Privatization and government preference

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Abstract

This paper uses a mixed oligopoly model to examine the relationship between the privatization of a public firm and government preferences for tax revenue. From a public choice viewpoint, we assume the government prefers tax revenue to the sum of consumer and producer surplus, whereas the public firm only cares about the sum of consumer and producer surplus. The results indicate that if the government sufficiently prefers tax revenue, it will not privatize the public firm.

1 Introduction

During the past two decades, a number of theoretical studies have concerned privatization.[1] Most of these studies use a mixed oligopoly model, where a public firm maximizing social welfare, or the sum of consumer and producer surplus, competes with private firms maximizing their own profits in Cournot or Stacklberg competition. Their main concern has been how the number of private firms affects the privatization of the public firm.

White (1996) and Fjell and Heywood (2004) introduced a production subsidy into the mixed oligopoly model. These works typically assume that both the government and the public firm are benevolent in that they maximize social welfare (the sum of consumer and producer surplus less the subsidy). As a result, they conclude that the government should subsidize production in the mixed oligopoly. However, from a public choice viewpoint, it does not seem adequate to assume that the government and the public firm are benevolent. Instead, it is reasonable to consider that the government and the public firm have the following two features.

First, the government may prefer tax revenue to the sum of consumer and producer surplus; that is, the government may not be benevolent.[2] The extreme case is a Leviathan government that seeks to maximize only tax revenue.[3] In this case, the government's optimal policy is to tax production rather than to subsidize it.[4]

Second, the objective of the public firm may differ from that of the government. One characteristic of bureaucracies is a division of functions between different agencies. In general, because the public firm is not a tax collection agency, the public firm does not care about tax revenue but instead cares about the sum of consumer and producer surplus.

In this analysis we assume that the government puts a larger weight on tax revenue than on the sum of consumer and producer surplus, whereas the public firm only cares about the sum of consumer and producer surplus. In this context, we examine the relationship between the privatization of the public firm and the government preferences for tax revenue.

The main results of this paper are as follows. First, the government

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[1]For a detailed survey, see De Fraja and Delbono (1990).

[2]Matsumura (1998) also assumes that the objective function of the government is not social welfare. Unlike our model, however, he assumes that the government puts a larger weight on consumer surplus than on producer surplus.

[3]For a detailed explanation of the Leviathan government, see Brennan and Buchannan (1980).

[4]Mujumdar and Pal (1998) and De Fraja (1991) are exceptions. Although they incorporate tax into a mixed oligopoly model, they do not investigate the optimal tax scheme.

sets a higher tax rate in a mixed oligopoly than in a privatized oligopoly. Second, whether the government privatizes the public firm depends on the government preference for tax revenue.

2 The model

2.1 The mixed oligopoly

We consider a mixed oligopoly model where a public firm (firm 0) and a private firm (firm 1) compete in a market. Both firms produce a homogeneous good and sell it in the market. An inverse demand function is given by $p = 1 - Q$, where p is the market price and Q is the total output. This total output is $Q = q_0 + q_1$, where q_0 and q_1 are the outputs of the public firm and the private firm, respectively. Both firms have the same cost function, $C(q_i) = kq_i^2/2$, $i = 0, 1$.[5] In what follows, we set $k = 1$ for simplicity. A specific tax rate is imposed on both firms.

It is assumed that both firms play a Cournot–Nash game; that is, they simultaneously choose output. The private firm chooses output so as to maximize the following profit function:

$$\pi_1 = (1 - Q)q_1 - \frac{q_1^2}{2} - tq_1, \tag{1}$$

where t is the specific tax rate. On the other hand, the public firm chooses output so as to maximize the sum of consumer and producer surplus:

$$W = \frac{Q^2}{2} + (1 - Q)Q - \frac{(q_0^2 + q_1^2)}{2} - T, \tag{2}$$

where $T(\equiv tQ)$ is the tax revenue.[6] From the first-order conditions, the Cournot–Nash equilibrium outputs of both firms can be obtained as follows:

$$q_0 = \frac{2(1 - t)}{5}, \tag{3}$$

$$q_1 = \frac{1 - t}{5}. \tag{4}$$

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[5]To focus on the tax revenue, neither efficiency nor the problem of entry is taken into consideration, so we ignore fixed cost.

[6]Following De Fraja (1991), in this paper we assume that the public firm only cares about the sum of consumer and producer surplus. If the public firm cares not only about the sum but also about the tax revenue or subsidy, T is cancelled out in equation (2), as in White (1996), Fjell and Heywood (2004), and Mujumdar and Pal (1998). Under this setting, even if the government puts a larger weight on the tax revenue than on the sum of both surpluses, it never privatizes the public firm.

