E c o n o m i c s   B u l l e t i n

Why not use standard panel unit root test for testing PPP

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Abstract

In this paper we show the consequences of applying a panel unit root test that assumes independence between the cross-sections when testing for a purchasing power parity relationship. The distribution of the tests investigated, including the IPS test of Im et al (2003), are influenced by a common stochastic trend which is usually not accounted for. The result is that the empirical size tends to one with the number of cross-sections. Hence, it is of crucial importance to account for this cross-sectional dependency.

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1 Introduction

There is a large amount of literature on testing purchasing power parity (PPP) due to the economic importance of the relationship. PPP states that in the long run the exchange rate adjusted price levels in two countries should be the same. Otherwise it is profitable to export/import goods. The most common way to test the PPP relationship is to apply the standard augmented Dickey-Fuller unit root test (ADF) to the real exchange rate $\varepsilon_t = \ln(P_{it}) - \ln(P_{jt}) + \ln(R_{ijt})$ where P_{it} (P_{jt}) is the price level in country i (j) and R_{ijt} is the exchange rate between country i and country j, all indexed for time period t. Shiller and Perron (1985) show that the power of the ADF test is very low for the number of observations encountered in real world data sets. Hence, PPP is often rejected, subsequent studies use a panel version with the purpose to increase the power. The most commonly used panel unit root tests are the ones of Levin and Lin (1992, 1993), LL, (later published as Levin, Lin and Chu, 2002) and Im, Pesaran and Shin (2003), IPS. Other tests have been proposed, see e.g. the surveys of Froot and Rogoff (1995), Banerjee (1999), Baltagi and Kao (2000) and Breitung and Pesaran (2007).

Therefore testing PPP using panel unit root tests are widely used, however, this paper shows that the inference used in these applications are likely to be wrong, i.e. the actual size may be very far from the nominal one. The base currency used introduces a common stochastic trend which is not accounted for in the distribution of the test statistics. Our paper analytically derives some useful expressions which help to understand the consequences of the common stochastic trend. E.g. the LL test is shown to diverge with the number of cross-sections. A Monte Carlo simulation is carried out to analyze the consequences for two panel unit root tests. The tests investigated in the Monte Carlo are the IPS and Hadri (2000), H, which is the panel unit roots version of the panel cointegration test of McCoskey and Kao (1998). The IPS test has a null hypothesis of unit root therefore we also have a look at the H test which has stationarity as null hypothesis. Strictly, the H test is a test of stationarity but, for simplicity, through out the paper the term testing for unit roots is used for all tests. The result is that for very small panels, $N = 2$, the size is approximately correct but for larger panels, $N \geq 10$, the size can be significantly distorted, i.e. the empirical size is much too large.

It should be noted that nowadays there are papers dealing with cross-sectional dependencies through the error term, see e.g. Banerjee, Marcellino and Osbat (2004, 2005) and through common factors, see e.g. Moon and Perron (2004) and Bai and Ng (2004). Further, O'Connell (1998) recognise the fact that PPP data would be cross-sectional dependent but fail to realize the true nature of this dependency (i.e. PPP implies a common stochastic trend and not only cross-sectional dependencies through the error term).

The paper is organized as follows: The next section introduces the "statistical" PPP model and this PPP specification is used throughout the paper. Section 3 analyzes the consequences for some panel unit root tests. The Monte Carlo simulation in Section 4 is used to show how large the consequences can be. Section 5 concludes the paper.

2 Null hypothesis of unit roots and the PPP

Testing the null hypothesis of a unit root in a univariate series is often based on the Dickey-Fuller type of equation (or the augmented type):

$$\Delta x_t = \rho x_{t-1} + e_t \tag{1}$$

which under the null hypothesis of a unit root ($\rho = 0$) becomes

$$\Delta x_t = e_t \tag{2}$$

A panel version may be based on, as LL and IPS,

$$\begin{bmatrix} \Delta x_{1t} \\ \Delta x_{2t} \\ \vdots \\ \Delta x_{Nt} \end{bmatrix} = \begin{bmatrix} \rho_1 x_{1t-1} \\ \rho_2 x_{2t-1} \\ \vdots \\ \rho_N x_{Nt-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \\ \vdots \\ e_{Nt} \end{bmatrix} \tag{3}$$

where N is the number of cross-sections, Δ is the first difference operator and e_{it} are disturbances with finite variance and independent and identically distributed over time. Note that ρ_i might or might not be equal to $\rho_j, i \neq j$. The panel null hypothesis is $\rho_1 = \rho_2 = ... = \rho_N = 0$. Under the alternative hypothesis, depending on which test used, some or all ρ_i are less than zero. From (3) it can be shown that there are N random walks in the system under the null hypothesis.

