $I_1 < I_2$ must hold as long as $\sigma > 0$. Hence, we have $HA(I_1, I_2, \sigma) + SUB(I_1, I_2, \sigma) < 0$ when $\sigma > 0$.

Figure 1: The Decomposition of the Effect of Uncertainty on Investment

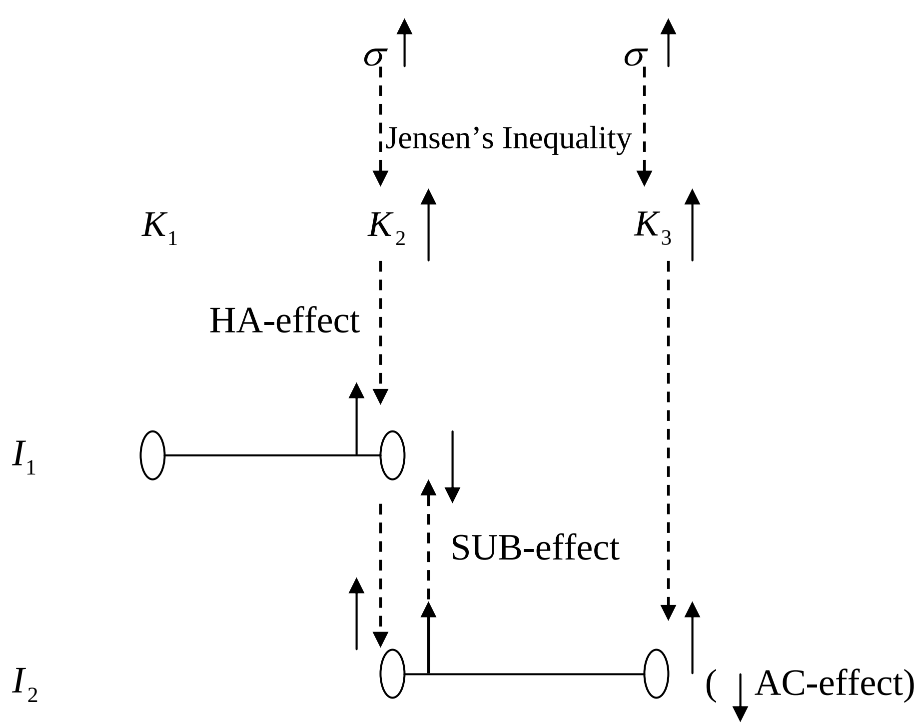
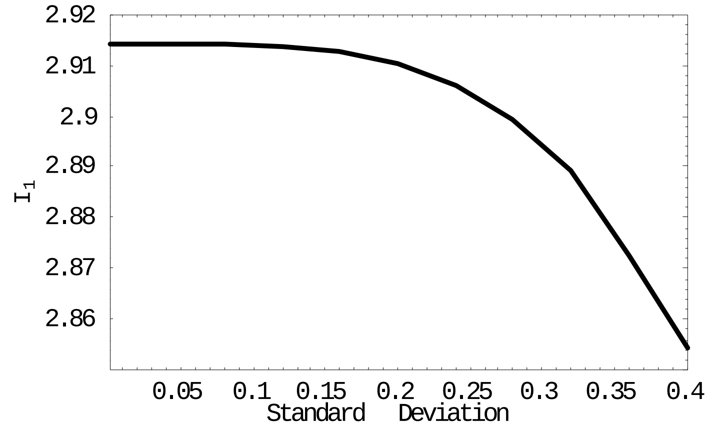
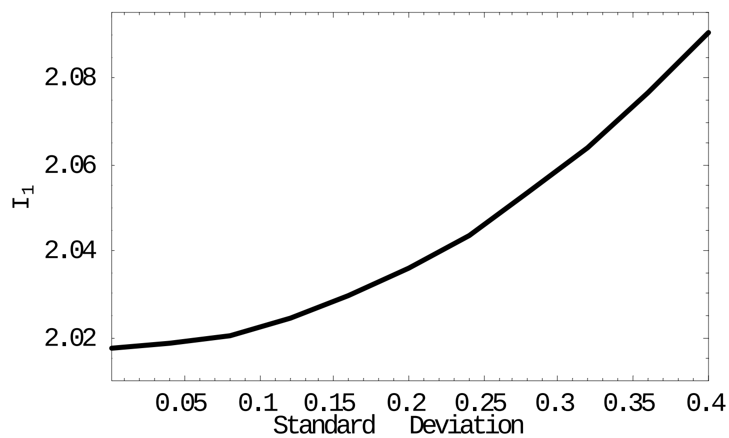


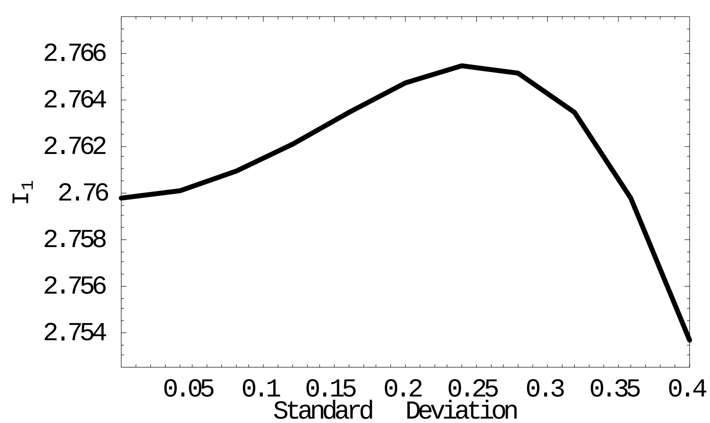
Figure 2: Investment as a Function of Uncertainty



(A) When $\gamma_1 = 0$



(B) When $\gamma_1 = 0.05$



(C) When $\gamma_1 = 0.005$