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Granger-Causality in the presence of structural breaks

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Abstract

The concept of Granger-Causality (GC) is widely used to draw inference concerning causality in applied economics. Stationary series are the term of reference used in GC testing, which is generally studied by means of a standard Dickey-Fuller test. We prove that, when the Data Generating Process (DGP) of the variables is either Broken-Trend Stationary (BTS) or Broken-Mean Stationary (BMS), correct inference can not be drawn from a standard Granger-Causality test and may identify inexistent causal relationships, even if the former variables are differenced. Asymptotic and finite-sample evidence in this sense is provided.

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1 Introduction

The roots of our modern concept of causality can be found in Aristotle's "efficient cause" ("the primary source of the change or rest"). Moreover, one of the explicit assumptions commonly made in regard to causality-in econometrics-is that the future can not cause the past. Granger's (1969) seminal article is currently a basic notion in the study of dynamic relationships between time series. Causality, in the Granger sense, is typically defined in terms of the predictability of a vector of variables one period ahead.[1] Paradoxically, GC tests are commonly practiced by means of in-sample $\mathcal{F}$-tests (Chao, Corradi, and Swanson 2001).

There are two different approaches taken in empirical works to drawing inference with regard to the causality among economic variables: the first is credited to Marshall-Neyman-Rubin, whilst the second is attributed to Wiener-Granger-Sims.[2] We focus on that of Wiener-Granger-Sims,[3] which, to the best of our knowledge, appears to be, the most widely accepted approach. The thorough study of GC during the last decades has led to the identification of a number of issues concerning temporal aggregation bias [which may arise when the time series are not collected frequently enough to fully capture the movements of economic variables; see McCrorie and Chambers (2006)], and measurement errors in variables (Andersson 2005), which may cause the GC test to fail to reject the null hypothesis of no Granger-Causality more often than it should (when the Granger-caused variable is measured with error).

Nonstationarity is a foremost issue in causality inference. Christiano and Ljungqvist (1988) computed a simulation experiment and found that drawing inference from the GC test with nonstationary variables-unit root with drift processes-may result in a non-standard distribution of the $\mathcal{F}$-statistic under the null hypothesis of no Granger-Causality. The authors further argue that when working with differenced variables, the GC may reflect the lack of power to detect GC but not GC itself. Sims, Stock, and Watson (1990) provide a theoretical basis for such findings: they assert that, when

[1] Assumption taken by Granger on his definition of causality (Granger 1969, Granger, Swanson, Watson, and Ghysels 2001).

[2] For a thorough study on the key differences between these, see (Lechner 2006) and (Heckman 1999).

[3] Wiener (1956), Granger (1969), and (Sims 1972).

the variables are not cointegrated and, moreover, are dominated by a deterministic trend, the $\mathcal{F}$ statistic does not converge to a standard distribution and includes nuisance parameters. Furthermore, if the variables do cointegrate, the $\mathcal{F}$-statistic converges to a standard distribution.[4] The problem of non-stationary variables in the performance of GC tests is best illustrated in the literature concerning the neutrality of money. There are a considerable number of works dealing with the money-output dynamic relationship. Hayo (1999), for instance, asserts that there is statistical evidence that money Granger-causes output, whether such inference is drawn from level or differentiated series. In contrast, Christiano and Ljungqvist (1988) argue that such a causal relationship can only be inferred by using log-level variables. Stock and Watson (1987) acknowledge the perplexing and sometimes contradictory results obtained in the literature-even when the specifications differ only slightly. By applying Sims, Stock, and Watson's (1990) results, the authors do not find evidence to support the neutrality of money (Mehra 1989). Since Perron's (1989) seminal paper, structural breaks have also played an important role in the unit-root testing literature. The existence of such breaks biases the usual tests for a unit root toward nonrejection. Deterministic-possibly broken-trends are not the sole possible DGP; the problem arises even in mean stationary series with level shifts, as documented in Perron and Vogelsang (1992). Such BMS processes may be useful in the study of the Power Purchasing Parity hypothesis, as well as in the evaluation of the behavior of price index time series in countries where a targeting policy is applied.

Stationarity and shifts are thus key elements in GC test literature. Most works dealing with GC use the ADF testing procedure to evaluate the stationarity of the variables (Mehra 1989, Foresti 2006, Hayo 1999, Giles, Tedds, and Werkneh 2002, McCrorie and Chambers 2006, Chao, Corradi, and Swanson 2001, Stock and Watson 1987). Therefore, in the next section, we study the stationarity inference drawn from a Dickey-Fuller test when the DGP is a BTS, a differenced BTS, and a BMS. We then study the asymptotic properties of the GC test when the variables used to perform it are generated by those same DGPs.

