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Generalized Taylor Rule and Determinacy of Growth Equilibrium

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Abstract

This paper re-examines equilibrium determinacy under the interest-rate control rules in a simple model of endogenous growth. We use a standard money-in-the-utility formulation with fixed labor supply and an Ak technology under which the balanced-growth path is unique and money is superneutral in the long run. We show that even in this environment the interest-rate feedback rule a la Taylor may produce indeterminacy of equilibrium if the monetary authority adjusts the nominal interest rate in response to the growth rate of real income as well as to the rate of inflation.

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1 Introduction

Many authors have explored whether the interest-rate control rule based on Taylor's (1993) idea contributes to reducing equilibrium indeterminacy which generates expectations-driven economic fluctuations. In this literature, it has been well known that an economy following Taylor's rule may easily produce multiple equilibria, if the model economy does not consider capital accumulation. For example, Benhabib et al. (2001a) reveal that an active interest-rate control under which the nominal interest rate is adjusted more than one-for-one with the rate of inflation, the competitive equilibrium is determinate. Conversely, under a passive interest-rate feedback rule which controls the nominal interest rate less than one-for-one with inflation, the competitive equilibrium tends to be indeterminate. At the same time, Benhabib et al. (2001b) demonstrate that those results would be reversed if the production function contains the stock of real money balances as an input.

In contrast to the models without capital, Meng and Yip (2004) confirm that the possibility of equilibrium indeterminacy under the Taylor rule is significantly reduced, if the economy allows capital accumulation.[1] Technically speaking, introducing capital adds a non-jumpable state variable to the model, which generally contributes to eliminating multiple converging paths. Meng and Yip (2004) also show that such a conclusion still holds, if monetary authority changes the nominal interest rate by observing the level of real income as well as inflation.[2]

This note reconsiders the issue of equilibrium determinacy under interest-rate control rules in the context of a simple growth model. We use a standard money-in-the-utility function model with an Ak technology and exogenous labor supply. In this setting, regardless of interest-rate control rules, money is superneutral on the balanced-growth path and the long-term growth rate of income is uniquely determined by the technology and preference parameters alone. In addition, if the monetary authority adjusts the nominal interest rate by observing the rate of inflation alone, such a monetary policy only affects the steady-state rate of inflation, and hence behaviors of consumption and capital will not respond to the monetary authority's behavior. However, if the monetary authority adopts Taylor's (1993) original proposal and controls nominal interest in response not only to inflation but also to the growth rate of income, then the balanced-growth path may exhibit indeterminacy: there is a continuum of equilibrium paths converging to the balanced-growth equilibrium. In this case, although the balanced-growth path satisfies superneutrality of money, the transition process is affected by the monetary policy. We reveal that, in addition to activeness of interest-rate control, the intertemporal substitutability in felicity also plays a key role for the presence of equilibrium

[1]Meng and Yip (2004) use a neoclassical monetary growth model based on the money-in-the-utility function formulation. Yip and Li (2004), on the other hand, show that if a cash-in-advance constraint applies to both investment and consumption so that money is not superneutral in the steady state, the interest-rate control rule may generate indeterminacy. See also Dupor (2001).

[2]Indeterminacy may emerge if the model introduces labor-leisure choice. As pointed out by Meng and Yip (2004), this possibility, however, requires that labor supply curve has a positive slope.

indeterminacy.[3]

2 The Model

We employ a standard money-in-the-utility-function modelling with an Ak technology. The representative household maximizes a discounted sum of utilities

$$U = \int_0^\infty e^{-\rho t} u(c, m)\, dt, \quad \rho > 0$$

subject to the flow budget and wealth constraints:

$$\dot{a} = ra - c - Rm,$$

$$a = k + m,$$

where c is consumption, m real money balances, k capital stock, a total wealth, r real interest rate and R denotes nominal interest rate. The initial holding of a is exogenously given. Here, we specify the instantaneous utility function in the following manner:

$$u(c, m) = \frac{\left(c^\gamma m^{1-\gamma}\right)^{1-\sigma}}{1-\sigma}, \qquad 0 < \gamma < 1, \quad \sigma > 0, \quad \sigma \neq 1.$$

Denoting the shadow value of a as q, we find that the optimization conditions include the following:

$$\frac{(1-\gamma)\, c}{\gamma m} = R, \tag{1}$$

$$\gamma c^{\gamma(1-\sigma)-1} m^{(1-\sigma)(1-\gamma)} = q, \tag{2}$$

$$\dot{q} = q(\rho - A), \tag{3}$$

together with the transversality condition: $\lim_{t\to\infty} e^{-\rho t} aq = 0$. Equation (1) means that the marginal rate of substitution between consumption and real money balances equal the nominal interest rate.

