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# Drift and equilibrium selection with human and computer players

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## *Abstract*

The theory of drift (Binmore and Samuelson 1999) concerns equilibrium selection in which second-order disturbances may have first-order effects in the emergence of one equilibrium over the other. We provided experimental evidence with human players supporting the model in Caminati, Innocenti and Ricciuti (2006). In this paper we test it with conditioning by computer players. When computers are removed and humans are matched against each other, the comparative static properties of the model are confirmed.

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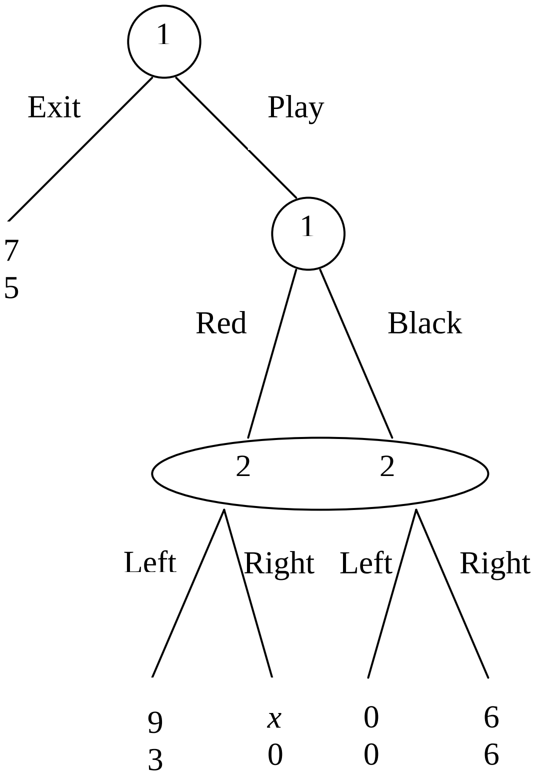
## 1. Introduction

A now standard argument used to select between multiple Nash equilibria in non-cooperative game situations, amounts to introducing perturbations (Selten 1975). In the context of evolutionary game theory (Maynard Smith 1982), perturbations may not concern the game itself, but only the learning process which brings the players to play equilibrium strategies. Convergence outcomes and their speed come to depend on the way in which learning is affected by disturbances. Errors may take the form of small stochastic shocks (Young, 1993, Kandori, Mailath and Rob 1993, Ellison 1993), or the less innocent form of systematic mistakes arising from a pay-off dependent, superficial scrutiny of the game influenced by ‘similar’ game situations experienced in life. Specifically, the theory of drift (Binmore and Samuelson 1999) suggests that the out-of-equilibrium states that are observed in repeated plays of a game reflect the dynamic coupling of different factors: *(i)* initial conditions sustained by prior beliefs about the behaviour of opponents; *(ii)* population ‘learning’ reflecting the frequency increase in each subpopulation of players of the strategies that are more profitable, given the available information on the actual play of the game; *(iii)* drift; that is, the expected subpopulation-average ‘mistake’ in a given game situation; *(iv)* residual stochastic perturbations.

This paper reports on experimental evidence concerning a modified version of the Dalek game in which the theory of drift predicts the conditions under which a strategy distribution in the population of players is stabilized near a Nash equilibrium component $E$ that is not locally asymptotically stable in the learning dynamics *(ii)*. Since stabilization requires that drift is compatible with (points to the interior of) $E$, the theory predicts that selection outcomes depend on the measure of $E$. A problem arising in this context is that the evidence for stabilization suggested by the data may not be entirely explained by the drift component *(iii)*; it may be also caused by the fact that the same pay-off change designed to alter the size of $E$ between experimental sessions, may also affect the initial conditions of the game and select them in the basin of attraction of a different equilibrium. To avoid the distortion brought by the dependence of initial conditions on payoffs, in our experiment the proper play of the game is preceded by a phase in which every subpopulation of players is conditioned to play a specific set of equilibrium strategies through a repeated matching with a virtual opponent.

## 2. Experimental design

The modified Dalek game proposed by Binmore and Samuelson (1999) is shown in the Figure below.



Player 1 moves first choosing either to Exit or to Play. In the first case the game ends with payoffs 7 for player 1 and 5 for player 2. In the second case player 1 makes the second choice between Red and Black and player 2 chooses between Left and Right without knowing the choice taken by player 1.

We considered two different game versions: in the first one, called A, the payoff $x$ of the strategy profile (Play-Red, Right) is 0; in the second one, called B, it is 7. Both games have the same two subgame perfect equilibria, (Play-Red, Left) and (Exit, Right). Binmore and Samuelson predict that the equilibrium (Exit, Right) will emerge more frequently when $x$ is small than when $x$ is large, and that the opposite will hold for (Play-Red, Right). Correspondingly, drift is less likely to be compatible with E in the former case.