Let $\tilde{p}_i$ denote the log of the price level in country i and r_{iN+1} the log of the exchange rate between country i and $N+1$. The PPP relationship states that $\tilde{p}_{it} - p_{N+1t} + r_{iN+1t} = \varepsilon_{it}$ should be a cointegrating relationship. To make the notation simpler we let $p_{it} = \ln\left(P_{it} * R_{iN+1t}\right)$, i.e. the price level in country i is in the currency of country $N+1$. Further, assume that all p_i and p_{N+1} are nonstationary. There are numerous empirical evidences that prices are nonstationary, see e.g. Culver and Papell (1997). We can also justify it from the same kind of argument that claims that stock prices are nonstationary. If there are cointegration between p_{it} and $p_{N+1,t}$ the same stochastic trend drives the two variables. As a consequence, $p_{jt}, i \neq j$, share the same trend, i.e. all prices are driven by the same stochastic trend. If there is no cointegration there are two stochastic trends, one for each price level. In a panel setup the economic model is

$$\begin{bmatrix} p_{1t} - p_{N+1t} \\ p_{2t} - p_{N+1t} \\ \vdots \\ p_{Nt} - p_{N+1t} \end{bmatrix} = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{Nt} \end{bmatrix} \tag{4}$$

where we test simultaneously if ε_{it} has a non-stationary behavior through a panel unit root test. As we do not estimate the cointegrating relationship testing for cointegration coincide with testing for unit roots. Under the null of no cointegration the N first prices are generated by

$$\begin{bmatrix} \Delta p_{1t} \\ \Delta p_{2t} \\ \vdots \\ \Delta p_{Nt} \end{bmatrix} = \begin{bmatrix} e_{1t} \\ e_{2t} \\ \vdots \\ e_{Nt} \end{bmatrix} \tag{5}$$

and the price level for country $N+1$ by

$$\begin{bmatrix} \Delta p_{N+1,t} \end{bmatrix} = \begin{bmatrix} e_{N+1,t} \end{bmatrix}. \tag{6}$$

Hence, the variables that are used in the panel unit root test are generated according to

$$\begin{bmatrix} \Delta \varepsilon_{1t} \\ \Delta \varepsilon_{2t} \\ \vdots \\ \Delta \varepsilon_{Nt} \end{bmatrix} = \begin{bmatrix} \Delta p_{1t} - \Delta p_{N+1,t} \\ \Delta p_{2t} - \Delta p_{N+1,t} \\ \vdots \\ \Delta p_{Nt} - \Delta p_{N+1,t} \end{bmatrix} = \begin{bmatrix} e_{1t} - e_{N+1,t} \\ e_{2t} - e_{N+1,t} \\ \vdots \\ e_{Nt} - e_{N+1,t} \end{bmatrix} \tag{7}$$

with covariance matrix

$$E \begin{bmatrix} e_{1t} - e_{N+1,t} \\ e_{2t} - e_{N+1,t} \\ \vdots \\ e_{Nt} - e_{N+1,t} \end{bmatrix} \begin{bmatrix} e_{1t} - e_{N+1,t} \\ e_{2t} - e_{N+1,t} \\ \vdots \\ e_{Nt} - e_{N+1,t} \end{bmatrix}' = \Omega - (\gamma \mathbf{1}' + \mathbf{1}\gamma') + \sigma^2 \mathbf{1}\mathbf{1}' = \Sigma. \tag{8}$$

where Ω is the covariance matrix for the first N price levels and σ^2 is the variance for the $N+1$ and $\mathbf{1}$ is an $N \times 1$ vector of ones. The expectation $E\left([e_{1t}, \ldots, e_{Nt}]' \, e_{N+1,t}\right)$ is γ. It is important to note that each equation of (7) contains one common tochastic trend besides the not common one. Further, for simplicity, we assume that the long run variance is the same.