[4] Dufour and Renault (1998) provide an excellent explanation of the theory underlying Granger-causality. They further research the notion of Granger-Causality where the horizon of predictability of a vector of variables is an arbitrary number of periods ahead (Lutkepohl 1993).

2 Asymptotic Results

We focus on two empirically relevant–in economics-DGPs: Broken-Trend Stationarity process (BTS), and Broken-Mean Stationary process (BMS), defined in equations (1), and (2), respectively. These processes have been used to model growing variables, both real and nominal, such as output, consumption, prices, and exchange rates (Perron 1989, Perron 1997, Lumsdaine and Papell 1997, Mehl 2000):

$$w_t = \mu_w + \beta_w t + \gamma_w DT_{wt} + u_{wt} \quad (1)$$
$$w_t = \mu_w + \theta_w DU_{wt} + u_{wt} \quad (2)$$

Where μ_w, β_w, and γ_w are constants, u_{wt} is white noise, and DU_{wt} and DT_{wt} are dummy variables allowing changes in the level and the slope, respectively; that is, $DU_{wt} = (1)\mathbf{1}(t > T_{b_w})$ and $DT_{wt} = (t - T_{b_w})\mathbf{1}(t > T_{b_w})$, where $\mathbf{1}(\cdot)$ is the indicator function, and T_{b_w} is the unknown date of the break in w. We denote the break fraction as $\lambda_w = (T_{b_w}/T) \in (0, 1)$, where T is the sample size.
In the previous section, we documented the extended use of the DF to pretest variables prior to the GC test. The DF test specification may vary, for which reason we study three different variations that are commonly practiced:

$$\Delta w_t = \tau_1 w_{t-1} + u_t \quad (3)$$
$$\Delta w_t = \alpha + \tau_2 w_{t-1} + u_t \quad (4)$$
$$\Delta w_t = \alpha + \tau_3 w_{t-1} + \delta t + u_t \quad (5)$$

Since Perron's (1989) seminal article, it has been well-known that structural breaks bias the DF test, which in turn over-accepts the null hypothesis of unit root. We investigate further along those lines, by studying the behavior of such bias:

Proposition 1 *Let y_t be a series generated by DGP (1) and the three DF specifications, (3,4, and 5), be estimated. We denote the DF t-statistic associated as $t_{\hat{\tau}_i}$ for $i = 1, 2, 3$; then as $T \longrightarrow \infty$:*

1. *Specification without trend or intercept (eq. 3): $t_{\hat{\tau}_1} = O_p\left(T^{1/2}\right)$; the sign of the asymptotic value of the t-statistic is always positive.*

2. *Specification with intercept (eq. 4):* $t_{\hat{\tau}_3} = O_p\left(T^{1/2}\right)$*; the sign of the asymptotic t-statistic depends upon the values of the parameters in DGP (1).*

3. *Specification with trend and intercept (eq. 5):*

   - *When* $\lambda \neq 0.5$, $t_{\hat{\tau}_3} = O_p\left(T^{1/2}\right)$ *and the sign of the asymptotic value of the t-ratio is positive (negative) when the break is in the first (second) half of the sample.*
   - *When* $\lambda = 0.5$, $t_{\hat{\tau}_3} = O_p\left(T^{-1/2}\right)$.

Where T *is the size of the sample.*

Proof: See Appendix A
Proposition 1 asserts that the BTS process may, under many circumstances, be mistakenly considered as a unit-root process; the t-ratio diverges to infinity in the specification with neither trend nor intercept, which will lead us, as the sample grows, to accept the null hypothesis of a unit root in the process. When the DF specification that includes an intercept is used, the asymptotic behavior of the t-test is unreliable; it might accept or reject the null hypothesis of a unit root depending on the values of DGP (1), as we can see in figure (1).