To test this prediction we introduced computers programmed to play two different strategies. In treatments denoted as A1 and B1 computers conditioned humans to play (Play-Red, Left), while in treatments A2 and B2 to play (Exit, Right). Computers always played the predetermined strategy ("full conditioning") to make the conditioning process as effective as possible. We informed subjects that they would start playing against computers and that computers would be removed during the experiment in an indefinite period.

A bothersome feature of the experimental design was that players 1 could exit and stop the game at the first stage. To avoid that players 2 were inactive for all the experiment when matched with a computer programmed to Exit we decided to make subjects play both roles.[1]

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[1] This methods differs from Roth and Shoumaker (1983) and Binmore et al. (1993) in which subjects play the conditioned strategy with a probability lower than 1 (and fixed roles), but in these cases exit was not one of the strategies played by the computer. This problem was solved in Caminati, Innocenti and Ricciuti (2006) by imposing timing without observability, i.e., the two players chose simultaneously but the player 2 was not informed of player 1's choice.

Subjects were drawn from the undergraduate population at Royal Holloway, University of London during the academic year 2003/04. We ran five sessions of game A and five sessions of game B. Each session lasted 60 periods and included twelve people, six being directed to equilibrium (Play-Red, Left) and six to equilibrium (Exit, Right). Computers were actually removed after 25 periods. Each session lasted approximately an hour) and the average payoff was about £ 9.50. During the sessions the subjects were seated at computer terminals in separate seats to prevent communication or visual contact among them. The experiment was computerized using the software Z-tree (Fishbacher 1999).[2]

## 3. Results

Following Binmore and Samuelson's theory, we tested the following two hypotheses:

a) in treatments A1 and A2, in which $x$ is equal to 0, and in treatment B1, with x equal to 7, subjects continue to play the conditioned strategies even after computers are removed;

b) in treatment B2, with $x$ equal to 7, the conditioned strategy (Exit, Right) is displaced by (Play-Red, Left) even after computers are removed.

Experimental results are shown in Table 1.

**Table 1 Experimental results (percentage of plays)**

| Treatment | Players | Conditioning | Prediction | Plays | Period 1 | Period 25 | Period 60 |
|---|---|---|---|---|---|---|---|
| A1 | Player 1 | Red | Red | Exit | 43.0 | 9.7 | 13.3 |
| | | | | Red | 49.4 | 88.9 | 86.7 |
| | | | | Black | 7.6 | 1.4 | 0.0 |
| | Player 2 | Left | Left | Left | 30.8 | 58.3 | 63.6 |
| | | | | Right | 69.2 | 41.7 | 34.3 |
| A2 | Player 1 | Exit | Exit | Exit | 84.0 | 96.0 | 70.7 |
| | | | | Red | 11.3 | 2.0 | 26.7 |
| | | | | Black | 4.7 | 2.0 | 2.6 |
| | Player 2 | Right | Right | Left | 47.4 | 25.0 | 36.4 |
| | | | | Right | 52.6 | 75.0 | 63.6 |
| B1 | Player 1 | Red | Red | Exit | 14.9 | 4.0 | 6.9 |
| | | | | Red | 85.1 | 92.0 | 93.1 |
| | | | | Black | 0.0 | 4.0 | 0.0 |
| | Player 2 | Left | Left | Left | 85.1 | 98.8 | 98.7 |
| | | | | Right | 14.9 | 1.2 | 1.3 |
| B2 | Player 1 | Exit | Red | Exit | 68.0 | 83.6 | 3.0 |
| | | | | Red | 30.5 | 16.4 | 97.0 |
| | | | | Black | 1.5 | 0.0 | 0.0 |
| | Player 2 | Right | Left | Left | 33.2 | 27.3 | 98.2 |
| | | | | Right | 66.8 | 73.7 | 1.8 |

By inspection, the first result is that the conditioning process was quite effective in bringing humans towards the desired choice. The second result is that in the treatments in which we expected the conditioned outcome to be stable (A1, A2, B1), it actually was. Finally, in experiment B2 players 1 conditioned to play Exit drastically changed in favour of Red when computers were removed and players 2 did actually play Left, as required by our hypothesis (b). Figures 1 and 2 show the choices of human players grouped in 12 sets of 5

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[2] Instructions are available upon request.

plays each. After computers' removal indicated by the vertical line, subjects converge very quickly on the predicted strategy profile Left-Red.

