E C O N O M I C S   B U L L E T I N

Bargaining over Managerial Contracts in Delegation Games: The Sequential Move Case

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Abstract

This paper examines the bargaining problem between firms' owners and managers over their managerial delegation contracts in a duopolistic market with differentiated-products. Assuming that delegated managers make every managerial decision in the market, we analyze how the managers' bargaining power affects social welfare and firms' profits for each case of sequential quantity competition and sequential price competition. We show that the relative increase in the managers' bargaining power leads to decrease in firms' profits but improves social welfare in each case, and that this result holds for any case of the degree of product differentiation. This shows that the existing results obtained for the simultaneous move case and a single homogeneous product case are robust in the sequential move cases.

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1 Introduction

This paper presents a theoretical analysis of the bargaining over managerial delegation contracts between owner-shareholders and firms' managers in a duopolistic market with differentiated-products. The analysis of strategic delegation contracts can be traced back to the seminal papers by Fershtman (1985), Fershtman and judd (1987), Sklivas (1987), and Vickers (1985). They independently showed that, in an oligopolistic market, when a firm hires a manager and delegates managerial decision to him with an incentive contract defined in terms of the firm's profit and quantity of sales, the firm often achieves a higher profit than in the case where the owner of the firm directly operate to maximize the profit of the firm. Recently, the recent literature on managerial delegation provides the analysis focusing on the bargaining between an owner and a manager of a firm for the purpose of exploring the issue of disclosure of managerial compensation required by modern corporate governance codes. The pioneering work by van Witteloostuijn *et al.* (2007) analyzes the following two-stage delegation-bargaining game in duopolistic market with a single homogeneous product. In the 1st stage, an owner and a manager in each firm negotiate over a compensation scheme formalized as an incentive contract à la Fershtman, Judd, and Sklivas, i.e. so-called FJS contract. Then, in the second stage, each firm's manager simultaneously chooses the quantity of output. In this model, they obtain that the managers' bargaining power has a positive affect on the equilibrium social welfare but leads to decrease in firms' profits. In the recent paper of Nakamura (2008), he maintains the assumption of simultaneous moves of firms and extends this model so as to deal with the case of a differentiated-products market. Then, he shows that the result by van Witteloostuijn *et al.* (2007) is robust with respect to the the form of firms' competition, quantity or price, and also to the degree of substitution of the products.

The purpose of this paper is to explore a further robustness result with respect to an order of firms' move. We extend the framework set up by van Witteloostuijn *et al.* (2007) not only to deal with the case of differentiated-products but also to consider the case of sequential decision-makings of firms' managers. For this purpose, we basically work with the model by van Witteloostuijn *et al.* (2007) and extend it as follows: (i) introducing the degree of substitution of products as in Nakamura (2008); (ii) changing the 2nd stage of the delegation game by van Witteloostuijn *et al.* (2007) from simultaneous move game into sequential move game. Consequently, our analysis can be regarded as the extension not only of van Witteloostuijn *et al.* (2007) but Nakamura (2008). Then, we examine how the managers' bargaining power has an affect on the equilibrium social welfare for each case of the quantity competition and the price competition. The result obtained in this paper is that the managers' bargaining power has a positive affect on the equilibrium social welfare and negative affect on firms' profits regardless of the degree of substitution of the goods and also of the form of competition, quantity or price, in our sequential move framework. In other words, we show that the results by van Witteloostuijn *et al.* (2007) and Nakamura (2008) are still robust when we consider the sequential move case.

The remainder of this paper is organized as follows. In Section 2, we present the basic setting. In Section 3, we examine the effect of the managers' bargaining power on social welfare and firms' profits for each case of quantity competition and price competition. Section 4 concludes with a few remarks. The proofs of the propositions are relegated in Appendix.

2 Model

We examine a managerial delegation in the Stackelberg duopoly with two differentiated goods, one for each product by a firm i ($i = 1, 2$). The inverse demand is $P_i = A - q_i - bq_j$, for each $i = 1, 2$ and $j \neq i$, where q_i denotes the output of firm i and $b \in [0, 1]$ denotes the degree of product differentiation. Each firm i has the same production technology represented as a linear cost function $c(q_i) = cq_i$ with $A > c \geq 0$. The profit function of a firm i ($= 1, 2$) is given as $\pi_i = P_i q_i - cq_i$.

Each firm is owned by a single private shareholder. Each owner delegates the output decision to a manager. The firm i's manager receives the payoff u_i defined in terms of an incentive contract w_i, à la Fershtman and Judd (1987) and Sklivas (1987), offered from the owner of the firm:

$$u_i = \pi_i + w_i q_i = (A - q_i - bq_j - c + w_i)q_i, \quad i, j = 1, 2 \text{ and } j \neq i. \tag{1}$$

We assume that the domain of admissible contracts w_i are restricted to those generating non-negative profit $\pi_i \geq 0$ for each firm $i = 1, 2$.

Our delegation model is formulated as the 2-stage game. In the 1st stage, an owner and a manager in each firm i ($= 1, 2$) negotiate over the content of the contract w_i. We follow van Witteloostuijn *et al.* (2007) and assume that they reach an agreement on the contract parameter which maximizes the (weighted) Nash product:

$$u_i^{\beta} \pi_i^{1-\beta}, \quad i = 1, 2, \tag{2}$$

where $\beta \in [0, 1)$ is the parameter which is interpreted as the manager's bargaining power. Then, in the 2nd stage, each manager sets the output of the firm to maximize her/his payoff. We examine two types of sequential competition: one is sequential quantity competition, and the other is sequential price competition. The case of the Cournot competition competition with homogeneous goods is analyzed in van Witteloostuijn *et al.* (2007). For the differentiated goods case, Nakamura (2008) examines the simultaneous quantity competition and the simultaneous price competition. In both papers, it is shown that increase in the manager's bargaining power β leads to decrease in the firm's profit and increase in social welfare. In this paper, we examine the robustness of these results for each of the sequential quantity competition and the sequential price competition. Throughout the paper, we suppose that the firm 2 is the follower of the sequential competitions.

