E C O N O M I C S   B U L L E T I N

Education policy in a general equilibrium model with heterogeneous agents.

Stephane Ciriani
Paris 2 Pantheon Assas University, Ermes Cnrs fre 2887

Abstract

This paper studies the impact of public intervention on educationfinance and economic growth in general equilibrium. I use a 3period overlapping generations model where human capitalinvestment is risky and individuals are heterogeneous with respectto their learning abilities. I show that subsidization of privatespending on education leads to a higher economic growth than purepublic education in the short run if initial inequality issufficiently low and in the long run if the dispersion of learningabilities is sufficiently low. The determination of the politicalequilibrium shows that there can exist a conflict betweendemocracy and economic growth.

I am grateful to Damien Gaumont, Philippe Cazenave, an anonymousreferee and an Associate Editor for helpful comments. The usualdisclaimer applies.
Citation: Ciriani, Stephane, (2007) "Education policy in a general equilibrium model with heterogeneous agents.." *Economics Bulletin*, Vol. 9, No. 1 pp. 1-7
Submitted: December 5, 2005. **Accepted:** January 4, 2007.
URL: http://economicsbulletin.vanderbilt.edu/2007/volume9/EB-05I20029A.pdf

1. Introduction.

This 3 period overlapping generations model with altruistic individuals studies the impact of public intervention on education finance and on economic growth when individuals are distributed with respect to their ability to learn. It examines the choice of an education regime in the case of risky human capital investment, endogenous labor supply and positive external benefits from education. The comparison between private and public education regimes in terms of growth and income inequality has already been explored in the literature (Glomm and Ravikumar (1992), Benabou (1996)), but the case of subsidization of private spending on education has received little attention. Gradstein and Justman (1997) introduce government subsidies to support private education and show that economic growth is stronger under subsidization than under private provision of education. The comparison of long-run growth under subsidization and under free public provision does not reveal any advantage for one regime or another.

The aim of this paper is to show that a subsidized regime can lead to a higher long-run growth than a pure public regime and a pure private regime. To do this, I extend the 2005 Zhang's model, to allow for random ability shocks on human capital in general equilibrium. Two modes of public intervention in education are considered: free uniform public provision and subsidization of private spending. I concentrate on the characteristics of subsidization over pure public education and pure private education. I study the link between economic growth and the democratic choice of an education regime and I determine the factors that favor both economic growth and public intervention.

I show that subsidization leads to a higher long-run economic growth than free uniform public education if individuals are weakly heterogeneous. Moreover, a conflict between economic growth and the democratic choice of education regime can arise. The next section sets up the model. Section 3 presents the general equilibrium. Section 4 compares the different regimes of education in terms of growth and income inequality both in the short and the long run. Section 5 studies the political equilibrium. Section 6 concludes.

2. Definition of the model.

2.1. Individuals and preferences.

Consider a 3 period overlapping generations economy. Individuals make their decisions at the beginning of their life-cycle. They are heterogeneous with respect to their inherited ability to learn ϵ_i and are indexed by $i \in [0, 1]$. They inherit human capital h_{it} from their parents. To rule out trivial steady-state equilibria, it is supposed that h_{it} is not zero. In the first period of their life-cycle, they accumulate human capital h_{it+1} through formal schooling. In the second period, adult, middle-aged individuals allocate their unit of time between work l_{it} and leisure $(1 - l_{it})$. They earn income $w_{it}h_{it}l_{it}$ where w_{it} is the wage rate, invest in the child's education e_{it} and save s_{it} for old age consumption d_{it+1}. In the third period, they retire and consume their savings plus interest income. For simplicity reason, population growth is constant over time or equivalently, one parent has one child. Individuals are altruistic towards their child and δ is the degree of intergenerational altruism, or the bequest motive. In order to avoid price anticipation over all future dynasties, I consider ad-hoc altruism. Individuals enjoy giving. They have additively logarithmic utility:

$$u(c_{it}, s_{it}, l_{it}, h_{it+1}) = \log c_{it} + \gamma \log d_{it+1} + \eta \log(1 - l_{it}) + \delta \log h_{it+1}. \quad (1)$$

The choice of a logarithmic function to represent preferences has several implications. First of all, savings only depends on wages, avoiding complexity generated by interest rate in the savings function. Second, all individuals have the same risk aversion under uncertainty. The term η captures the excess burden associated with the finance of public education[1]. Consumption of the young and old individuals are given by the following equations:

$$c_{it} + s_{it} + e_{it} = w_{it}h_{it}l_{it}, \quad (2)$$

$$d_{it+1} = (1 + r_{it+1})s_{it}. \quad (3)$$

The human capital of a child, h_{it+1}, depends on learning abilities ϵ_i and on individual and average spending on education e_t and $\overline{e}_t$. It is defined by a standard Cobb-Douglas technology:

$$h_{it+1} = \epsilon_{it} e_{it}^{\alpha} \overline{e}_t^{\beta}, \quad (4)$$

[1] Gradstein and Justman (1997) assume that the excess burden varies directly with the taste for leisure.

with $0 < \alpha+\beta < 1$. Let superscript (p) denote pure private provision of education, (g) free uniform public provision of education and (z) subsidization of education. Under private provision, education expenditures are defined by: $e_{it}^{p} = \theta^{p} w_t^{p} h_{it}^{p} l^{p}$ where θ^{p} is the share of income that parent (i) chooses to allocate to the education of his child. Under free uniform public provision, the government sets a uniform level of spending on education: $e_t^{g} = \overline{e}_t = \theta^{g} l^{g} w_t^{g} \overline{h}_t^{g}$ where θ_t^{g} is the tax rate and $\overline{h}_t^{g} = \overline{h}_t$ is the average human capital. Under subsidization, private provision is supported by a subsidy at the proportional rate z which is financed by a proportional income tax θ^{z}. Education expenditures are defined by: $e_t^{z} = (\theta^{z}/z)\, w_t^{z} \overline{h}_t^{z} l^{z}$.