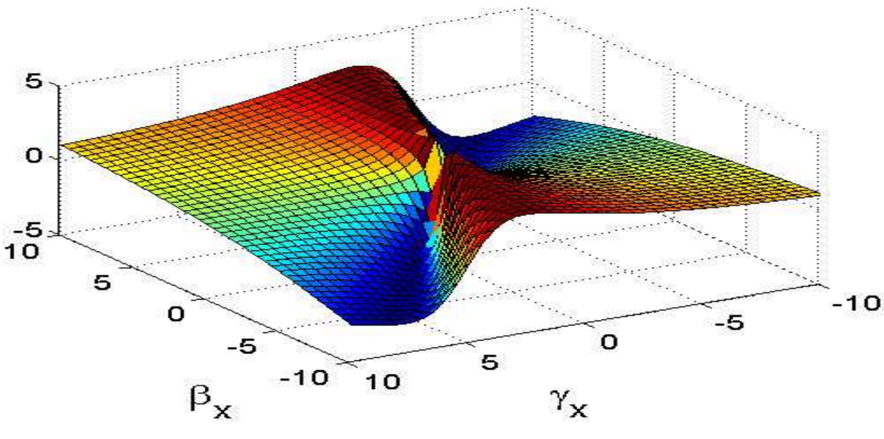

Figure 1: $t_{\hat{\tau}_2}$ against β_x and γ_x in DGP (1) when $\lambda = 0.36$.

Finally, in the specification with trend and intercept, the t-ratio diverges to infinity except in the unlikely event of a break located

exactly in the middle of the sample.[5] Whichever the case, the acceptance or rejection of the null hypothesis is dependent upon the location of the break. The test is thus unreliable under this DF specification.
It might be expected that anyone seeking to find stationary series by using a DF test might spuriously find unit roots processes and thus difference the series in order to render these stationary. First-differenced BTS series are then:

$$\Delta w_t = \beta_w + \gamma_w DU_{w,t} + \epsilon_{x,t} \tag{6}$$

for $w = x, y$, where $DU_{wt} = \mathbf{1}(t > T_{b_w})$ and $\epsilon_{w,t} = (u_{w,t} - u_{w,t-1})$
We then obtain the asymptotics of the DF test on the differenced series:

Proposition 2 *Let w_t be a series generated by equation (1) and first-differenced as in eq. (6); denote $t_{\hat{\tau}_i}$ for $i = 1, 2, 3$ the Dickey-Fuller t-ratio associated to $\hat{\tau}_i$ in equations, (3), (4), and (5). Then, as $T \longrightarrow \infty$ $t_{\hat{\tau}_i} = O_p\left(T^{1/2}\right)$ for $i = 1, 2, 3$. Moreover, the sign of the asymptotic value of $t_{\hat{\tau}_i}$ is always negative.*

Proof: See Appendix A
The above proposition indicates that, asymptotically, one would reject the hypothesis of a unit root in the differenced BTS series. As with differenced BTS processes, the same conclusion is drawn when the underlying DGP is a BMS. Further details can be found in Ventosa-Santaularia and Gomez (2006).

3 Granger-Causality

The basic methodology of the GC test in a bivariate case can be found in most text books. A time-series variable, x_t, is said to fail to Granger-Cause another variable, y_t, if the Mean-Squared Error (MSE) of a forecast of y_{t+s}, based on $\Psi_t^{yx} = \{x_t, x_{t-1}, \ldots, y_t, y_{t-1}, \ldots\}$ is equal to the MSE of a forecast based on $\Psi_t^{y} = \{y_t, y_{t-1}, \ldots\}$. In practice, as mentioned previously, GC tests are commonly instrumented by means of in-sample $\mathcal{F}$-tests. Due to of computational

[5] When the break is in the middle of the sample, the statistic collapses to zero. This would lead us to reject the null hypothesis and determine that there is a unit root in the series.

limitations, we focus on the most simple GC regression specification test:

$$v_t = \gamma_1 v_{t-1} + \gamma_2 z_{t-1} + u_{ut} \quad (7)$$
$$v_t = \gamma_1 v_{t-1} + u_{rt} \quad (8)$$

Where v_t, z_t are generated indistinctly by DGPs (1) and/or (2); the former DGP may or may not be differenced as in (6). Eq. (7) accounts for the unrestricted model whilst eq. (8) is the restricted model. A "multiple" linear-restriction test, i.e. an $\mathcal{F}$ test, is then calculated on the hypothesis $\mathcal{H}_0 : \gamma_2 = 0$ against the alternative, $\mathcal{H}_A : \gamma_2 \neq 0$. The test statistic is constructed as follows:

$$\mathcal{F} = \frac{RRSS - URSS}{URSS/(T-1)} \quad (9)$$

where $RRSS$ and $URSS$ are the sum of least squares residuals from the restricted and the unrestricted models, respectively. We study the asymptotic performance of the GC test when the variables are generated either by DGP (1) or (2) and obtain the following results:

Theorem 1 *Let y_t, and x_t be two independent series generated indistinctly by DGPs (1) or (2); the former may or may not be differentiated as in eq. (6); let the $\mathcal{F}$ statistic (eq. 9) be constructed based on the sum of residuals in (7) and (8). Then, as $T \to \infty$:*

$$\mathcal{F} = O_p(T)$$

Proof: See Appendix A
The properties of the $\mathcal{F}$-statistic ensure its positiveness; therefore, according to theorem (1), we will reject the null hypothesis of no Granger-causality between the variables in all cases as the sample size grows.
We present finite-sample evidence of the behavior of the GC test. This Monte Carlo simulation exercise is twofold. On the one hand, the $\mathcal{F}$-statistic associated with the GC test is computed and compared with the corresponding critical value at a 5% level. Surprisingly, nonsense rejection of the null hypothesis of no Granger-Causality occurs almost systematically with such small sample sizes,

i.e. those of 50 observations or fewer [see Table (1)]. On the other hand, we computed an experiment where the "Granger-caused" variable pertains to the historical Nelson and Plosser macroeconomic dataset. More precisely, we applied the GC to test the hypothesis that a simulated BTS variable (whether differentiated or not) Granger-causes either the Real GNP or the monetary aggregate M_2. The experiment, repeated $10,000$ times, revealed that it is highly possible to reject the null of no Granger-Causality between these variables and simulated data [see Table (1)].The specification of the Monte Carlo experiment, together with the details of the historical dataset, can be found in appendix B.

T	50	100	250	500
BTS vs BTS	0.99	1.00	1.00	1.00
ΔBTS vs BTS	1.00	1.00	1.00	1.00
BTS vs ΔBTS	0.37	0.98	1.00	1.00
ΔBTS vs ΔBTS	0.93	1.00	1.00	1.00
Real and simulated data				
GNP vs BTS	1.00			
ΔGNP vs BTS	1.00			
ΔGNP vs ΔBTS	1.00			
M_2 vs BTS	1.00			
ΔM_2 vs BTS	1.00			
ΔM_2 vs ΔBTS	1.00			

Table 1: Rejection rates of the $\mathcal{F}$-statistic GC test.

4 Concluding Remarks

As a limited concept of causality, circumscribed to the predictability of a variable relative to the information included in another, Granger-Causality has proved to be a remarkably useful tool in applied economics. It provides an important methodological tool in the answering of the typical scientific question "What happens to y when x moves?" Here we have proved that a simple GC test fails to accept the null hypothesis of no Granger-Causality between two independent Broken-Trend or Broken-Mean time series, whether the former series are differentiated or not. Hence, it should be clear that such tests should not be used when the series appear to be generated by this DGP. Sadly, it is also well-documented that the most popular unit-root test, the Dickey-Fuller, provides biased results when the true DGP is a BTS. Further unit-root research using more sophisticated tests is therefore recommended, although the identification of structural breaks remains elusive. Hence, a warning call becomes evident: pre-testing for unit roots and deterministic trends (with breaks) is imperative before drawing inference as regards Granger-Causality.

A Proof of Propositions 1 and 2, and Theorem 1

We present a guide on how to obtain the order in probability of a certain combination of DGPs; namely x_t and y_t are generated by DGP (1). The expressions needed to compute the asymptotic value of the GC $\mathcal{F}$ statistic are:[6]. Since x_t is independent from y_t, we do not use the lag of x_t to calculate the asymptotics, but rather the variable in levels, with no loss of generality.

[6]All sums are from $t = 1$ to T unless otherwise specified. $z = x, y$; $\lambda_d = \lambda_x - \lambda_y$; we assume, for simplicity, that $\lambda_x > \lambda_y$.