Our use of the Nash product in the 1st stage of the delegation game should be justified with some elaboration. The Nash bargaining solution is widely-used solution concept in the literature on the bargaining-delegation game partly because it is the only one solution concept that satisfies Nash's (1950) moderate conditions and also because it is known that the prescription given by the Nash bargaining solution can be arbitrarily approximated by the subgame perfect Nash equilibrium in the alternate-offer game in Rubinstein (1982) and Binmore *et al.* (1986) with sufficiently large discount factor $\delta \in [0, 1]$ of a player. Some readers may wonder whether the convexity of the payoff possibility set, one of the analytical assumptions in Nash's (1950) formulation of the bargaining problem, is ensured in our model. Nevertheless, we need not be troubled by this. Kaneko (1980) proposes the direct extension of the Nash bargaining solution for the bargaining problem with compact but not necessarily convex payoff possibility set and shows that his extension is the only one solution (set-valued function) that satisfies his moderate conditions which are similar to those in Nash (1950).[1] As discussed by Myerson (1991, p. 374), the effective negotiations should be those satisfies moderate conditions suggested by an impartial arbitration. From this point of view, Kaneko's (1980) axiomatization of the extended Nash solution strongly support our use of the Nash bargaining solution without any checking the convexity of the payoff possibility set. As will be shown later, the boundedness of payoff possibility set is ensured in our model.

3 Results

3.1 Sequential quantity competition

First, we examine how managerial power β affects social welfare and profits of the firms for the case of the sequential quantity competition. We derive the subgame perfect equilibrium (hereafter, SPNE) of the bargaining-delegation game by the backward induction. From the first-order condition (hereafter, FOC) of the payoff-maximizaton by the follower, i.e. the manager of the firm 2, we have

$$\frac{\partial u_2}{\partial q_2} = 0 \Leftrightarrow A - q_2 - bq_1 - c + w_2 = q_2 \tag{3a}$$

$$\Leftrightarrow q_2(q_1, w_2) = \frac{1}{2}(A - c - bq_1 + w_2). \tag{3b}$$

Given (3b), the manager of the firm 1 chooses the output q_1 to maximize her/his payoff u_1. The FOC is given as:

$$\frac{\partial u_1}{\partial q_1} = 0 \Leftrightarrow A - q_1 - b\left[\frac{(A - c - bq_1 + w_2)}{2}\right] - c + w_1 = \frac{2 - b^2}{2} q_1. \tag{4}$$

From (3b) and (4), the equilibrium outputs in the 2nd stage are determined as functions of the contract parameters (w_1, w_2):

$$\begin{cases} q_1(w_1, w_2) = \frac{(A-c)(-2+b)-2w_1+bw_2}{2(-2+b^2)} & \text{(5a)} \\ q_2(w_1, w_2) = \frac{(A-c)(-4+2b+b^2)+2bw_1-(4-b^2)w_2}{4(-2+b^2)}. & \text{(5b)} \end{cases}$$

Next, we derive the equilibrium contracts in the 1st stage. Note that, from the FOCs (3b) and (4), we have

$$\begin{cases} u_1 = \left(\frac{2-b^2}{2}\right) q_1^2 \quad \text{and} \quad \pi_1 = \left(\frac{2-b^2}{2}\right) q_1^2 - w_1 q_1 & \text{(6a)} \\ u_2 = q_2^2 \quad \text{and} \quad \pi_2 = q_2^2 - w_2 q_2\,. & \text{(6b)} \end{cases}$$

From (6a) and (6b), we have

$$\pi_1 = \left(\frac{2-b^2}{2}\right) q_1(q_1 - w_1) \quad \text{and} \quad \pi_2 = q_2(q_2 - w_2), \tag{7}$$

and moreover, by (5a) and (5b), we can check that $\frac{\partial^2 \pi_1}{\partial w_1^2} = \frac{1}{-2+b^2} < 0$ and $\frac{\partial^2 \pi_2}{\partial w_2^2} = \frac{-16+16b^2-3b^4}{8(-2+b^2)^2} < 0$ for all $b \in [0, 1]$. Thus, π_i is a strictly concave quadratic function of w_i with two different real solutions

[1] In Kaneko (1980), the axiomatic foundation is established for the symmetric version of the extension of the Nash bargaining solution. Nevertheless, from the proof of his characterization theorem, it can be easily checked that the asymmetric version is characterized when the symmetricity condition is dropped.

for the case of $\pi_i = 0$, which ensures that the admissible domain of w_i, denoted by Ω_i, is a compact interval with non-empty interior for each $i = 1, 2$. Since u_i and π_i are continuous on Ω_i, the payoff possibility set defined by the pair of attainable payoff to a manager and attainable profit of a firm, i.e. the pair $(u_i(\Omega_i), \pi_i(\Omega_i))$, is well-defined as a compact set of $\mathbb{R}^2$. Furthermore, by (6a) and (6b), there exists $(s, t) \in (u_i(\Omega_i), \pi_i(\Omega_i))$ such that $s > 0$ and $t > 0$. Then, from Kaneko's (1980) characterization, the use of the (weighted) Nash product as a solution concept is supported by his moderate conditions.

Using (6a) and (6b), the FOC of the maximization of the Nash product is obtained as follows:

$$\begin{aligned}\frac{\partial}{\partial w_1}\left(u_1^{\beta}\cdot\pi_1^{1-\beta}\right) = 0 &\Leftrightarrow \frac{\partial}{\partial w_1}\left\{\left[\left(\tfrac{2-b^2}{2}\right)q_1^2\right]^{\beta}\cdot\left[\left(\tfrac{2-b^2}{2}\right)q_1^2 - w_1 q_1\right]^{1-\beta}\right\} = 0\\ &\Leftrightarrow \left[-(1-\beta)w_1 + (2-b^2)q_1\right]\tfrac{\partial q_1}{\partial w_1} - (1-\beta)q_1 = 0;\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{\partial}{\partial w_2}\left(u_2^{\beta}\cdot\pi_2^{1-\beta}\right) = 0 &\Leftrightarrow \frac{\partial}{\partial w_2}\left[q_2^{2\beta}\cdot\left(q_2^2 - w_2 q_2\right)^{1-\beta}\right] = 0\\ &\Leftrightarrow \left[-(1-\beta)w_2 + 2q_2\right]\tfrac{\partial q_2}{\partial w_2} - (1-\beta)q_2 = 0.\end{aligned} \tag{9}$$

By (8) and (9), the equilibrium contracts (w_1^*, w_2^*) are determined as follows:

$$\begin{cases} w_1^* = \frac{(A-c)(2-b^2)\beta\left(8-4b(1+\beta)+2b^2(-2+\beta)+b^3(1+\beta)\right)}{16-4b^2(4+\beta^2)+b^4(3-\beta+2\beta^2)} & \text{(10a)}\\ w_2^* = \frac{(A-c)\left(16\beta-8b\beta(1+\beta)+4b^2(1-3\beta+\beta^2)+2b^3(-1+\beta+2\beta^2)-b^4(1-3\beta+2\beta^2)\right)}{16-4b^2(4+\beta^2)+b^4(3-\beta+2\beta^2)}. & \text{(10b)}\end{cases}$$