2.2. Production and distribution.

It is assumed that the initial distribution of human capital[2] is lognormal, of mean μ_{i0} and variance σ_{i0}^2. Learning abilities are also lognormal and independently distributed in each generation with mean zero and variance $\sigma_{\epsilon}^2 = \phi^2$. The distribution of abilities in each generation is constant over time, independent of the initial distribution of human capital $\Phi_{i0}(h_{i0})$, and is not correlated across generations. Average human capital is defined by:

$$\overline{h}_t = \int_0^{\infty} h_t d\Phi_t(h_t) = \exp(\mu_t + \sigma_t^2). \tag{5}$$

Average abilities are defined by: $E(\epsilon_{it}) = \exp\left(\mu_\epsilon + \sigma_\epsilon^2\right) = \exp \phi^2$, with mean $\mu_\epsilon = 0$. Firms produce a single good with physical capital K_t and labor defined in efficiency units L_t. Physical capital totally depreciates after one period. The production function is assumed to be Cobb-Douglas $Y_t = AK_t^{\varphi} L_t^{1-\varphi}$, with $0 < \eta < 1$ and $A > 0$ a constant productivity parameter. Labor in efficiency units is $L_t = l_t \overline{h}_t = \int_0^{\infty} l_t h_t d\Phi_t(h_t)$, and $k_t = K_t / \int_0^{\infty} l_t h_t d\Phi_t(h_t)$ is the physical capital per effective worker. Competitive firms maximize profit $\pi_t = AK_t^{\varphi} L_t^{1-\varphi} - (1+r_t)K_t - w_t L_t$. Under perfect competition, factors are paid to their marginal product. Profit maximization leads to the following wage rate per units of effective labor and interest rate:

$$w_t \equiv w(k_t) = A(1-\varphi)k_t^{\varphi}, \tag{6}$$

$$1 + r_t \equiv 1 + r(k_t) = A\varphi k_t^{\varphi-1}. \tag{7}$$

Denoting $\lambda_t(s_t)$ the distribution of savings s_t, the equilibrium condition on capital market (and the dynamics of the model) are defined by:

$$K_{t+1} = \int_0^{\infty} s_{it} d\lambda_{it}(s_{it}). \tag{8}$$

Physical capital depends on the distribution of savings.

3. The determination of general equilibrium.

The aim of this Section is to compare the optimal allocation of resources under subsidization and under pure private regime, then under subsidization and under free uniform public regime. The results are presented in Proposition 1:

Proposition 1.

(i) Comparing subsidization (z) and pure private provision of education (p) allows to show that the share of income devoted to education is always higher under subsidization over time $(\theta^{z}/z > \theta^{p})$, if there are external benefits from education $(\beta > 0)$. The optimal labor supplies are equal under both subsidized and private regimes $(l^{z} = l^{p})$.

(ii) Comparing subsidization (z) and free uniform public education (g) allows to show that the optimal labor supply is always higher under subsidization over time $(l^{z} > l^{g})$, and that the share of national income allocated to education is the same under free uniform education and under subsidization $(\theta^{z}/z = \theta^{g})$.

The Proof is presented in the 3 following Sections.

[2]Galor and Tsiddon (1997) use a general distribution for human capital.

3.1. The private education regime.

I first consider the case of pure private education. All middle-aged individuals choose the same allocation of time between work and leisure. They also choose the same spending on education, since they have the same risk aversion. The human capital of a child is defined by:

$$h_{it+1}^{p} = \epsilon_{it}(\theta^{p} l^{p})^{\alpha+\beta} (h_{it}^{p})^{\alpha} (\overline{h}_{t}^{p})^{\beta} (w_{t}^{p})^{\alpha+\beta}. \tag{9}$$

Solving the concave program of a parent defined by (1), (2), (3) and (4), I obtain the optimal labor supply: $l^{p} = (1+\gamma+\delta\alpha)/[(1+\gamma+\eta+\delta\alpha)]$ and the optimal share of income devoted to education: $\theta^{p} = \delta\alpha/[(1+\gamma+\delta\alpha)]$. Optimal savings are defined by: $s_{it}^{p} = v_{s}^{p} w_{t}^{p} h_{it}^{p}$ where $v_{s}^{p} = \gamma(1-\theta^{p}) l^{p} = \gamma(1+\gamma)/[1+\gamma+\eta+\delta\alpha]$ captures the impact of the control variables on savings. Following (8), the capital market equilibrium requires: $K_{t+1}^{p} = \int_{0}^{\infty} s_{it}^{p} d\lambda_{it}(s_{it}) = v_{s}^{p} w_{t}^{p} \int_{0}^{\infty} h_{it}^{p} d\Phi_{it}(h_{it}) = v_{s}^{p} w_{t}^{p} \overline{h}_{t}^{p}$.