The government's payoff is given by:

$$U = W + (1 + \alpha)T, \tag{5}$$

where α is the parameter representing the weight of the government preference for the tax revenue. Since we are interested in the case where the government puts a larger weight on T than on W, we set $\alpha \geq 0$.[7] If $\alpha = 0$, the government puts the same weight on W and T. In this case, because the government's payoff represents social welfare, the government is benevolent. To the contrary, the greater α becomes, the more the government cares about T. In particular, if α approximates infinity, the government cares only about T; that is, the government is a Leviathan.

From (2), (3) and (4) into (5), the government's payoff can be rewritten as follows:

$$U = \frac{1}{25}(1 - t)[8 + t(7 + 15\alpha)]. \tag{6}$$

When the government chooses t so as to maximize (6), the optimal tax rate in the mixed oligopoly can be obtained as follows:

$$t_m^* = \frac{15\alpha - 1}{2(7 + 15\alpha)}. \tag{7}$$

If the weight of the government preference for the tax revenue is sufficiently large (in the case of $\alpha > 1/15$), the optimal tax rate becomes positive. Conversely, when it is small (in the case of $0 \leq \alpha < 1/15$), the optimal tax rate becomes negative; in the case of $\alpha = 1/15$, the optimal tax rate is zero. We find that the greater the weight of the government preference for the tax revenue, the higher the tax rate the government imposes. This result is intuitive and straightforward.

From (3), (4), (6) and (7), we can show the following equilibrium outcomes in the mixed oligopoly:

$$q_{0m}^* = \frac{3(1 + \alpha)}{7 + 15\alpha}, \tag{8}$$

$$q_{1m}^* = \frac{3(1 + \alpha)}{2(7 + 15\alpha)}, \tag{9}$$

$$Q_m^* = \frac{9(1 + \alpha)}{2(7 + 15\alpha)}, \tag{10}$$

$$U_m^* = \frac{9(1 + \alpha)^2}{4(7 + 15\alpha)}. \tag{11}$$

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[7]If $\alpha < 0$, we can consider the case where the government puts a larger weight on W than on T. However, we exclude this case because it is inconsistent with reality.

As shown, every equilibrium outcome depends only on α. It should be noted that as α becomes larger, the output of each firm, and therefore the total output, decreases. This is because from (7), the optimal tax rate is positively correlated with α. In addition, it is clear that the government's payoff is U-shaped with respect to α.

2.2 The privatized oligopoly

Let us turn to the case of a privatized oligopoly where the public firm is privatized without cost. The privatized firm now plans to maximize its profit by choosing output. As in the case of the mixed oligopoly, both firms play a Cournot–Nash game. The outputs of both firms are expressed as follows:

$$q_0 = q_1 = \frac{1-t}{4}. \tag{12}$$

From (2), (5) and (12), the government's payoff can be written as follows:

$$U = \frac{1}{16}(1-t)[5 + t(3 + 8\alpha)]. \tag{13}$$

When the government chooses t so as to maximize (13), the optimal tax rate in the privatized oligopoly can be obtained as follows:

$$t_p^* = \frac{4\alpha - 1}{3 + 8\alpha}. \tag{14}$$

If $\alpha > 1/4$, the optimal tax rate becomes positive; if $0 \leq \alpha < 1/4$, the optimal tax rate becomes negative and therefore the government subsidizes both firms; and if $\alpha = 1/4$, the optimal tax rate becomes zero. This optimal tax rate in the privatized oligopoly is also increasing in α as in the mixed oligopoly.

From (12), (13) and (14), the outputs of both firms, the total output and the government's payoff in the equilibrium can be written as

$$q_{0p}^* = q_{1p}^* = \frac{1+\alpha}{3 + 8\alpha}, \tag{15}$$

$$Q_p^* = \frac{2(1+\alpha)}{3 + 8\alpha}, \tag{16}$$

$$U_p^* = \frac{(1+\alpha)^2}{3 + 8\alpha}, \tag{17}$$

respectively. As α becomes large, the optimal tax rate increases, and the outputs of both firms thereby decrease. In addition, the government's payoff is U-shaped with respect to α. These features are similar to the mixed oligopoly case.

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2.3 Comparisons

In this subsection, we compare the mixed and privatized oligopoly equilibria. The differences in the optimal tax rates (or, possibly, the optimal subsidy rate), the total outputs and the government's payoffs can be calculated as

$$t_m^* - t_p^* = \frac{11(1+\alpha)}{2(3+8\alpha)(7+15\alpha)} > 0, \tag{18}$$

$$Q_m^* - Q_p^* = \frac{(1+\alpha)(12\alpha-1)}{2(3+8\alpha)(7+15\alpha)}, \tag{19}$$

$$U_m^* - U_p^* = \frac{(1+\alpha)^2(12\alpha-1)}{4(3+8\alpha)(7+15\alpha)}, \tag{20}$$

respectively. These results lead to the following two findings.