We assume that the production function is specified as

$$y = Ak, \tag{4}$$

where y denotes aggregate output. The commodity market is assumed to be competitive so that the real interest rate is determined by

$$r = A. \tag{5}$$

[3] When the nominal interest rate responds to inflation alone in an Ak growth model, intermediacy would emerge either if labor supply is endogenous or if a cash-in- advance constraint applies to investment as well: see Mino and Itaya (2004 and 2007) and Suena and Yip (2005). In those cases, money is not superneutral on the balanced- growth path, which is different from our present formulation where monetary policy cannot affect long-term economic growth.

We ignore capital depreciation and thus the market equilibrium condition for commodity is $y = \dot{k} + c$, which yields

$$\frac{\dot{k}}{k} = A - z, \tag{6}$$

where $z = c/k$.

Following Taylor (1993), we assume that the monetary authority adjusts the nominal interest rate by observing the level of real income as well as the rate of inflation. Since we deal with a growing economy in which real income continues expanding, we consider that the monetary authority changes the nominal interest rate in response not to the level of income but to the growth rate of income.[4] The monetary policy rule is thus specified as

$$R = \phi(\pi) + \eta(g). \quad \phi' > 0, \quad \eta' > 0, \tag{7}$$

where g denotes the growth rate of income. From (4) and (6), g is given by

$$g = \frac{\dot{y}}{y} = \frac{\dot{k}}{k} = A - z.$$

In view of the Fisher condition, the relation and nominal and real interest rates is described by

$$r + \pi = R. \tag{8}$$

From (7) we obtain:

$$A + \pi = \phi(\pi) + \eta(A - z), \tag{9}$$

which yields

$$\frac{d\pi}{dz} = \frac{\eta'(A - z)}{\phi'(\pi) - 1}.$$

As a result, the relation between π and z is expressed as

$$\pi = \pi(z), \tag{10}$$

where

$$\text{sign } \pi'(z) = \text{sign } [\phi'(\pi) - 1].$$

Namely, the equilibrium rate of inflation is positively (resp. negatively) related to the consumption-capital ratio, z, if the monetary authority actively (resp. passively) responds to a change in the rate of inflation.

[4] In our notation, Taylor's principle is expressed as $R = 1.5(\pi - \pi^*) + 0.5y$ (or $R = 1.5(\pi - \pi^*) + 1.0y$), where π^* denotes the target rate of inflation.

3 Policy Rules and Aggregate Stability

To derive a complete dynamic system, first note that from (1), (8) and (9) we obtain

$$\frac{c}{m} = \frac{\gamma}{1-\gamma}[A + \pi(z)].$$

Taking the time derivatives of the both sides of the above, we obtain

$$\frac{\dot{c}}{c} - \frac{\dot{m}}{m} = \frac{\pi'(z)\,\dot{z}}{A + \pi(z)}. \tag{11}$$

Using (2) and (3), we derive:

$$[\gamma(1-\sigma) - 1]\frac{\dot{c}}{c} + (1-\sigma)(1-\gamma)\frac{\dot{m}}{m} = \rho - A. \tag{12}$$

Eliminating $\dot{m}/m$ from (11) and (12) yields

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}(A-\rho) - \left(\frac{1}{\sigma} - 1\right)(1-\gamma)\frac{\pi'(z)\,\dot{z}}{A + \pi(z)}. \tag{13}$$

Since it holds that $\dot{z}/z = \dot{c}/c - \dot{k}/k$, equations (6) and (13) present the following:

$$\frac{\dot{z}}{z} = \frac{1}{\sigma}(A-\rho) - \left(\frac{1}{\sigma} - 1\right)(1-\gamma)\frac{\pi'(z)\,\dot{z}}{A + \pi(z)} - A + z.$$

The above is rewritten as

$$\frac{\dot{z}}{z} = \frac{\frac{1}{\sigma}(A-\rho) - A + z}{\Gamma(z)}, \tag{14}$$

where

$$\Gamma(z) = 1 + \left(\frac{1}{\sigma} - 1\right)(1-\gamma)\frac{\pi'(z)\,z}{A + \pi(z)}.$$

Equation (14) gives a complete dynamic equation that summarizes the dynamic behavior of our economy.