3.2. The pure public education regime.

Under free uniform public provision, middle-aged individuals determine individually the optimal labor supply. They choose the tax rate θ^{g} through majority voting. The human capital of a child is defined by:

$$h_{it+1}^{g} = \epsilon_{it}(\theta_{i}^{g} l_{i}^{g})^{\alpha+\beta} (\overline{h}_{t}^{g})^{\alpha+\beta} (w_{t}^{g})^{\alpha+\beta}. \tag{10}$$

All individuals prefer the same optimal tax rate defined by: $\theta^{g} = \delta(\alpha+\beta)/[(1+\gamma+\delta(\alpha+\beta))]$. The optimal tax rate θ^{g} is higher than the optimal share of income devoted to education under the private regime, as it depends positively on the magnitude of the learning externality (β). The optimal labor supply is defined by: $l^{g} = (1+\eta)/(1+\gamma+\eta)$, and is lower than under the private regime, as l^{p} depends positively on the bequest motive (δ) and on the elasticity of individual spending on education (α). This is due to the transformation of the human capital technology under each regime. Optimal savings and capital market equilibrium are defined by: $s_{it}^{g} = v_{s}^{g} w_{t}^{g} h_{t}^{g}$ and $K_{t+1}^{g} = v_{s}^{g} w_{t}^{g} \overline{h}_{t}^{g}$ with $v_{s}^{g} = \gamma(1+\gamma)/[(1+\gamma+\eta)(1+\gamma+\delta(\alpha+\beta))]$.

3.3. The subsidized education regime.

Let us consider that a subsidy (z) supports private education. The subsidy is financed by a proportional income tax θ^{z}. Middle-aged individuals vote on the subsidy and individually decide on the allocation of time between labor and leisure. The human capital of a child is defined by:

$$h_{it+1}^{z} = \epsilon_{it}((\theta^{z}/z)\, l^{z}))^{\alpha+\beta} (h_{it}^{z})^{\alpha} (\overline{h}_{t}^{z})^{\beta} (w_{t}^{z})^{\alpha+\beta}. \tag{11}$$

The parents choose the same allocation of time between work and leisure than under the private regime. The optimal labor supply is the same than under the private regime. Therefore, it is higher than under free uniform provision: ($l^{z} = l^{p} > l^{g}$). The optimal share of national income allocated to education is the same than under free uniform provision as it is also chosen by majority voting: (θ^{z}/z)$= \theta^{g}$. It is then higher than under private provision. Optimal savings and capital market equilibrium are defined by: $s_{it}^{z} = v_{s}^{z} w_{t}^{z} h_{it}^{z}$ and $K_{t+1}^{z} = v_{s}^{z} w_{t}^{z} \overline{h}_{t}^{z}$ with $v_{s}^{z} = \gamma(1+\gamma+\delta\alpha)/[(1+\gamma+\eta+\delta\alpha)(1+\gamma+\delta(\alpha+\beta))]$.

4. The growth rates, the steady-state human capital and the evolution of inequality.

4.1. The mean and dispersion of human capital.

This subsection studies the distribution of human capital under each regime. Denote the mean of $\log(h_t)$ by μ_t. Under subsidization, the dynamics of the mean are given by the following expression:

$$\mu_{t+1}^{z} = \omega^{z} + \varphi\,(\alpha+\beta) \log k_{t}^{z} + (\alpha+\beta)\,\mu_{t}^{z} + \beta\sigma_{t}^{2}/2, \tag{12}$$

where $\omega^{z} = (\alpha+\beta)\log((\theta^{z}/z) l^{z} A(1-\varphi))$ captures the effect of the control variables on mean and average human capital [3]. The dynamics of the variance are given by:

$$\sigma_{t+1}^{2} = \alpha^{2}\sigma_{t}^{2} + \phi^{2} = \sigma_{\infty}^{2} + \alpha^{2t}(\sigma_{0}^{2} - \sigma_{\infty}^{2}), \tag{13}$$

[3] For Benabou (1996), the control variables of the human capital technology capture both the optimal allocation of resources and the private incentives to invest in education.

where $\sigma_\infty^2 = \phi^2/(1-\alpha^2)$ is the variance in any steady states. Using (4) and (5), the growth rate of average human capital is defined by:

$$\overline{h}_{t+1}^z/\overline{h}_t^z = \exp\{\omega^z + \tfrac{\phi^2}{2} + \varphi\,(\alpha+\beta)\log k_t^z - (1-\alpha-\beta)\log \overline{h}_t^z - \alpha(1-\alpha)\tfrac{\sigma_t^2}{2}\}. \quad (14)$$

Under subsidization, inequality has a direct negative effect on the growth rate of average human capital[4] as the human capital of a child is increasing and concave in the parental human capital ($\alpha < 1$). Under the pure public regime, inequality is defined by the dispersion of abilities, $\sigma_t^2 = \phi^2$, which is stable over time. The growth rate of average human capital is defined by:

$$\overline{h}_{t+1}^g/\overline{h}_t^g = \exp\{\omega^g + \tfrac{\phi^2}{2} + \varphi\,(\alpha+\beta)\log k_t^g - (1-\alpha-\beta)\log \overline{h}_t^g\}. \quad (15)$$

Under the private regime, the growth rate is defined by:

$$\overline{h}_{t+1}^p/\overline{h}_t^p = \exp\{\omega^p + \tfrac{\phi^2}{2} + \varphi\,(\alpha+\beta)\log k_t^p - (1-\alpha-\beta)\log \overline{h}_t^p - \alpha(1-\alpha)\tfrac{\sigma_t^2}{2}\}. \quad (16)$$

The stochastic components of average human capital dynamics are similar under subsidization and under pure private education. Inequalities are independent of the rate of subsidization of private education.