Substituting the equilibrium contracts (w_1^*, w_2^*) into (5a) and (5b), we obtain the equilibrium outputs (q_1^*, q_2^*) as follows:

$$\begin{cases} q_1^* = \frac{(A-c)(1+\beta)\left(8-4b(1+\beta)+2b^2(-2+\beta)+b^3(1+\beta)\right)}{16-4b^2(4+\beta^2)+b^4(3-\beta+2\beta^2)} & \text{(11a)}\\ q_2^* = \frac{(A-c)(4-b^2)(1+\beta)(4-b(2+b)+(b-2)b\beta)}{2\left(16-4b^2(4+\beta^2)+b^4(3-\beta+2\beta^2)\right)}. & \text{(11b)}\end{cases}$$

Then, substituting the equilibrium outputs (11a) and (11b) into the firms' profit functions, we can derive the equilibrium profits (π_1^*, π_2^*):

$$\begin{cases} \pi_1^* = \frac{(A-c)^2(-2+b^2)(-1+\beta^2)\left(8-4b(1+\beta)+2b^2(\beta-2)+b^3(1+\beta)\right)^2}{2\left(16-4b^2(4+\beta^2)+b^4(3-\beta+2\beta^2)\right)^2}, & \text{(12a)}\\ \pi_2^* = \frac{(A-c)^2(16-16b^2+3b^4)(\beta^2-1)\left(4-2b(1+\beta)+b^2(\beta-1)\right)^2}{4\left(16-4b^2(4+\beta^2)+b^4(3-\beta+2\beta^2)\right)^2}, & \text{(12b)}\end{cases}$$

Social welfare in the equilibrium which is, as usual, measured by the sum of consumer surplus $\frac{1}{2}({q_1^*}^2 + 2bq_1^*q_2^* + {q_2^*}^2)$ and producer surplus $\pi_1^* + \pi_2^*$ is obtained as:

$$SW = \frac{(A-c)^2(1+\beta)\left[\begin{array}{c}-512(-3+\beta)+512b(-2-\beta+\beta^2)-128b^2(17-11\beta+5\beta^2+\beta^3)+256b^3(5+2\beta-2\beta^2+\beta^3)+32b^4(31-36\beta+22\beta^2+\beta^3)\\ -16b^5(29+5\beta-11\beta^2+13\beta^3)+8b^6(-21+38\beta-29\beta^2+4\beta^3)+4b^7(13-\beta-5\beta^2+9\beta^3)-b^8(-1+\beta)^2(-7+5\beta)\end{array}\right]}{8\left(16-4b^2(4+\beta^2)+b^4(3-\beta+2\beta^2)\right)^2}. \tag{13}$$

Now, for the equilibrium profits (π_1^*, π_2^*) and social welfare, we obtain the following result.

Proposition 1. *For all $(b, \beta) \in [0, 1] \times [0, 1)$, $\frac{\partial \pi_i^*}{\partial \beta} \leq 0$ for each $i = 1, 2$ (equality holds only when $(b, \beta) = (0, 0)$), and $\frac{\partial SW}{\partial \beta} > 0$, that is, in the case of the Stackelberg competition, if the managers'* *bargaining power increases, then profitability of the firms decreases but social welfare increases in any case of* $(b, \beta) \in [0,1] \times [0,1)$.

Proof. See Appendix. □

From the above proposition, we can conclude that it can be said that the result obtained by van Witteloostuijn *et al.* (2007) robustly holds still in the case of the sequential quantity competition for any case of the degree of substitution of the goods $\beta \in [0,1)$ excepting $\beta = 0$.

3.2 Sequential price competition

Next, we examine the case of the sequential price competition. From the inverse demand functions $P_i = A - q_i - bq_j$ for $i, j = 1, 2$ and $j \neq i$, the demand functions are derived as:

$$q_i = \frac{A(b-1) + P_i - bP_j}{b^2 - 1}, \quad i, j = 1, 2 \text{ and } j \neq i. \tag{14}$$

In what follows, we limit ourselves to the case of $b \in [0, 1)$.

In the 2nd stage, the managers choose the prices of their own products to maximize their payoffs in a sequential order of decision-making. The price set by the manager in the firm 2, the follower, satisfies the following FOC:

$$\frac{\partial u_2}{\partial P_2} = 0 \Leftrightarrow (1 - b^2)\left[\frac{A(b-1) + P_2 - bP_1}{b^2 - 1}\right] = P_2 - c + w_2 \tag{15a}$$

$$\Leftrightarrow P_2 = \frac{1}{2}\left(A(1-b) + bP_1 + c - w_2\right). \tag{15b}$$

Given that the firm 2 sets the price satisfying (15b), the manager of the firm 1 chooses the price P_1 to maximize her/his payoff:

$$\frac{\partial u_1}{\partial P_1} = 0 \Leftrightarrow \left[\frac{2(1-b^2)}{2-b^2}\right]\left\{\frac{A(b-1) - P_1 - b\left[\frac{A(1-b)+bP_1+c-w_2}{2}\right]}{b^2-1}\right\} = P_1 - c + w_1 \tag{16a}$$

$$\Leftrightarrow P_1 = \frac{(A-c)(2+b-b^2) - (2-b^2)w_1 - bw_2}{2(2-b^2)}. \tag{16b}$$

Substituting (16b) into (15b), the equilibrium prices in the 2nd stage are obtained as follows:

$$\begin{cases} P_1(w_1, w_2) = \frac{(A-c)(2+b-b^2)-(2-b^2)w_1-bw_2}{2(2-b^2)} & (17a) \\ P_2(w_1, w_2) = \frac{(A-c)(4-2b-3b^2+b^3)+c(8-4b^2)+(b^3-2b)w_1+(b^2-4)w_2}{4(2-b^2)}. & (17b) \end{cases}$$

From (14), (17a), and (17b), the equilibrium outputs are determined as:

$$\begin{cases} q_1(w_1, w_2) = \frac{(A-c)(-2+b+b^2)-(2-b^2)w_1+bw_2}{4(-1+b^2)} & (18a) \\ q_2(w_1, w_2) = \frac{(A-c)(1-b)(4+2b-b^2)-(2b-b^3)w_1+(4-3b^2)w_2}{4(2-3b^2+b^4)}. & (18b) \end{cases}$$

Next, we move to the derivation of the equilibrium contracts. Note that, from the FOCs (15a) and

(16a), the objectives of the managers and the profits of the firms can be rewritten as follows:

$$\begin{cases} u_1 = \left(\frac{2(1-b^2)}{2-b^2}\right)q_1^2 \quad \text{and} \quad \pi_1 = \left(\frac{2(1-b^2)}{2-b^2}\right)q_1^2 - w_1 q_1 & \text{(19a)} \\ u_2 = (1-b^2)q_1^2 \quad \text{and} \quad \pi_1 = (1-b^2)q_1^2 - w_1 q_1 \,. & \text{(19b)} \end{cases}$$

By the same argument as in Sec.3.1, the use of the (weighted) Nash product as a solution concept of the owner-manager negotiation is supported by Kaneko's (1980) conditions.