It is easy to see that either if $0 < \sigma < 1$ and $\pi'(z) > 0$ or if $\sigma > 1$ and $\pi'(z) < 0$, then

$$\Gamma(z) > 0,$$

so that a unique balanced-growth path in which z is determined by

$$\frac{1}{\sigma}(A-\rho) - A + z^* = 0 \tag{15}$$

is unstable. This means that the economy always stays on the balanced-growth path, which means that the economy exhibits global determinacy. Notice that both active control ($\phi' > 1$ so that $\pi'(z)$ is positive) and passive control ($\phi' < 1$ so that $\pi'(z)$ is negative) may yield determinacy depending on the magnitude of σ.

In contrast, either if $\sigma > 1$ and $\pi'(z) > 0$ or if $\sigma < 1$ and $\pi'(z) < 0$, then it is possible to hold $\Gamma(z) < 0$ and thus $d(\dot{z}/z)/dz < 0$ on the balanced-growth path. In this case, we see that the balanced-growth path is stable and it exhibits local indeterminacy.

To sum up, we have shown:

Proposition 1 *Suppose that the interest rate control rule is given by* (7). *Then either if* $\phi'(\pi) > 0$ *and* $0 < \sigma < 1$ *or if* $\phi'(\pi) < 1$ *and* $\sigma > 1$, *the balanced-growth path satisfies global determinacy.*

Proposition 2 *The necessary and sufficient condition for global indeterminacy is:*

$$1 + \left(\frac{1}{\sigma} - 1\right) \frac{\eta'(A - z)(1 - \gamma) z}{[\phi'(\pi) - 1][A + \pi(z)]} < 0, \tag{16}$$

where z^* *and* π^* *are their steady-state values.*[5]

Intuitive implication of the above results is as follows. Suppose that the economy is initially in the balanced-growth equilibrium where capital, consumption and real money balances grow at a common rate of $g^* = (1/\sigma)(A - \rho)$. Suppose further that, due to a sunspot-driven expectations change, households anticipate a rise in the rate of capital accumulation and that the consumption-capital ratio, z, will decline. Then, for example, if $0 < \sigma < 1$ and $\phi' > 1$, equation (13) indicates that the growth rate of consumption will decrease.[6] This means that consumption growth is insufficient to meet the output expansion caused by the expected acceleration of capital formation. Hence, the initial expectations are not self fulfilled, implying that the balanced-growth path itself is a unique competitive equilibrium and the economy has no transition process. Conversely, if (16) is satisfied, (13) indicates that consumption growth is enhanced. Therefore, there would be enough consumption demand for the expected increase in production, so that the initial expectations are self fulfilled. If this is the case, there exists a infinite number of converting trajectories at least around the balanced-growth equilibrium: the economy can be out of the balanced-growth equilibrium and monetary disturbances affect the dynamic behavior of the economy.

To be more concrete, let us specify the policy-rule function in such a way that

$$R = \pi^* \left(\frac{\pi}{\pi^*}\right)^{\phi} + A \left(\frac{g}{g^*}\right)^{\eta}, \quad \phi > 0, \quad \eta > 0, \tag{17}$$

where π^* is the target rate of inflation and g^* denotes the balanced-growth rate determined by $g^* = (1/\sigma)(A - \rho)$. In this specification, the target rate of inflation is set by the monetary authority and (8) is satisfied on the balanced-growth path where $g = g^*$ and $\pi = \pi^*$. Given this specification, equation (9) becomes

$$A + \pi = \pi^* \left(\frac{\pi}{\pi^*}\right)^{\phi} + A \left(\frac{A - z}{A - z^*}\right)^{\eta},$$

which yields

$$\frac{d\pi}{dz} = \frac{\frac{A\eta}{A - z^*} \left(\frac{A - z}{A - z^*}\right)^{\eta - 1}}{\phi \left(\frac{\pi}{\pi^*}\right)^{\phi - 1} - 1}.$$

[5]Global indeterminacy emerges if (16) is satisfied for all $z \in (0, A)$, which imposes further restrictions on $\phi(\pi)$ and $\eta(g)$ functions.

[6]In this situation the substitution effect of a change in the nominal interest rate dominates the income effect, which depresses growth of consumption demand.