4.2. Growth rates and short run comparison.

This subsection is dedicated to the comparison of economic growth under alternative regimes in the short run. I study the characteristics of the subsidized regime over the pure private regime and the free uniform public regime. I first compare subsidization with pure private provision of education. I use the converging geometric sequence[5]:

$$\Psi_{t+1}\,(z,p) = \log(\overline{h}_{t+1}^z/\overline{h}_t^z) - \log(\overline{h}_{t+1}^p/\overline{h}_t^p). \quad (17)$$

The growth rate differential is defined by the first term of the sequence: $\Psi_1(z,p) = \log(\overline{h}_1^z/\overline{h}_0^z) - \log(\overline{h}_1^p/\overline{h}_0^p)$. Using (4) and the expressions of education spending in Subsection 2.1, the growth rate differential can be rewritten as follows:

$$\Psi_1(z,p) = (\alpha+\beta)(\log(\theta^z/z) - \log(\theta^p)). \quad (18)$$

As initial inequality is the same under both regimes, $\Psi_1(z,p)$ depends on the tax differential as labor supplies are equal under both regimes ($l^z = l^p$). Due to Proposition 1, the share of income devoted to education is higher under subsidization than under pure private provision if there are positive external benefits from education ($\beta > 0$). This ensures that $\Psi_1(z,p)$ is positive. Subsidization leads to a higher growth than pure private provision in the short run as $\Psi_1(z,p)$ is positive over time. Moreover, differentiating $\Psi_1(z,p)$ shows that the advantage of subsidization over pure private provision in terms of growth increases with the external benefits from education (β) but decreases with the bequest motive (δ). I then compare subsidization with free uniform public provision. Using the geometric sequence, the growth rate differential is defined by:

$$\Psi_1(z,g) = (\alpha+\beta)\log\left(\tfrac{l^z}{l^g}\right) - \alpha(1-\alpha)\tfrac{\sigma_0^2}{2}. \quad (19)$$

Following Proposition 1, the optimal labor supply is lower under the free uniform regime, then $\log\left(\frac{l^z}{l^g}\right)$ is positive. I can now define a threshold value for initial inequality:

$$\widetilde{\sigma}_0^2/2 := \tfrac{(\alpha+\beta)}{\alpha(1-\alpha)}\log\left(\tfrac{l^z}{l^g}\right) = \tfrac{(\alpha+\beta)}{\alpha(1-\alpha)}\log\left(\tfrac{(1+\gamma+\delta\alpha)(1+\eta)}{(1+\gamma+\eta+\delta\alpha)(1+\gamma+\eta)}\right). \quad (20)$$

For every value of σ_0^2 lower than $\widetilde{\sigma}_0^2$, subsidization leads to a higher growth than free uniform provision. Differentiating $\Psi_1(z,g)$, the advantage of subsidization over free uniform provision in the short run increases with the externality (β), the bequest motive (δ) and the excess burden (η). In the short run, a large saving motive (γ) increases the advantage of subsidization over private provision and reduces the advantage of subsidization over free uniform provision. Collecting the results leads to the following proposition:

[4] See Guaitoli (2000) for a formal analysis of the negative effects of inequality on the accumulation of human capital.

[5] See Gradstein and Justman (1997) for further information about the properties of this sequence.

Proposition 2.

(i) The comparison of subsidization (z) with pure private provision of education (p) shows that subsidization (z) always lead to a higher economic growth in the short run. The advantage of subsidization in terms of growth increases with the external effect (β) and decreases with the bequest motive (δ).

(ii) The comparison of subsidization (z) with free uniform public provision of education (g) shows that subsidization (z) leads to a higher economic growth in the short run if initial inequality (σ_0^2) is sufficiently low (lower than a positive threshold). The advantage of subsidization in terms of growth increases with the external effect (β), the bequest motive (δ) and the excess burden (η).

Remark 1. The comparison of private provision with free uniform provision shows that a large inequality favors public intervention in terms of short-run growth. The growth rate differential: $(\alpha+\beta)\log\left(\frac{\theta^p l^p}{\theta^g l^g}\right)$ - $\alpha(1-\alpha)\frac{\sigma_0^2}{2}$ decreases with $\left(\sigma_0^2\right)$ while the effect of (β) is ambiguous and a large (δ) favors the private provision of education.

Proof: see the Appendix.

4.3. The steady-state levels of human capital and physical capital per effective worker.

This subsection studies the characteristics of subsidization over the private regime and the free uniform public regime in the long run. The balanced growth path is characterized by a unique steady-state growth rate for both physical capital and human capital, as well as a stationary variance σ_∞^2. Let us denote $g_{\overline{h}_t} = \overline{h}_{t+1}/\overline{h}_t$ the growth rate of average human capital. The balanced growth path satisfies the following condition: $g_{K_t}=g_{\overline{h}_t}=g_\infty$. Using (14), the steady state of average human capital $\overline{h}_t$ under subsidization is given by:

$$\overline{h}_\infty^z = \exp\{[\omega^z + \varphi(\alpha+\beta)\log\overline{k}_\infty^z + \tfrac{\phi^2}{2(1+\alpha)}]\}/\left(1-\alpha-\beta\right), \tag{21}$$

where $\log\overline{k}_\infty^z = \left(\log v_s^z + \log(1-\varphi) - \log\overline{l}\right)/(1-\varphi)$ is the steady-state physical capital per effective worker[6]. Under free uniform provision, the steady state is defined by:

$$\overline{h}_\infty^g = \exp\{[\omega^g + \varphi(\alpha+\beta)\log\overline{k}_\infty^g + \tfrac{\phi^2}{2}]\}/\left(1-\alpha-\beta\right). \tag{22}$$

Under pure private provision, the steady state is defined by:

$$\overline{h}_\infty^p = \exp\{[\omega^p + \varphi(\alpha+\beta)\log\overline{k}_\infty^p + \tfrac{\phi^2}{2(1+\alpha)}]\}/\left(1-\alpha-\beta\right). \tag{23}$$