Using (19a) and (19b), the FOC of the maximization of the Nash product is obtained as follows:

$$\frac{\partial}{\partial w_1}\left(u_1^\beta \cdot \pi_1^{1-\beta}\right) = 0 \Leftrightarrow \frac{\partial}{\partial w_1}\left\{\left[\left(\frac{2(1-b^2)}{2-b^2}\right)q_1^2\right]^\beta \cdot \left[\left(\frac{2(1-b^2)}{2-b^2}\right)q_1^2 - w_1 q_1\right]^{1-\beta}\right\} = 0$$
$$\Leftrightarrow \left[-(1-\beta)w_1 + \left(\frac{4(1-b^2)}{2-b^2}\right)q_1\right]\frac{\partial q_1}{\partial w_1} - (1-\beta)q_1 = 0; \tag{20}$$

$$\frac{\partial}{\partial w_2}\left(u_2^\beta \cdot \pi_2^{1-\beta}\right) = 0 \Leftrightarrow \frac{\partial}{\partial w_2}\left\{\left[(1-b^2)q_2^2\right]^\beta \cdot \left[(1-b^2)q_2^2 - w_2 q_2\right]^{1-\beta}\right\} = 0$$
$$\Leftrightarrow \left[-(1-\beta)w_2 + 2(1-b^2)q_2\right]\frac{\partial q_2}{\partial w_2} - (1-\beta)q_2 = 0. \tag{21}$$

Solving the system of equations (20) and (21), we obtain the equilibrium contracts (w_1^*, w_2^*), which, in turn, allow us to determine the equilibrium outputs (q_1^*, q_2^*) in terms of the degree of substitution $b \in [0,1)$ and the managers' bargaining power $\beta \in [0,1)$:

$$\begin{cases} w_1^* = \frac{2(A-c)(1-b)\beta\left(8-4b(-1+\beta)-2b^2(2+\beta)+b^3(-1+\beta)\right)}{16-4b^2(4+\beta^2)+b^4(3+\beta+2\beta^2)} & \text{(22a)} \\ w_2^* = \frac{(A-c)(1-b)\left(16\beta-8b(-1+\beta)\beta-4b^2(1+3\beta+\beta^2)+b^4(1+3\beta+2\beta^2)-2b^3(1+\beta-2\beta^2)\right)}{16-4b^2(4+\beta^2)+b^4(3+\beta+2\beta^2)}. & \text{(22b)} \end{cases}$$

$$\begin{cases} q_1^* = \frac{(A-c)(b^2-2)(1+\beta)\left(8-4b(-1+\beta)-2b^2(2+\beta)+b^3(-1+\beta)\right)}{2(1+b)\left(16-4b^2(4+\beta^2)+b^4(3+\beta+2\beta^2)\right)} & \text{(23a)} \\ q_2^* = \frac{(A-c)(4-3b^2)(1+\beta)\left(4+2b(1-\beta)-b^2(1+\beta)\right)}{2(1+b)\left(16-4b^2(4+\beta^2)+b^4(3+\beta+2\beta^2)\right)}. & \text{(23b)} \end{cases}$$

We are now ready to examine how the bargaining power β affects the profits and social welfare in the case where the price competition takes place in the 2nd stage. By the equilibrium outputs (23a) and (23b), the equilibrium profits (π_1^*, π_2^*) and social welfare SW are determined as:

$$\begin{cases} \pi_1^* = \frac{(A-c)^2(1-b)(-2+b^2)(-1+\beta^2)\left(8-4b(-1+\beta)-2b^2(2+\beta)+b^3(-1+\beta)\right)^2}{2\left(16-4b^2(4+\beta^2)+b^4(3+\beta+2\beta^2)\right)^2}, & \text{(24a)} \\ \pi_2^* = \frac{(A-c)^2(1-b)(4-3b^2)^2(1+\beta)^2\left(-4+2b(-1+\beta)+b^2(1+\beta)\right)^2}{4\left(16-4b^2(4+\beta^2)+b^4(3+\beta+2\beta^2)\right)^2}, & \text{(24b)} \end{cases}$$

$$SW = \frac{(A-c)^2(1+\beta)\begin{bmatrix} -512(-3+\beta)+512b(-1+\beta)^2-128b^2(25-7\beta+\beta^2+\beta^3)+128b^3(-7+\beta)(-1+\beta)^2+32b^4(73-12\beta+4\beta^2+9\beta^3) \\ -16b^5(-1+\beta)^2(-31+9\beta)-8b^6(87+2\beta+7\beta^2+24\beta^3)+4b^7(-1+\beta)^2(-23+11\beta)+b^8(71+17\beta+17\beta^2+39\beta^3) \end{bmatrix}}{8(1+b)\left(16-4b^2(4+\beta^2)+b^4(3+\beta+2\beta^2)\right)^2}. \tag{25}$$

The following proposition tells that the result obtained in the case of the sequential quantity competition still robustly holds.

Proposition 2. *For all* $(b,\beta) \in [0,1) \times [0,1)$, $\frac{\partial \pi_i^*}{\partial \beta} \leq 0$ *for each* $i = 1,2$ *(equality holds only when* $(b,\beta) = (0,0)$*), and* $\frac{\partial SW}{\partial \beta} > 0$*, that is, in the case of the price competition, if the managers' bargaining* *power increases, then profitability of the firms decreases but social welfare increases in any case of* $(b, \beta) \in [0, 1) \times [0, 1)$.

Proof. See Appendix. □

Our Propositions 1 and 2 and the existing results by Nakamura (2008) together conclude that the result in van Witteloostuijn *et al.* (2007) is completely robust with respect to the form of competition, sequential or simultaneous move, and to the degree of substitution of the goods. More precisely, in the current framework of the managerial delegation in the private duopoly, we always observe that increase in the managers' bargaining power leads to the decrease in the firms' profits but improves social welfare regardless of the form of competition and of the degree of substitution of the goods. We summarize these results in Table 1.