$$
\begin{aligned}
\sum t &= \frac{1}{2}\left(T^2 + T\right) \\
\sum t^2 &= \frac{1}{6}\left(2T^3 + 3T^2 + T\right) \\
\sum DT_{zt} &= \frac{1}{2}\left[T^2\left(1-\lambda_z\right)^2 + T\left(1-\lambda_z\right)\right] \\
\sum DT_{zt-1} &= \sum DT_{zt} - T\left(1-\lambda_z\right) \\
\sum DT_{zt}^2 &= \frac{1}{6}\left(2T^3\left(1-\lambda_z\right)^3 + 3T^2\left(1-\lambda_z\right)^2 + T\left(1-\lambda_z\right)\right) \\
\sum DT_{zt-1}^2 &= \sum DT_{zt}^2 - T^2\left(1-\lambda_z\right)^2 \\
\sum DT_{zt}t &= \lambda_z T \sum DT_{zt} + \sum DT_{zt}^2 \\
\sum DT_{zt-1}t &= \sum DT_{zt}t - \lambda_z T^2\left(1-\lambda_z\right) - \sum DT_{zt} \\
\sum DT_{zt}DT_{zt-1} &= \sum DT_{zt}^2 - \sum DT_{zt} \\
\sum DT_{yt}DT_{xt} &= \lambda_d T \sum DT_{xt} - \sum DT_{xt}^2 \\
\sum DT_{yt-1}DT_{xt} &= \sum DT_{yt}DT_{xt} - \sum DT_{xt}
\end{aligned}
$$

As for the stochastic sums, most results can be found in Phillips (1986), Durlauf and Phillips (1988) and Phillips and Ouliaris (1990). The remainder can be easily obtained:

$$
\begin{aligned}
\sum u_{yt-1} &= \sum u_{yt} - u_{yT} \\
\sum u_{yt-1}^2 &= \sum u_{yt}^2 - u_{yT}^2 \\
\sum u_{yt-1}t &= \sum u_{yt}t + \sum u_{yt} - u_{yT} - Tu_{yt} \\
\sum DT_{yt-1}u_{yt-1} &= \sum DT_{yt}u_{yt} - T\left(1-\lambda_y\right)u_{yT} \\
\sum DT_{yt-1}u_{yt} &= \sum DT_{yt}u_{yt} - \sum_{t=\lambda_y T+1}^{T} u_{yt} \\
\sum DT_{yt}u_{yt-1} &= \sum DT_{yt}u_{yt} + u_{y\lambda_y T} + \sum_{t=\lambda_y T+1}^{T} u_{yt} - \\
&\quad u_{yT} - T\left(1-\lambda_y\right)u_{yT}
\end{aligned}
$$

These elements allow for the programming of all those sums required to study the asymptotic behavior of the restricted and unrestricted regressions: $\sum y_t^2$, $\sum y_t x_t$, $\sum y_t y_{t-1}$, $\sum y_{t-1}^2$, and, $\sum y_{t-1} x_t$; For example:

$$\begin{aligned}\sum y_t^2 &= \mu_y^2 T + \beta_y^2 \sum t^2 + \gamma_y^2 \sum DT_{yt}^2 + \underbrace{\sum u_{yt}^2}_{O_p(T)} + 2\mu_y\beta_y \sum t \\ &+ 2\mu_y\gamma_y \sum DT_{yt} + 2\mu_y \underbrace{\sum u_{yt}}_{O_p\left(T^{\frac{1}{2}}\right)} + 2\beta_y\gamma_y \sum DT_{yt} t \\ &+ 2\beta_y \underbrace{\sum u_{yt} t}_{O_p\left(T^{\frac{3}{2}}\right)} + 2\gamma_y \underbrace{\sum DT_{yt} u_{yt}}_{O_p\left(T^{\frac{3}{2}}\right)}\end{aligned}$$

These programs can be downloaded from:
http://www.ventosa-santaularia.com/GC_vsvv_2008.zip.

B Data Generating Processes of the Simulations

The parameter values used for each simulation included in this article are as follows:

Figure 1:	$x_t = \mu_x + \beta_x t + \gamma_x DT_{xt} + u_{xt}$
	$\mu_x = 7$; $\beta_x \in [-10, 10]$; $\gamma_x \in [-10, 10]$
	$\lambda_x = 0.36$; $u_{xt} \sim \mathcal{N}(0, 1)$
Table 1:	$w_t = \mu_w + \beta_w t + \gamma_w DT_{wt} + u_{wt}$
	$\mu_x = 0.75$; $\mu_y = 0.5$; $\beta_x = 0.03$
	$\beta_y = 0.05$; $\gamma_x = -0.01$; $\gamma_y = 0.02$
	$\lambda_x = 0.3$; $\lambda_y = 0.7$; $u_{wt} \sim \mathcal{N}(0, 1)$

The variables GNP (1909-1988) and M_2 (1889-1988) correspond to the Real Gross National Product (Billions of 1958 dollars) and the money aggregate (Billions of dollars, annual averages), respectively. Both series are in natural logs. We use the Nelson and Plosser macroeconomic dataset updated to 1988 by Herman van Dijk, which can be found at the JBES 1994 dataset archives. The data are annual.

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