When we evaluate the above on the balanced growth path where $z = z^*$ and $\pi = \pi^*$, in view of (11), we obtain

$$\left.\frac{d\pi}{dz}\right|_{z=z^*} \equiv \pi'(z^*) = \frac{1}{\phi - 1}\left[\frac{\sigma A \eta}{A - \rho}\right].$$

Using the above, we find that

$$\begin{aligned}\Gamma(z^*) &= 1 + \left(\frac{1}{\sigma} - 1\right)(1-\gamma)\frac{\pi'(z^*)\, z^*}{A + \pi(z^*)} \\ &= 1 + \frac{(1-\sigma)(1-\gamma) A\eta}{\sigma(A+\pi^*)(\phi-1)(A-\rho)}\left[A - \frac{1}{\sigma}(A-\rho)\right]. \qquad (18)\end{aligned}$$

Therefore, $\Gamma(z^*)$ is strictly negative, if and only if

$$\frac{\eta(1-\sigma)}{\phi - 1} < -\frac{\sigma(A+\pi^*)(A-\rho)}{A(1-\gamma)\left[A - \frac{1}{\sigma}(A-\rho)\right]} \; (< 0) \qquad (19)$$

The necessary conditions to hold this inequality are $\sigma < 1$ and $\phi < 1$ or $\sigma > 1$ and $\phi > 1$. If one of these conditions are met, the possibility of indeterminacy increases as η has a larger value, that is, the monetary authority is more sensitive to a divergence between the actual growth rate and the long-run target rate of income expansion.

As an numerical example, let us set:

$$A = 0.07, \quad \rho = 0.04, \quad \gamma = 0.7, \quad \pi^* = 0.02.$$

Then the relation between ϕ, σ and η that satisfies $\Gamma(z^*) = 0$ in (18) is given by

$$\phi = 1 + \frac{7.77(\sigma - 1)\left[0.07(\sigma-1) + 0.04\right]}{\sigma^2}\eta. \qquad (20)$$

Panels (a) and (b) in Figure 1 depict the graphs between ϕ and η under given levels of σ. Figures 1 (a) assumes that $\sigma = 2.0$ so that the balanced growth rate is $g^* = (1/\sigma)(A - \rho) = 0.015$, while Figure (b) sets $\sigma = 0.5$ and thus $g^* = 0.06$. As these figures demonstrate, in both cases the region of the value of ϕ under which indeterminacy emerges is enhanced as η increases. Figure 2 shows the graph of (20) with a given η. Since in this figure z^* has a negative value for $0 < \sigma < 0.428$, we focus on the region where $\sigma > 0.428$. Again, the graph means that an increase in η enhances the region of indeterminacy in the (ϕ, σ) space.

4 Conclusion

This paper re-examines whether the interest-rate feedback rule according to Taylor (1993) eliminates expectations-driven fluctuations in an endogenously growing economy. To focus on the role of monetary policy rule, we have used an Ak model with fixed labor supply in which money is superneutral on the balanced-growth path.

Even in such a simple setting, the interest control rule may generate indeterminacy of equilibrium, if the monetary authority adjusts the nominal interest rate in response to the growth rate of income as well as to the rate of inflation. It is shown that the key elements for indeterminacy conditions are the sensitivity of nominal interest to inflation and the intertemporal rate of substitution in felicity.

For expositional simplicity, this paper examines the issue in a continuous-time model. As is well known, in discrete time settings, both the timing of money holding and the time perspective of the monetary authority (for example, forward-looking vs. current-looking rules) are also relevant for determinacy of equilibrium.[7] Examining the role of generalized Taylor rule in alternative formulations of discrete-time monetary growth models deserves further investigation.