Table 1: Affect of managers' bargaining power on profit and social welfare

		Order of firms' moves	
		simultaneous	*sequential*
Quantity competition	*homogeneous goods*	van Witteloostuijn *et al.* (2007) $\frac{\partial \pi_i}{\partial \beta} < 0$ and $\frac{\partial SW}{\partial \beta} > 0$	Proposition 1 $\frac{\partial \pi_i}{\partial \beta} \leq 0$ and $\frac{\partial SW}{\partial \beta} > 0$ (equality holds when $(b, \beta) = (0, 0)$)
	heterogeneous goods, $b \in [0, 1]$	Nakamura (2007) $\frac{\partial \pi_i}{\partial \beta} < 0$ and $\frac{\partial SW}{\partial \beta} > 0$	
Price competition	*homogeneous goods*	Nakamura (2007) $\frac{\partial \pi_i}{\partial \beta} < 0$ and $\frac{\partial SW}{\partial \beta} > 0$	Proposition 2 $\frac{\partial \pi_i}{\partial \beta} \leq 0$ and $\frac{\partial SW}{\partial \beta} > 0$ (equality holds when $(b, \beta) = (0, 0)$)
	heterogeneous goods, $b \in [0, 1)$		

4 Concluding Remarks

In this paper, we examined how managers' bargaining power β affects the profits and social welfare in the private duopoly framework originally set up by van Witteloostuijn *et al.* (2007). In particular, we extend their original framework so as to deal with the case of differentiated products. Then, we obtained that the result by van Witteloostuijn *et al.* (2007) is still robust with respect to the form of the firms' competition in the market stage, quantity competition or price competition, and also to the degree of substitution of the goods in our sequential move framework. This also shows that the result of Nakamura's (2008) is robust with respect to the order of firms move, simultaneous or sequential.

Two interesting extension of the model remain. Our analysis as well as van Witteloostuijn *et al.* (2007) and Nakamura (2008) is carried out in the duopoly setting. Once we establish the robustness of the effect of the managers' bargaining power on the firms' profits and social welfare, the natural question to ask is whether or not this robustness holds in the oligopoly setting with more than two firms. We leave this issue for future research.

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Appendix

Proof of Proposition 1

Let $\phi(b,\beta) = 16 - 4b^2(4+\beta^2) + b^4(3-\beta+2\beta^2)$. Then, the partial derivatives, $\frac{\partial \pi_1^*}{\partial \beta}$, $\frac{\partial \pi_2^*}{\partial \beta}$, and $\frac{\partial SW}{\partial \beta}$, are obtained as:

$$\frac{\partial \pi_1^*}{\partial \beta} = \Xi_1(A,b,c)\cdot[\phi(\beta,b)]^{-3}\cdot\left[\sum_{i=0}^{10}\phi_{1,i}(b,\beta)\right], \tag{26}$$

where $\Xi_1(A,b,c) = (A-c)^2(2-b^2)$, $\phi_{1,0}(b,\beta) = -1024\beta$, $\phi_{1,1}(b,\beta) = 512b(-1+2\beta+3\beta^2)$, $\phi_{1,2}(b,\beta) = -256b^2(-2-10\beta+6\beta^2+3\beta^3)$, $\phi_{1,3}(b,\beta) = 128b^3(6-20\beta-21\beta^2+6\beta^3+\beta^4)$, $\phi_{1,4}(b,\beta) = -64b^4(11+32\beta-42\beta^2-16\beta^3+2\beta^4)$, $\phi_{1,5}(b,\beta) = -32b^5(14-63\beta-48\beta^2+33\beta^3+4\beta^4)$, $\phi_{1,6}(b,\beta) = 16b^6(20+42\beta-93\beta^2-25\beta^3+9\beta^4)$, $\phi_{1,7}(b,\beta) = 8b^7(16-78\beta-45\beta^2+52\beta^3+3\beta^4)$, $\phi_{1,8}(b,\beta) = -4b^8(14+19\beta-75\beta^2-16\beta^3+10\beta^4)$, $\phi_{1,9}(b,\beta) = 2b^9(-7+33\beta+15\beta^2-25\beta^3)$, and $\phi_{1,10}(b,\beta) = b^{10}(1+\beta)^2(4-11\beta+3\beta^2)$;

$$\frac{\partial \pi_2^*}{\partial \beta} = \Xi_2(A,b,c)\cdot[\phi(\beta,b)]^{-3}\cdot\left[\sum_{i=0}^{8}\phi_{2,i}(b,\beta)\right], \tag{27}$$

where $\Xi_2(A,b,c) = \frac{1}{2}(A-c)^2(-16+16b^2-3b^4)$, $\phi_{2,0}(b,\beta) = 256\beta$, $\phi_{2,1}(b,\beta) = -128b(-1+2\beta+3\beta^2)$, $\phi_{2,2}(b,\beta) = 64b^2(-2-8\beta+6\beta^2+3\beta^3)$, $\phi_{2,3}(b,\beta) = -32b^3(4-16\beta-15\beta^2+6\beta^3+\beta^4)$, $\phi_{2,4}(b,\beta) = 16b^4(8+17\beta-33\beta^2-10\beta^3+2\beta^4)$, $\phi_{2,5}(b,\beta) = 8b^5(5-33\beta-15\beta^2+25\beta^3+2\beta^4)$, $\phi_{2,6}(b,\beta) = -4b^6(9+14\beta-45\beta^2+6\beta^4)$, $\phi_{2,7}(b,\beta) = 4b^7(-1+10\beta-10\beta^3+\beta^4)$, and $\phi_{2,8}(b,\beta) = b^8(\beta-1)^2(2+9\beta+\beta^2)$;

$$\frac{\partial SW}{\partial \beta} = \Xi_s(A,c,\beta)\cdot[\phi(\beta,b)]^{-3}\cdot\left[\sum_{i=0}^{12}\phi_{s,i}(\beta,b)\right], \tag{28}$$