I first compare subsidization and private provision. Let us denote $\Psi_\infty(z,p) = \log(\overline{h}_\infty^z/\overline{h}_\infty^p)$ and $\Psi_\infty(z,g) = \log(\overline{h}_\infty^z/\overline{h}_\infty^g)$ the growth rate differentials. The function $\Psi_\infty(z,p)$ is defined by:

$$\Psi_\infty(z,p) = \left(\tfrac{\alpha+\beta}{1-\alpha-\beta}\right)\left\{\log\left(\tfrac{(\theta^z/z)}{\theta^p}\right) + \tfrac{\varphi}{1-\varphi}\log\left(\tfrac{1-(\theta^z/z)}{1-\theta^p}\right)\right\}. \tag{24}$$

Differentiating $\Psi_\infty(z,p)$ shows that the advantage of subsidization over pure private provision increases in the long run when the bequest motive (δ) decreases and the saving motive (γ) increases. The sign of the derivative of $\Psi_\infty(z,p)$ with respect to (β) depends on some specific parameter values. The effect of external benefits from education on $\Psi_\infty(z,p)$ is then ambiguous. The excess burden has no effect on the growth differential as $\Psi_\infty(z,p)$ does not depend on the optimal labor supply. The bequest motive is more important under the pure private regime than under subsidization, where each individual receives a share of national income to support private education. I then compare growth rates under subsidization and free uniform provision. I define $\Psi_\infty(z,g)$ as follows:

$$\Psi_\infty(z,g) = \tfrac{1}{(1-\alpha-\beta)}\left\{\tfrac{(\alpha+\beta)}{1-\varphi}\log\left(\tfrac{l^z}{l^g}\right) - \tfrac{\alpha\phi^2}{2(1+\alpha)}\right\}. \tag{25}$$

Due to Proposition 1, the optimal labor supply is higher under subsidization than under free uniform public provision, then $\log\left(\frac{l^z}{l^g}\right)$ is positive. I define the threshold value for the dispersion of abilities:

$$\widetilde{\phi}^2/2 := \tfrac{(1+\alpha)(\alpha+\beta)}{\alpha(1-\varphi)}\log\left(\tfrac{l^z}{l^g}\right) = \tfrac{(1+\alpha)(\alpha+\beta)}{\alpha(1-\varphi)}\log\left(\tfrac{(1+\gamma+\delta\alpha)(1+\gamma+\eta)}{(1+\gamma+\eta+\delta\alpha)(1+\gamma)}\right). \tag{26}$$

[6]The determination of the steady states is given in the Appendix

For every value of ϕ^2 lower than $\widetilde{\phi}^2$, subsidization generates a higher economic growth than free uniform public provision in the long run[7]. Differentiating $\Psi_\infty(z, g)$ shows that the advantage of subsidization over free uniform education increases with the learning externality (β), the bequest motive (δ) and the excess burden (η). It decreases with the saving motive (γ) and the dispersion of abilities (ϕ^2). The excess burden of funding education is more important under the free uniform public regime because spending on education is supported by government. Each individual receives an equal share of average income. Income tax raised to fund education benefits individuals with income below average. This generates an excess burden which in turn reduces the optimal labor supply ($l^g < l^z = l^p$). The free uniform regime generates a larger excess burden than subsidization. The resulting distribution of the cost of education under free uniform public provision is less costly for any individual with income above the average. Both $\Psi_\infty(z, p)$ and $\Psi_\infty(z, g)$ also increase with the elasticity of physical capital (φ). In the long run, a large saving motive (γ) favors subsidization over private provision while it mitigates the advantage of subsidization over free uniform provision. A large bequest motive (δ) favors private provision over public intervention, as the weight of the bequest motive on the balanced growth path is more important when education is provided privately.

Proposition 3.

(i) *In the long run, the subsidization of private education* (z) *leads to a higher economic growth than free uniform public provision* (g) *if the dispersion of abilities in the population* (ϕ^2) *is sufficiently low (lower than a positive threshold).*

(ii) *The comparison of subsidization* (z) *with pure private provision,* (p) *shows that the advantage of subsidization in terms of economic growth decreases with the bequest motive* (δ), *increases with the saving motive* (γ), *and does not depend on the excess burden* (η). *An increase in the positive external benefits from education* (β) *does not necessary favor subsidization over private provision in terms of growth.*

(iii) *The comparison of subsidization* (z) *with free uniform public provision* (g) *shows that the advantage of subsidization in terms of growth increases with the bequest motive* (δ), *the external benefits from education* (β), *and the excess burden* (η), *while it decreases with the saving motive* (γ).

Remark 2. The comparison of pure private provision with free uniform provision depends on some specific parameter values. The growth rate differential is defined by:

$$\Psi_\infty(p, g) = \tfrac{\alpha+\beta}{1-\alpha-\beta} \log \left\{ \log\left(\tfrac{\theta^p l^p}{\theta^g l^g}\right) + \tfrac{\varphi}{1-\varphi} \log\left(\tfrac{(1-\theta^p) l^p}{(1-\theta^g) l^g}\right) - \tfrac{\alpha\phi^2}{2(1+\alpha)} \right\}. \tag{27}$$

It is an increasing function of the bequest motive (δ) and the excess burden (η). The impact of the positive learning externality (β) and of the saving motive (γ) are ambiguous.

Proof: See the Appendix.