Under the alternative of cointegration the data generating process is

$$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{Nt} \end{bmatrix} = \begin{bmatrix} p_{1t} - p_{N+1,t} \\ p_{2t} - p_{N+1,t} \\ \vdots \\ p_{Nt} - p_{N+1,t} \end{bmatrix} \tag{9}$$

where $p_{N+1,t}$ is generated according to (6). Note here that there is only one stochastic trend driving all prices.

Some notation: The Brownian motion generated by $\varepsilon_{it} = p_{it} - p_{N+1,t}$ is denoted $B_i(\Sigma) = W_i(\Sigma) - W(\Sigma)_{N+1}$ and when $B_i(\Sigma)$ is normalized to have unit covariance matrix $B_i = W_i - W_{N+1}$. Further, $\rightarrow$ denotes the limit when $T \rightarrow \infty$.

3 Consequences for some tests of omitted cross-sectional dependencies

3.1 LL

The LL test in the version of Levin and Lin (1993) is based on the regression, $i = 1, ..., N$,

$$\varepsilon_{it} = \rho\varepsilon_{it-1} + e_{it} \tag{10}$$

where e_{it} is independent across i and t, and are identically distributed with mean 0 and variance σ^2. The null hypothesis is $\rho = 1$. The panel estimator proposed for this simple model is

$$\hat{\rho} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\varepsilon_{it}\varepsilon_{it-1}}{\sum_{i=1}^{N}\sum_{t=1}^{T}\varepsilon_{it-1}^2} \tag{11}$$

with $t-$statistic

$$t_\rho = \frac{\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left[\frac{1}{\sigma^2 T}\sum_{t=1}^{T}\varepsilon_{it-1}e_{it}\right]}{\frac{\hat{\sigma}}{\sigma}\left[\frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{\sigma^2T^2}\sum_{t=1}^{T}\varepsilon_{it-1}^2\right)\right]^{1/2}} \tag{12}$$

They showed that the $t-$statistic converges to a standard normal distribution under the assumption of independent random walks. To see the consequences in the PPP case, the three parts of the $t-$statistic are analyzed. First it is obvious that $\hat{\sigma} = \sqrt{\sum \varepsilon_i^2/T-1}$ is a consistent estimator of σ, hence $\hat{\sigma}/\sigma \to 1$ with T. The inner part of the denominator tends, with T, to

$$\begin{aligned}\frac{1}{\sigma^2T^2}\sum_{t=1}^{T}\varepsilon_{it-1}^2 &= \frac{1}{\sigma^2T^2}\sum_{t=1}^{T}\left(p_{it-1}-p_{N+1t-1}\right)^2 \\ &= \frac{1}{\sigma^2T^2}\sum_{t=1}^{T}\left(p_{it-1}^2 - 2p_{it-1}p_{N+1t-1} + p_{N+1t-1}^2\right) \\ &\to \int W_i^2 dr - 2\int W_iW_{N+1}dr + \int W_{N+1}^2 dr \end{aligned} \tag{13}$$

where W_i and W_{N+1} are Brownian motions with unit variance. Here, we have assumed that the long run variance is the same. Considering the N asymptotics, the limits are

$$\frac{1}{N}\sum_{i=1}^{N}\int W_i^2 dr \to 0.5 \tag{14}$$

$$\frac{1}{N}\sum_{i=1}^{N}\int W_iW_{N+1}dr \to 0 \tag{15}$$

and the trivial

$$\frac{1}{N}\sum_{i=1}^{N}\int W_{N+1}^{2}dr=\int W_{N+1}^{2}dr. \tag{16}$$

The limit of the numerator is

$$\begin{aligned}\frac{1}{\sigma^2 T}\sum_{t=1}^{T}\varepsilon_{it-1}e_{it} &= \frac{1}{\sigma^2 T}\sum_{t=1}^{T}\left(p_{it-1}-p_{N+1}\right)\left(e_{it}-e_{yt}\right)\\ &= \frac{1}{\sigma^2 T}\sum_{t=1}^{T}\left(p_{it-1}e_{it}-p_{it-1}e_{yt}-p_{N+1t-1}e_{it}+p_{N+1t-1}e_{yt}\right)\\ &\to \int W_i dW_i-\int W_i dW_{N+1}-\int W_{N+1}dW_i+\int W_{N+1}dW_{N+1}\end{aligned} \tag{17}$$