where $\Xi_s(A,c,\beta) = \frac{1}{4}(A-c)^2(\beta-1)$, $\phi_{s,0}(b,\beta) = -8192$, $\phi_{s,1}(b,\beta) = 12288b(1+\beta)$, $\phi_{s,2}(b,\beta) = -2048b^2(-7+9\beta+3\beta^2)$, $\phi_{s,3}(b,\beta) = 1024b^3(-26-18\beta+9\beta^2+\beta^3)$, $\phi_{s,4}(b,\beta) = -256b^4(31-141\beta-30\beta^2+6\beta^3)$, $\phi_{s,5}(b,\beta) = -256b^5(-86-27\beta+63\beta^2+4\beta^3)$, $\phi_{s,6}(b,\beta) = 64b^6(15-405\beta-21\beta^2+37\beta^3$, $\phi_{s,7}(b,\beta) = -64b^7(136-18\beta-153\beta^2+\beta^3)$, $\phi_{s,8}(b,\beta) = -16b^8(-27-522\beta+84\beta^2+73\beta^3)$, $\phi_{s,9}(b,\beta) = 16b^9(104-63\beta-150\beta^2+17\beta^3)$, $\phi_{s,10}(b,\beta) = 12b^{10}(-11-98\beta+41\beta^2+16\beta^3)$, $\phi_{s,11}(b,\beta) = 4b^{11}(-31+33\beta+51\beta^2-13\beta^3)$, and $\phi_{s,12}(b,\beta) = b^{12}(11+51\beta-39\beta^2-7\beta^3)$.

First, we show that $\frac{\partial \pi_i^*}{\partial \beta} < 0$ for all $(b,\beta) \in (0,1]\times[0,1)$ for each $i = 1,2$, and $\frac{\partial \pi_i^*}{\partial \beta} = 0$ when $(b,\beta) = (0,0)$. The latter is straightforward, and thus, we only prove the former. It is easily checked that, for all $b \in (0,1]$, $\Xi_1(A,b,c) > 0$ and $\Xi_2(A,b,c) < 0$. In what follows, we show that $\phi(b,\beta) > 0$, $\sum_{i=0}^{10}\phi_{1,i}(b,\beta) < 0$, and $\sum_{i=0}^{8}\phi_{2,i}(b,\beta) > 0$ for all $(b,\beta) \in (0,1]\times[0,1)$. These functions are quite complicated, and we compute the values of the functions on $(b,\beta) \in (0,1]\times[0,1)$. Note that $\phi(1,1) = \sum_{i=0}^{10}\phi_{1,i}(1,1) = \sum_{i=0}^{8}\phi_{2,i}(1,1) = 0$. Thus, together with this fact, Figures 1 to 8 where the values of these three functions on the domain $(0,1]\times[0,1)$ are plotted confirm that $\phi(b,\beta) > 0$, $\sum_{i=0}^{10}\phi_{1,i}(b,\beta) < 0$, and $\sum_{i=0}^{8}\phi_{2,i}(b,\beta) > 0$ for all $(b,\beta) \in (0,1]\times[0,1)$. Thus, by (26) and (27), we can conclude that $\frac{\partial \pi_i^*}{\partial \beta} < 0$ for all $(b,\beta) \in (0,1]\times[0,1)$ for each $i = 1,2$.

We next show that $\frac{\partial SW}{\partial \beta} > 0$ for all $(b,\beta) \in [0,1]\times[0,1)$. Note that $\Xi_s(A,c,\beta) < 0$ for all $\beta \in [0,1)$. Thus, from the above argument, we are enough to show that $\sum_{i=0}^{12}\phi_{s,i}(b,\beta) < 0$ for all $(b,\beta) \in [0,1]\times[0,1)$. From the fact that $\sum_{i=0}^{12}\phi_i^s(1,1) = 0$ and Figures 9 and 10, we can check that $\sum_{i=0}^{12}\phi_{s,i}(b,\beta) < 0$ for all $(b,\beta) \in [0,1] \times [0,1)$. Hence, by (28), $\frac{\partial SW}{\partial \beta} > 0$ holds for any $(b,\beta) \in [0,1] \times [0,1)$.

Proof of Proposition 2

Let $\phi^B(b,\beta) = 16 - 4b^2(4+\beta^2) + b^4(3+\beta+2\beta^2)$, and also $\Xi_1(b,\beta)$, $\Xi_2(b,\beta)$, and $\Xi_s(b,\beta)$, be the same as considered in the proof of Proposition 1, i.e. $\Xi_1(A,b,c) = (A-c)^2(2-b^2)$, $\Xi_2(A,b,c) = \frac{1}{2}(A-c)^2(-16+16b^2-3b^4)$, and $\Xi_s(A,c,\beta) = \frac{1}{4}(A-c)^2(\beta-1)$. Then, the partial derivatives, $\frac{\partial \pi_1^*}{\partial \beta}$, $\frac{\partial \pi_2^*}{\partial \beta}$, and $\frac{\partial SW}{\partial \beta}$, are obtained as:

$$\frac{\partial \pi_1^*}{\partial \beta} = \left(\frac{1-b}{1+b}\right) \cdot \Xi_1(A,b,c) \cdot \left[\phi^B(\beta,b)\right]^{-3} \cdot \left[\sum_{i=0}^{10} \phi_{1,i}^B(b,\beta)\right], \tag{29}$$

where $\phi_{1,0}^B(b,\beta) = -1024\beta$, $\phi_{1,1}^B(b,\beta) = 512b(-1-2\beta+3\beta^2)$, $\phi_{1,2}^B(b,\beta) = -256b^2(2-10\beta-6\beta^2+3\beta^3)$, $\phi_{1,3}^B(b,\beta) = 128b^3(6+20\beta-21\beta^2-6\beta^3+\beta^4)$, $\phi_{1,4}^B(b,\beta) = 64b^4(11-32\beta-42\beta^2+16\beta^3+2\beta^4)$, $\phi_{1,5}^B(b,\beta) = -32b^5(14+63\beta-48\beta^2-33\beta^3+4\beta^4)$, $\phi_{1,6}^B(b,\beta) = -16b^6(20-42\beta-93\beta^2+25\beta^3+9\beta^4)$, $\phi_{1,7}^B(b,\beta) = 8b^7(16+78\beta-45\beta^2-52\beta^3+3\beta^4)$, $\phi_{1,8}^B(b,\beta) = 4b^8(14-19\beta-75\beta^2+16\beta^3+10\beta^4)$, $\phi_{1,9}^B(b,\beta) = 2b^9(-7-33\beta+15\beta^2+25\beta^3)$, and $\phi_{1,10}^B(b,\beta) = -b^{10}(-1+\beta)^2(4+11\beta+3\beta^2)$;