5. The democratic choice of education and the economic growth.

This subsection studies the mode of public intervention which can emerge in a democratic society. The education regime is determined by majority voting and the preferences of the median voter are decisive. To characterize the factors that favor the choice of subsidization over free uniform provision, I compare the indirect utility under each regime. Denote u^z_{m0} and u^g_{m0} the indirect utility of the median voter under subsidization and under free uniform provision in period ($t = 0$). Under subsidization, the indirect utility depends on median and mean human capital:

$$u^z_{m0} = (1+\gamma)\log c^z_0 + \gamma \log\gamma + \eta\log(1-l^z_0) + \delta\,(\alpha+\beta)\log\omega^z + \delta\alpha\log h_{m0} + \delta\beta\log\overline{h}_0. \tag{28}$$

Under free uniform provision, the indirect utility depends on the mean:

$$u^g_{m0} = (1+\gamma)\log c^g_0 + \gamma\log\gamma + \eta\log(1-l^g_0) + \delta\,(\alpha+\beta)\log\omega^g + \delta(\alpha+\beta)\log\overline{h}_0. \tag{29}$$

[7]Our results differs from Vidal and Brauninger (2000) who show that pure public education leads to a higher long-run growth than a subsidized private education regime

In order to choose between the two education regimes, the median voter considers the following utility differential: $d(z,g)_{m0} = u^z_{m0} - u^g_{m0}$. Following the results in Proposition 1 and (4), (11), (12) and (13), $d(z,g)_{m0}$ is defined by:

$$d(z,g)_{m0} = (1+\gamma+\delta(\alpha+\beta))\log\left(\frac{l_0^z}{l_0^g}\right) + \eta\log\left(\frac{1-l_0^z}{1-l_0^g}\right) - \delta\alpha\log\left(\frac{h_{m0}}{\overline{h}_0}\right), \tag{30}$$

where the ratio of the median to the mean of human capital is $h_{m0}/\overline{h}_0 = \exp(\sigma_0^2/2) = \exp(\phi^2/2\left(1-\alpha^2\right))$. Along the steady state path, the variance satisfies the following condition: $\sigma_0^2 = \sigma_\infty^2 = \phi^2/(1-\alpha^2)$. The median voter prefers free public provision over subsidization if the utility differential is negative:

$$\phi^2/2 < \left(1-\alpha^2\right)\left\{(1+\gamma+\delta(\alpha+\beta))\log\left(\frac{l_0^z}{l_0^g}\right) + \eta\log\left(\frac{1-l_0^z}{1-l_0^g}\right)\right\}/\delta\alpha. \tag{31}$$

Subsidization is adopted if the dispersion of abilities is sufficiently low. Moreover, differentiation of (30) shows that the utility differential is an increasing function of the learning externality (β) and the bequest motive (δ). It is also a decreasing function of the dispersion of abilities $\left(\phi^2\right)$. Following Proposition 2 and Proposition 3, the deterministic factors that favor both the preference of the median voter for subsidization and the advantage of subsidization over free uniform provision in terms of growth are a large external effect (β) and a large bequest motive (δ). While a rise in (η) favors subsidization over free uniform provision in terms of growth, it is not certain that it would increase the preference of the median voter for subsidized education. An increase in (η), or equivalently, in the efficiency cost of raising taxes to fund public education, has an ambiguous effect on the utility differential. This can generate a conflict between the democratic choice of education provision and economic growth. Proposition 4 collects all the results.

Proposition 4.

(i) A large external effect (β), a large bequest motive (δ) and a small dispersion of abilities in the population $\left(\phi^2\right)$ favor both the preference of the median voter for subsidization (z) over free uniform public provision (g) and economic growth.

(ii) The comparison of subsidization (z) with free uniform public provision (g) shows that an increase in the excess burden (η) has a positive effect on the growth differential but an ambiguous effect on the utility differential. This results in a possible conflict between the democratic choice of an education regime and economic growth.

Proof: see the Appendix.

6. Concluding remarks.

This paper shows that the subsidization of private education leads to a higher economic growth than free uniform public provision in the short run if initial inequality is low and in the long run if individuals are weakly heterogeneous in terms of learning abilities. Subsidization also leads to a higher growth than pure private provision in the short run. In the long run, a sufficiently low dispersion of abilities favors subsidization over pure private provision. Large external benefits from education and a strong bequest motive favor both economic growth and the preference of the median voter for subsidies over free public education. A conflict between economic growth and the democratic choice of an education regime can arise if there is an increase in the size of the excess burden. While this favors subsidization in terms of economic growth, it does not necessarily strengthen the preference of the median voter for subsidies over free uniform public provision. A low dispersion of abilities is also necessary for subsidization to emerge as the preferred education regime.

Appendix: **Proofs of Proposition 2, Proposition 3 and Proposition 4:**

1. Proof of Proposition 2 and proof of Proposition 3

In this Appendix I show how I obtain the expressions that allow to compare each regime in terms of growth and inequality. Let us Denote $k_t = K_t/l\overline{h}_t$ the ratio of physical capital to effective labor, $g_{K_t} = K_{t+1}/K_t$ the growth rate of physical capital and $g_{\overline{h}_t} = \overline{h}_{t+1}/\overline{h}_t$ the growth rate of average human capital. I consider the case of subsidized education. Physical capital is defined by: $K^z_{t+1} = \int_0^\infty s^z_{it} d\lambda_{it}$. Using the expression of savings in Subsection 3.3, straightforward algebraic manipulations lead to the following expression of savings:

$$K^z_{t+1} = v^z_s w_t \int_0^\infty h^z_{it} d\Phi_{it}(h_{it}) = v^z_s A(1-\varphi)\left(K^z_t\right)^\varphi l^{-\varphi} (\overline{h}^z_t)^{1-\varphi} = v^z_s A(1-\varphi) l^{-1} \left(K^z_t\right)^\varphi l^{1-\varphi} (\overline{h}^z_t)^{1-\varphi}, \quad (32)$$

where $v^z_s = \gamma(1-\theta^z/z)l^z/(1+\gamma)$ captures the impact of the control variables on savings. Dividing (32) by K^z_t, I obtain the growth rate of physical capital:

$$g_{K^z_t} = K^z_{t+1}/K^z_t = v^z_s A(1-\varphi) l^{-1} (K^z_t/l\overline{h}^z_t)^{\varphi-1}. \quad (33)$$