First, as long as α is finite, the optimal tax rate in the mixed oligopoly is always higher than that in the privatized oligopoly. In addition, the difference in the optimal tax rates between the mixed and privatized oligopoly cases becomes smaller, when α becomes larger. Note that if α approaches infinity, the optimal tax rates in both the mixed and privatized oligopoly cases are the same: $t_m^* = t_p^* = 1/2$ (see Figure 1).[8]
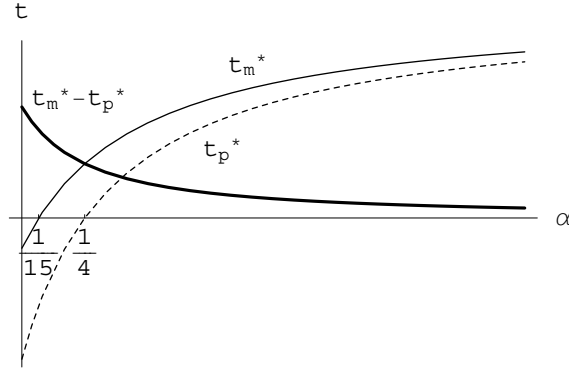

Figure 1: The comparison of the optimal tax rates

Second, if $\alpha > 1/12$, both total output and the government's payoff in the mixed oligopoly are larger than those in the privatized oligopoly. In this

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[8]If α approaches infinity, the government chooses the tax rate so as to maximize the tax revenue. The first-order condition is $d(tQ)/dt = Q(1+\epsilon) = 0$, where ϵ represents the elasticity of output with respect to the tax rate. Because the elasticities obtained in the mixed and privatized oligopoly cases are the same, these tax rates are also identical.

case, the government does not have an incentive to privatize the public firm. In contrast, if $0 \le \alpha < 1/12$, both total output and the government's payoff in the mixed oligopoly are smaller than those in the privatized oligopoly, so the government will privatize the public firm. Obviously, if $\alpha = 1/12$, $Q_m^* = Q_p^* = 13/22$ and $U_m^* = U_p^* = 169/528$ hold, then the government is indifferent to the choice regarding privatization. We have summarized these results in Figure 2 and 3.
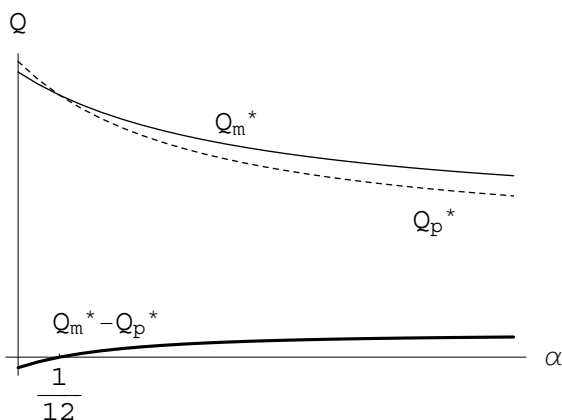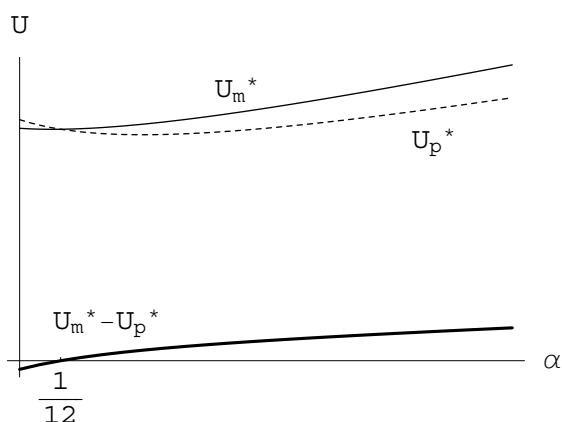

Figure 2: The comparison of total outputs

Figure 3: The comparison of the government's payoffs

It should be noted that when total output in the mixed oligopoly is larger than that in the privatized oligopoly, the government's payoff in the mixed oligopoly is also larger than that in the privatized oligopoly. From (3), (4) and (12), the difference in total outputs can be represented as a function of t_m and t_p: $Q_m - Q_p = (1 - 6t_m + 5t_p)/10$. Therefore, we can see that the difference is decreasing in t_m and increasing in t_p. If the government puts a

sufficiently larger weight on tax revenue than on the sum of both surpluses, i.e., if $\alpha > 1/12$, the difference in the optimal tax rates is small. In this case, the government does not privatize the public firm. In contrast, if the government puts a moderately larger weight on tax revenue than on the sum of both surpluses, i.e., if $0 \leq \alpha < 1/12$, the government will privatize the public firm, because the difference in the optimal tax rates is sufficiently large.

3   Conclusion

In this analysis we used a mixed oligopoly model to examine the relationship between the privatization of a public firm and the government preferences for tax revenue. If the government sufficiently prefers the tax revenue to the sum of the consumer and producer surplus, it will not privatize the public firm. This result may indicate that differences in the progress of privatization in various countries depend on government preferences for tax revenue.

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