It is well known that $\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\int W_i dW_i \to N\left(0,0.5\right)$. The quantities $\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\int W_i dW_{N+1}$ and $\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\int W_{N+1}dW_i$ both have mean 0 and variance 0.5 but with more kurtosis than what would be implied by a normal distribution. The part that influences the statistic the most is the last in equation (17),

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\int W_{N+1}dW_{N+1}=\sqrt{N}\int W_{N+1}dW_{N+1} \tag{18}$$

Because $\int W_{N+1}dW_{N+1}$is either positive of negative (or zero with probability zero) it tends to $-\infty$ or ∞ with N. This implies that the $t-$statistic tend to $-\infty$ or ∞ with N. This result is consitent with Lemma 1 in Moon and Perron (2004). Defining the new test statistic

$$\tilde{t}_{\rho}=\frac{t_{\rho}}{\sqrt{N}} \tag{19}$$

would have the asymptotic distribution

$$\tilde{t}_{\rho}\to\frac{\int W_{N+1}dW_{N+1}}{\left[0.5+\int W_{N+1}^{2}dr\right]^{1/2}}. \tag{20}$$

3.2 IPS

With no lags, the asymptotic version of the IPS test is

$$\Psi_{\bar{t}}=\frac{\sqrt{N}\left(\bar{t}_N-E\left[t_i|\rho_i=1\right]\right)}{\sqrt{Var\left[\bar{t}_N|\rho_i=1\right]}}$$

When the $t-$statistics are independent $Var\left[\bar{t}_N|\rho_i=1\right]=\frac{1}{N}\sum_{i=1}^{N}Var\left[t_i|\rho_i=1\right]$. We know that the expectation of the mean remains the same if the $t-$statistics are dependent as in the case of PPP. A closer look at the individual $t-$statistics reveals

$$t_{i\rho} = \frac{\frac{1}{\sigma^2 T}\sum_{t=1}^{T}\varepsilon_{it-1}e_{it}}{\frac{\hat{\sigma}}{\sigma}\left(\frac{1}{\sigma^2 T^2}\sum_{t=1}^{T}\varepsilon_{it-1}^2\right)^{1/2}} \tag{21}$$

$$\to \frac{\int W_i dW_i - \int W_i dW_{N+1} - \int W_{N+1} dW_i + \int W_{N+1} dW_{N+1}}{\left(\int W_i^2 dr - 2\int W_i W_{N+1} dr + \int W_{N+1}^2 dr\right)^{1/2}} \tag{22}$$

The variance of $\bar{t}_N$ would be complicated and we have not found an analytical expression but it is easily seen that the mean is positive.

3.3 The H test

The panel cointegration test of McCoskey and Kao (1998) is easily modified to become a panel test for unit roots as done in Hadri (2000). The LM test statistic is

$$LM = \frac{T^{-2}\sum_{i=1}^{N}\sum_{t=1}^{T}S_{it}^2}{s^2} \tag{23}$$

where S_{it} is the partial sum of the ith variable,

$$S_{it} = \sum_{j=1}^{t} e_{ij} \tag{24}$$

and s^2 is a consistent estimator (assuming iid errors) of the long run variance,

$$s^2 = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} e_{it}^2 \tag{25}$$

The panel test is the standardized LM test statistic,

$$\sqrt{N}\frac{(LM - \mu_{LM})}{\sqrt{s_{LM}^2}} \tag{26}$$

where μ_{LM} and s_{LM}^2 are the expectation and the variance of the LM test statistic. It can be shown, see e.g. Shin (1994), that the LM test statistic is distributed as the quantity $\int W^2$.