Taking logs, I obtain the following expression:

$$\log(g_{K^z_t}) = \log(K^z_{t+1}/K^z_t) = \log v^z_s + \log A(1-\varphi) - \log l - (1-\varphi)\log k^z_t. \quad (34)$$

I study the balanced growth path. Imposing $g_{K^z_t} = g_{\overline{h}^z_{it}} = g_\infty$ in (34) and in (14) in Subsection 4.1 leads to the steady-state physical capital per worker:

$$\log\left(k^z_\infty\right) = \tfrac{1}{1-\varphi}\left\{\log v^z_s + \log A(1-\varphi) - \log l\right\}. \quad (35)$$

Plugging the expression of $\log\left(k^z_\infty\right)$ into (14), I obtain the steady-state average human capital $\overline{h}^z_\infty$ in (21). The steady-state average human capital can be rewritten as:

$$\overline{h}^z_\infty = \exp\{[\omega^z + \tfrac{\varphi(\alpha+\beta)}{1-\varphi}\left(\log v^z_s + \log A(1-\varphi) - \log l\right) + \tfrac{\phi^2}{2(1+\alpha)}]/\left(1-\alpha-\beta\right)\}. \quad (36)$$

The optimal allocation of resources is given by:

$$\omega^z = \left[(\alpha+\beta)\log\left(\delta(\alpha+\beta)(1+\gamma+\delta\alpha)(1+\gamma)A(1-\varphi)\right)\right]/\left[(1+\gamma+\delta(\alpha+\beta))\right]. \quad (37)$$

Similarly, the steady-state average human capital under free uniform public education is obtained by plugging $\log\left(k^g_\infty\right)$ into (15) and the steady-state average human capital under private education is obtained by plugging $\log\left(k^p_\infty\right)$ into (16). This leads to (22) and (23) in Subsection 4.3. The optimal allocation of resources under free uniform public provision is defined by:

$$\omega^g = [(\alpha+\beta)\log\left(\delta(\alpha+\beta)A(1-\varphi)(1+\gamma)\right]/[(1+\gamma+\delta(\alpha+\beta)(1+\gamma+\eta)], \quad (38)$$

and the optimal allocation of resources under private provision is given by:

$$\omega^p = [(\alpha+\beta)\log(\delta\alpha(1-\varphi)]/[(1+\gamma+\eta+\delta\alpha))]. \quad (39)$$

I turn to the comparison of short-run growth rate differentials. They are defined by: $\Psi_1(z,p) = \log(\overline{h}^z_1/\overline{h}^p_1)$, and $\Psi_1(z,g) = \log(\overline{h}^z_1/\overline{h}^g_1)$. Given the initial state defined by: $\overline{h}^z_0 = \overline{h}^p_0 = \overline{h}^g_0 = \overline{h}_0$, using (14), (15), (16), (17) and Proposition 1, (18) and (19) can be rewritten as:

$$\Psi_1(z,p) = (\alpha+\beta)\left\{\log\left(\tfrac{\delta(\alpha+\beta)}{1+\gamma+\delta(\alpha+\beta)}\right) - \log\left(\tfrac{\delta\alpha}{1+\gamma+\delta\alpha}\right)\right\}, \quad (40)$$

$$\Psi_1(z,g) = (\alpha+\beta)\left\{\log\left(\tfrac{1+\gamma+\delta\alpha}{1+\gamma+\delta\alpha+\eta}\right) - \log\left(\tfrac{1+\gamma}{1+\gamma+\eta}\right)\right\} - \alpha(1-\alpha)\tfrac{\sigma_0^2}{2}. \quad (41)$$

First of all, straightforward calculations allow to verify the results in Proposition 2 (i) and (ii). Second, differentiating both expressions with respect to (β), (γ), (δ), and (ϕ^2) allows to show that: $\partial\Psi_1(z,p)/\partial\beta > 0$, $\partial\Psi_1(z,p)/\partial\delta < 0$, $\partial\Psi_1(z,p)/\partial\gamma > 0$ and $\partial\Psi_1(z,g)/\partial\beta > 0$, $\partial\Psi_1(z,g)/\partial\delta > 0$, $\partial\Psi_1(z,g)/\partial\gamma < 0$ and

$\partial\Psi_1(z,g)/\partial\eta > 0$. The comparison of pure private education and free uniform education depends on the following expressions:

$$\Psi_1(p,g) = (\alpha+\beta)\left\{\log\left(\tfrac{\alpha}{\alpha+\beta}\right) + \log\left(\tfrac{1+\gamma+\delta\alpha+\delta\beta}{1+\gamma+\delta\alpha+\eta}\right) - \log\left(\tfrac{1+\gamma}{1+\gamma+\eta}\right)\right\} - \tfrac{\alpha\phi^2}{2(1+\alpha)}. \qquad (42)$$