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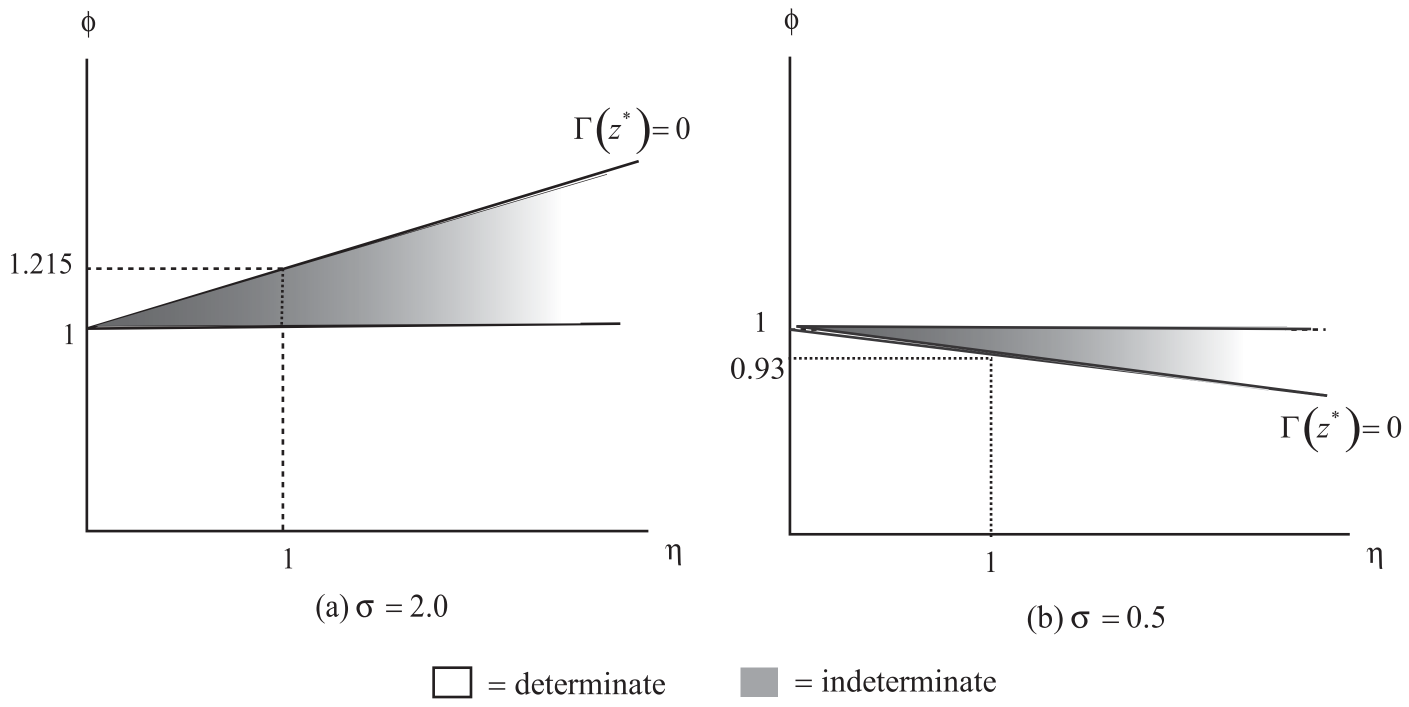

Figure 1

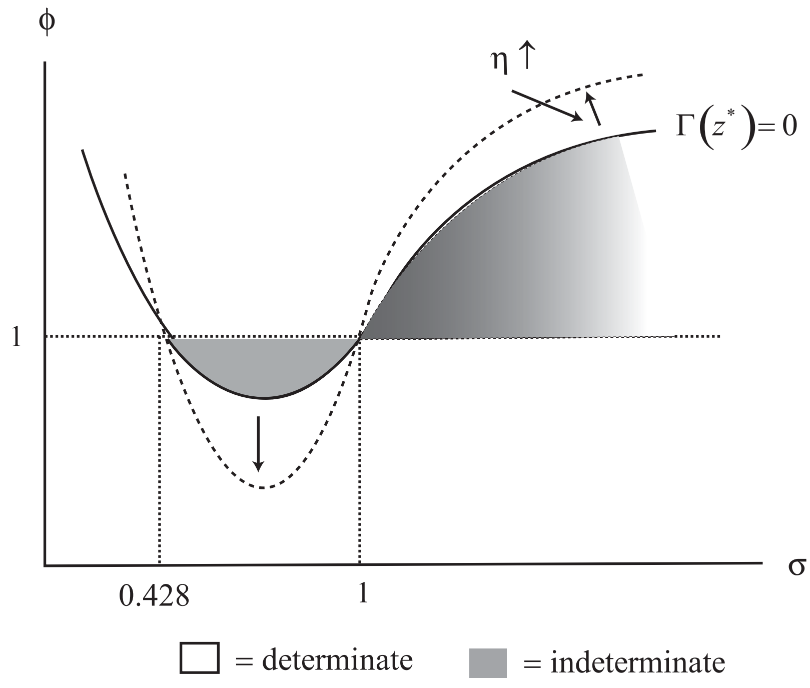

Figure 2