$$\frac{\partial \pi_2^*}{\partial \beta} = \left(\frac{1-b}{1+b}\right) \cdot \Xi_2(A,b,c) \cdot \left[\phi^B(\beta,b)\right]^{-3} \cdot \left[\sum_{i=0}^{8} \phi_{2,i}^B(b,\beta)\right], \tag{30}$$

where $\phi_{2,0}^B(b,\beta) = 256\beta$, $\phi_{2,1}^B(b,\beta) = -128b(-1-2\beta+3\beta^2)$, $\phi_{2,2}^B(b,\beta) = 64b^2(2-8\beta-6\beta^2+3\beta^3)$, $\phi_{2,3}^B(b,\beta) = -32b^3(4+16\beta-15\beta^2-6\beta^3+\beta^4)$, $\phi_{2,4}^B(b,\beta) = -16b^4(8-17\beta-33\beta^2+10\beta^3+2\beta^4)$, $\phi_{2,5}^B(b,\beta) = 8b^5(5+33\beta-15\beta^2-25\beta^3+2\beta^4)$, $\phi_{2,6}^B(b,\beta) = 4b^6(9-14\beta-45\beta^2+6\beta^4)$, $\phi_{2,7}^B(b,\beta) = 4b^7(-1-10\beta+10\beta^3+\beta^4)$, and $\phi_{2,8}^B(b,\beta) = -b^8(\beta+1)^2(2-9\beta+\beta^2)$;

$$\frac{\partial SW}{\partial \beta} = \left(\frac{1-b}{1+b}\right) \cdot \Xi_s(A,c,\beta) \cdot \left[\phi^B(\beta,b)\right]^{-3} \cdot \left[\sum_{i=0}^{12} \phi_{s,i}^B(\beta,b)\right], \tag{31}$$

where $\phi_{s,0}(b,\beta) = -8192$, $\phi_{s,1}(b,\beta) = 4096b(-1+3\beta)$, $\phi_{s,2}(b,\beta) = -2048b^2(-11-3\beta+3\beta^2)$, $\phi_{s,3}(b,\beta) = 1024b^3(10-30\beta-3\beta^2+\beta^3)$, $\phi_{s,4}(b,\beta) = 256b^4(-93-57\beta+54\beta^2+2\beta^3)$, $\phi_{s,5}(b,\beta) = -256b^5(36-111\beta-27\beta^2+8\beta^3)$, $\phi_{s,6}(b,\beta) = -64b^6(-193-189\beta+171\beta^2+17\beta^3$, $\phi_{s,7}(b,\beta) = 64b^7(58-192\beta-81\beta^2+21\beta^3)$, $\phi_{s,8}(b,\beta) = 16b^8(-205-270\beta+234\beta^2+45\beta^3)$, $\phi_{s,9}(b,\beta) = -48b^9(14-53\beta-32\beta^2+7\beta^3)$, $\phi_{s,10}(b,\beta) = b^{10}(404+672\beta-540\beta^2-168\beta^3)$, $\phi_{s,11}(b,\beta) = 4b^{11}(11-51\beta-39\beta^2+7\beta^3)$, and $\phi_{s,12}(b,\beta) = b^{12}(-17-39\beta+21\beta^2+11\beta^3)$.

The rest of the proof is similar to that of Proposition 1. Note that $\frac{1-b}{1+b} > 0$ for all $b \in [0,1)$. Thus, the same conclusion as in the proof of Proposition 1 can be obtained by Figures 11 to 20.

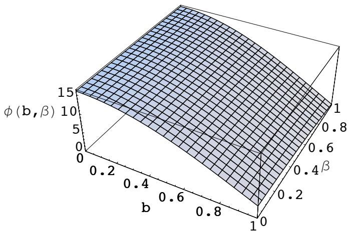

Figure 1: $\phi(\beta, b)$ with $b \in [0, 1]$ and $\beta \in [0, 1)$

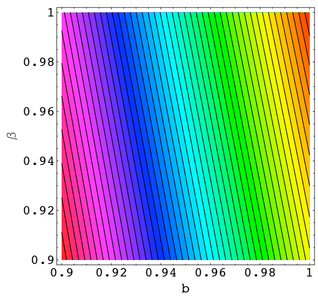

Figure 2: Contours of $\phi(\beta, b)$ with $b \in [0, 1]$ and $\beta \in [0.95, 1)$

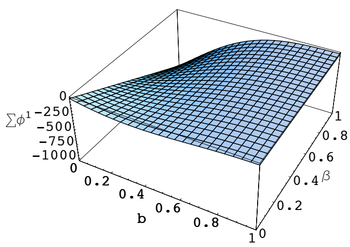

Figure 3: $\sum_{i=0}^{12} \phi_i^1(\beta, b)$ with $b \in [0, 1]$ and $\beta \in [0, 1)$

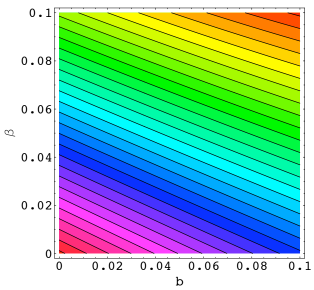

Figure 4: Contours of $\sum_{i=0}^{12} \phi_i^1(\beta, b)$ on $(b, \beta) \in [0, 0.1]^2$

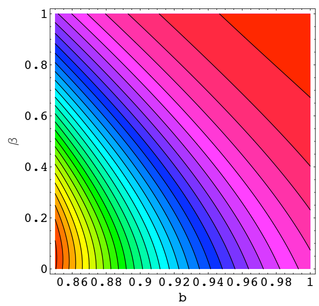

Figure 5: Contours of $\sum_{i=0}^{12} \phi_i^1(\beta, b)$ on $(b, \beta) \in [0.85, 1] \times [0, 1)$

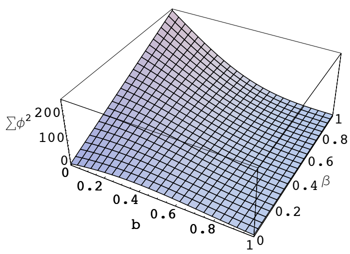

Figure 6: $\sum_{i=0}^{12} \phi_i^2(\beta, b)$ with $b \in [0, 1]$ and $\beta \in [0, 1)$

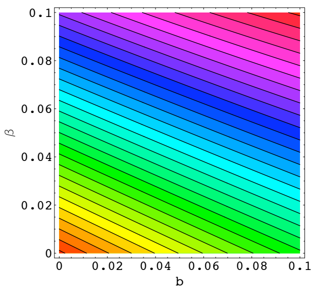

Figure 7: Contours of $\sum_{i=0}^{12} \phi_i^2(\beta, b)$ on $(b, \beta) \in [0, 0.1]^2$