Differentiating $\Psi_1(p,g)$ with respect to the parameters leads to the statements in Remark 1. I turn to the long-run analysis: using (14), (15), (16), (17), (21), (22), (23) and Proposition 1, (24) and (25) can be rewritten as:

$$\Psi_\infty(z,p) = \tfrac{(\alpha+\beta)}{(1-\alpha-\beta)(1-\varphi)}\left\{\log\left(\tfrac{\delta(\alpha+\beta)}{1+\gamma+\delta(\alpha+\beta)}\right) - \log\left(\tfrac{\delta\alpha}{1+\gamma+\delta\alpha}\right)\right\}, \qquad (43)$$

$$\Psi_\infty(z,g) = \tfrac{1}{(1-\alpha-\beta)}\left\{\tfrac{\alpha+\beta}{1-\varphi}\log\left(\tfrac{1+\gamma+\delta\alpha}{1+\gamma+\delta\alpha+\eta}\right) - \log\left(\tfrac{1+\gamma}{1+\gamma+\eta}\right) - \tfrac{\alpha\phi^2}{2(1+\alpha)}\right\}. \qquad (44)$$

Straightforward calculation allows to verify the results in Proposition 3 (i). Differentiating both expressions leads to the results of Proposition 3 (ii) and (iii): $\partial\Psi_\infty(z,p)/\partial\delta < 0$, $\partial\Psi_\infty(z,p)/\partial\gamma > 0$ and $\partial\Psi_\infty(z,g)/\partial\beta > 0$, $\partial\Psi_\infty(z,g)/\partial\delta > 0$, $\partial\Psi_\infty(z,g)/\partial\gamma < 0$ and $\partial\Psi_\infty(z,g)/\partial\eta > 0$. The learning externality (β) favors subsidization over private provision in the short run. In the long run, its impact is ambiguous as the sign of $\partial\Psi_\infty(z,p)/\partial\beta$ depends on some specific parameter values. The comparison of private provision and free uniform provision depends on the sign of $\Psi_\infty(p,g)$. Using (17) and Proposition 1, (27) can be rewritten as:

$$\Psi_\infty(p,g) = \tfrac{\alpha+\beta}{1-\alpha-\beta}\left\{\log\tfrac{\alpha}{\alpha+\beta} + \tfrac{1}{1-\varphi}\log\left(\tfrac{1+\gamma+\delta\alpha+\delta\beta}{1+\gamma+\delta\alpha}\right) + \tfrac{\varphi}{1-\varphi}\left(\log\left(\tfrac{1+\gamma+\delta\alpha}{1+\gamma+\delta\alpha+\eta}\right) - \log\left(\tfrac{1+\gamma}{1+\gamma+\eta}\right)\right)\right\}. \qquad (45)$$

Differentiation of growth rate differential leads to the statements of Remark 2.

2. Proof of Proposition 4.

Let us denote $c_0^z = \gamma\left((1-\theta_0^z/z)\,w_0h_0l_0^z\right)/(1+\gamma)$ the optimal consumption subsidization and $c_0^g = \gamma\left((1-\theta_0^g)\,w_0h_0l_0^g\right)/(1+\gamma)$ the optimal consumption under free uniform public regime. Using $l_0^z = (1+\gamma+\delta\alpha)/(1+\gamma+\delta\alpha+\eta)$ and $l_0^g = (1+\gamma)/(1+\gamma+\eta)$, and noting that $\theta_0^z/z = \theta_0^g$, I obtain the expression of the utility differential $d(z,g)_{m0} = u_{m0}^z - u_{m0}^g$. I rewrite (30) as:

$$d(z,g)_{m0} = (1+\gamma+\delta(\alpha+\beta))\left\{\log\left(\tfrac{1+\gamma+\eta}{1+\gamma+\delta\alpha+\eta}\right) - \log\left(\tfrac{1+\gamma}{1+\gamma+\eta}\right)\right\} + \eta\log\left(\tfrac{1+\gamma+\eta}{1+\gamma+\delta\alpha+\eta}\right) - \tfrac{\delta\alpha\phi^2}{2(1-\alpha^2)}. \qquad (46)$$

Differentiating (46) gives the results of Proposition 4 (i): $\partial d(z,g)_{m0}/\partial\beta > 0$, $\partial d(z,g)_{m0}/\partial\delta > 0$, and $\partial d(z,g)_{m0}/\partial\phi^2 < 0$. Differentiation of (46) also reveals that the sign of $d(z,g)_{m0}/\partial\gamma$ and the sign of $\partial d(z,g)_{m0}/\partial\eta$ depend on some specific parameter values. The effect of a rise in (η) on the utility differential is ambiguous. This leads to the result of Proposition (4) (ii).

References.

Benabou, R. (1996) "Heterogeneity, Stratification and Growth: Macroeconomic Implications of Community Structure and School Finance" *American Economic Review*, **86**, 584-609.

Glomm, G., and B. Ravikumar (1992) "Public Versus Private Investment in Human Capital: Endogenous Growth and Income Inequality" *Journal of Political Economy* **100**, 818-834.

Gradstein M., and M. Justman (1997) "Democratic Choice of an Education System: Implications for Growth and Income Distribution" *Journal of Economic Growth* **2**, 169-183.

Galor O. and D. Tsiddon (1997) "The Distribution of Human Capital and Economic Growth" *Journal of Economic Growth* **3**, 93-124.

Guaitoli, D. (2000) "Human Capital Distribution, Growth and Convergence" *Research in Economics* **54**, 331-350.

Vidal J.P., and M. Brauninger (2000) "Private versus Public Financing of Education and Endogenous Growth" *Journal of Population Economics* **13**, 387-401.

Zhang J. (2005) "Income Ranking and Convergence with Physical and Human Capital and Income Inequality" *Journal of Economic Dynamics and Control* **29**, 547-566.