As seen from equation (23) , in the PPP case the asymptotic distribution would be

$$LM \to \frac{\sum_{i=1}^{N}\int B(\Sigma)_i^2}{s^2} \tag{27}$$

$$= \frac{\sum_{i=1}^{N}\left(\int W(\Sigma)_i^2 dr - 2\int W(\Sigma)_i W(\Sigma)_{N+1} dr + \int W(\Sigma)_{N+1}^2 dr\right)}{s^2} \tag{28}$$

so variance would be affected. If we assume that all prices have the same variance

$$LM \to \sum_{i=1}^{N} \left(\int W_i^2 dr - 2 \int W_i W_{N+1} dr + \int W_{N+1}^2 dr \right). \tag{29}$$

4 A Monte Carlo simulation

To evaluate the consequences of the presence of cross-sectional dependencies of the PPP type a small Monte Carlo simulation is carried out. For simplicity we assume that all variables have the same variance but we change the correlation between the base price and the other prices, $\Phi = -0.75, -0.25, 0, 0.25, 0.75$. We choose $N = 2, 5, 10, 50, 100, 200$ and 400. This will allow us to observe what the sizes converge to and how fast. The length of the random walks approximating the Brownian motions is 800 and the number of replicates is 100000. A 5% nominal size is used throughout.

The results of the Monte Carlo simulation is computed in Tables (1) and (2) for IPS and the H test respectively. The tables show the sizes of the tests when the original test procedure are used. For the IPS test the mean and variance used are asymptotic versions of those presented in Im, Pesaran and Shin (2003). After standardization, the IPS test statistic is compared to the Gaussian distribution. The test of Levin and Lin (1993) is not simulated as the distribution is shown to be divergent.

Tables (1) – (2) in here

The results from the Monte Carlo simulation are that the empirical size for low values of N is hardly affected however for higher values the empirical size seems much distorted. The empirical size is much bigger than the nominal size. When the correlation decreases from 0.75 the effect on the empirical size becomes further distorted. The IPS test performs badly with a empirical size of over 50% for $N = 400$ and correlation less than 0.75. For both tests the empirical size seems to slowly tend to one with the number of cross-sections.

5 Conclusion

In this paper we have shown the consequences to the distribution of some panel unit root test statistics when testing the PPP theory. All the tests investigated are influenced by a large extent. In most cases the empirical size becomes much too large, rarely it is not influenced at all. The empirical size usually increases with the number of cross-sections in the panel and when the correlation between the base price level and the other price levels decreases in absolute value. The simulation shows that the empirical size tends to one for the three test statistics although slowly.

For practical purposes the results of this paper have two major implications when testing the PPP hypothesis. Firstly, the empirical size of a panel unit root test is likely to be far from the nominal. One reason for using a panel test is to increase the power of the test but the increased size makes it difficult to judge if a rejection of the null hypothesis depends on increased power or to a to large empirical size. Secondly, as the distribution heavily depends on the correlation between the stochastic trends, these estimates are not available. Therefore in practice it is very difficult to correct the empirical size when using tests that do not consider the cross-sectional dependencies implied by PPP. It is interesting to note that the results hold irrespectively of tests with a null of unit root or for no unit root null.

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Tables

$\Phi\backslash N$	2	5	10	50	100	200	400
0.75	0.0314	0.0324	0.0351	0.0606	0.0899	0.1395	0.2163
0.25	0.0330	0.0448	0.0652	0.2271	0.3564	0.4719	0.5299
0	0.0350	0.0563	0.0991	0.3347	0.4479	0.5193	0.5535
-0.25	0.0406	0.0791	0.1489	0.4109	0.4826	0.5319	0.5562
-0.75	0.0611	0.1673	0.2808	0.4508	0.4887	0.5134	0.5238

Table 1: Empirical size of the IPS test when testing for PPP. Nominal size is 5%.

$\Phi\backslash N$	2	5	10	50	100	200	400
0.75	0.0436	0.0414	0.0396	0.0715	0.1042	0.1526	0.1764
0.25	0.0507	0.0626	0.0812	0.1621	0.2006	0.2429	0.2537
0	0.0541	0.0747	0.1004	0.1908	0.2248	0.2614	0.2691
-0.25	0.0602	0.0881	0.1195	0.2087	0.2413	0.2705	0.2810
-0.75	0.0712	0.1154	0.1528	0.2352	0.2616	0.2840	0.2934

Table 2: Empirical size of the Hadri (2000) LM test when testing for PPP. Nominal size is 5%.