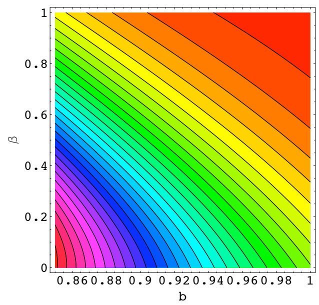

Figure 8: Contours of $\sum_{i=0}^{12} \phi_i^2(\beta, b)$ on $(b, \beta) \in [0.85, 1] \times [0, 1)$

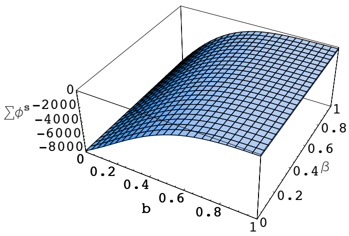

Figure 9: $\sum_{i=0}^{12} \phi_i^s(\beta, b)$ with $b \in [0, 1]$ and $\beta \in [0, 1)$

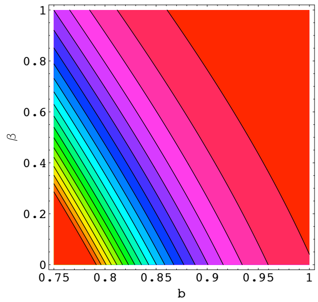

Figure 10: Contours of $\sum_{i=0}^{12} \phi_i^s(\beta, b)$ with $b \in [0.75, 1]$ and $\beta \in [0, 1)$

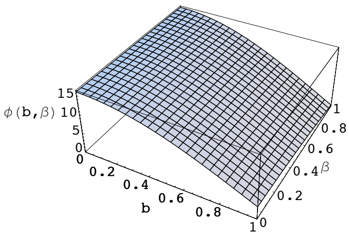

Figure 11: $\phi^B(\beta, b)$ with $b \in [0, 1]$ and $\beta \in [0, 1)$

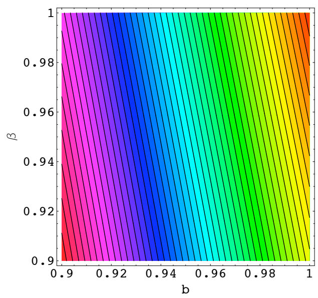

Figure 12: Contours of $\phi^B(\beta, b)$ with $b \in [0.9, 1)$ and $\beta \in [0.9, 1)$

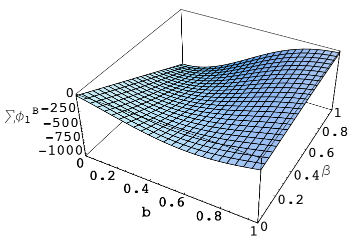

Figure 13: $\sum_{i=0}^{10} \phi_{1,i}^B(\beta, b)$ with $b \in [0, 1]$ and $\beta \in [0, 1)$

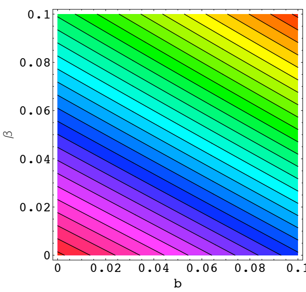

Figure 14: Contours of $\sum_{i=0}^{10} \phi_{1,i}^B(\beta, b)$ on $(b, \beta) \in [0, 0.1]^2$

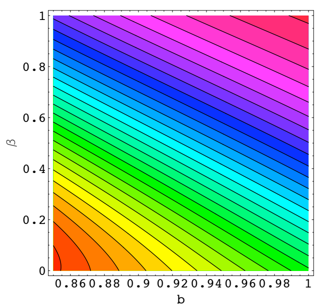

Figure 15: Contours of $\sum_{i=0}^{10} \phi_{1,i}^B(\beta, b)$ on $(b, \beta) \in [0.85, 1] \times [0, 1)$

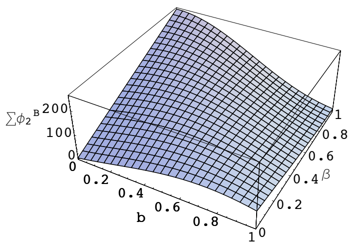

Figure 16: $\sum_{i=0}^{8} \phi_{2,i}^{B}(\beta, b)$ with $b \in [0, 1]$ and $\beta \in [0, 1)$

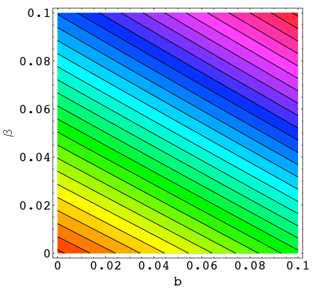

Figure 17: Contours of $\sum_{i=0}^{8} \phi_{2,i}^{B}(\beta, b)$ on $(b, \beta) \in [0, 0.1]^2$

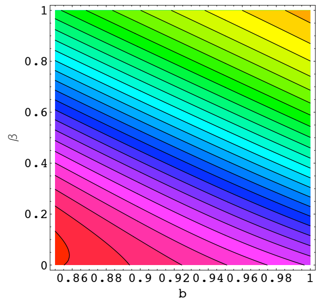

Figure 18: Contours of $\sum_{i=0}^{8} \phi_{2,i}^{B}(\beta, b)$ on $(b, \beta) \in [0.85, 1] \times [0, 1)$

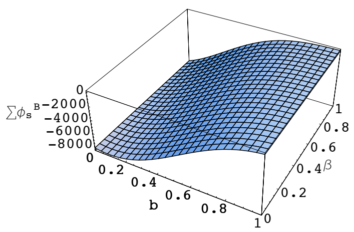

Figure 19: $\sum_{i=0}^{12} \phi_{s,i}^{B}(\beta, b)$ with $b \in [0, 1]$ and $\beta \in [0, 1)$

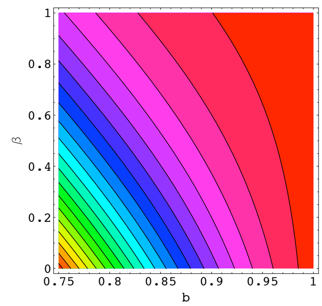

Figure 20: Contours of $\sum_{i=0}^{12} \phi_{s,i}^{B}(\beta, b)$ with $b \in [0.75, 1]$ and $\beta \in [0, 1)$