**Fig. 1 Choices of players 1 in treatment B2**

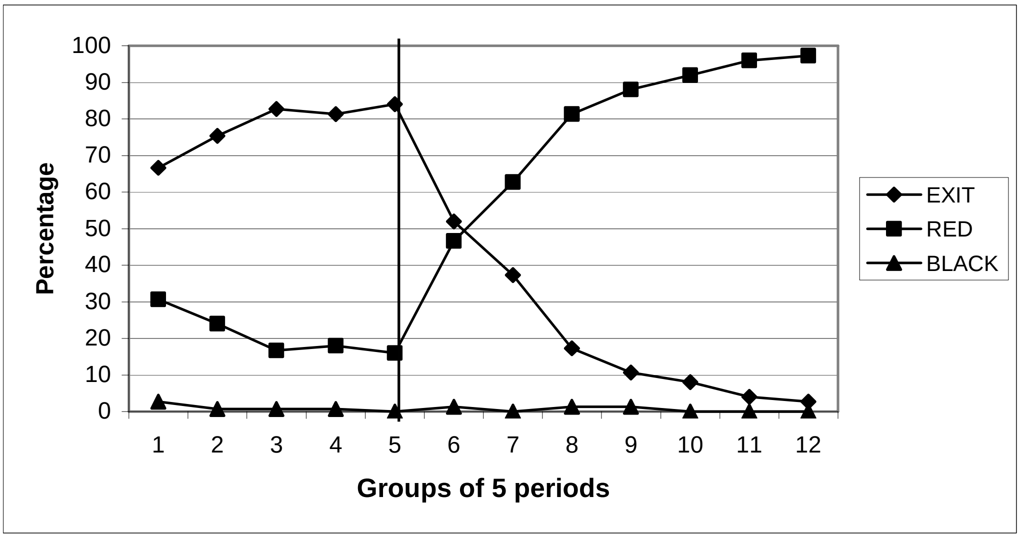
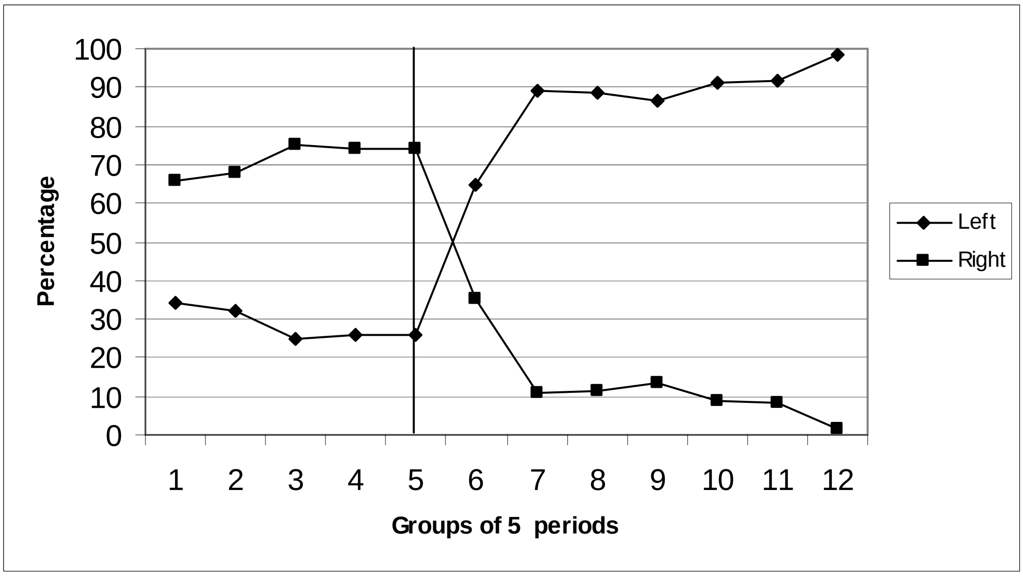


**Fig. 2 Choices of players 2 in treatment B2**



A formal test of the comparative statics implied by our hypotheses can be done by applying a $\chi^2$ test for independent samples. In this case we modify the previous hypotheses in the following way:

a*) there is a significant difference in the number of players 1 choosing Exit and Play-Red in treatments A1 and A2. There is no significant difference in the number of players 1 choosing Exit and Play-Red in experiments B1 and B2.

b*) there is a significant difference in the number of players 2 choosing Left and Right in treatments A1 and A2. There is no significant difference in the number of players 2 choosing Left and Right in experiments B1 and B2.

In this case we have two groups (represented by a pair of treatments) and either three (for players 1) and two (for players 2) possible choices. The null hypothesis states that the two groups do not differ with respect to the relative frequency with which group members fall in several categories. To test this hypothesis we count the number of cases from each group which fall in the various categories, and compare the proportion of cases from one group in the various categories with the proportion of cases from the other group. The $\chi^2$ has degrees of freedom equal $df = (r - 1)(k - 1)$ where $r$ is the number of rows and $k$ is the number of columns. The number of columns is always equal to two, while the number of rows is equal to three (Exit, Red, and Black) for players 1, and equal to two (Left and Right) for players 2. Therefore the numbers of degrees of freedom are two and one, respectively.

When we consider the full conditioning treatment, the statistics concerning the distribution of players 1 in treatments A1 and A2 is equal to 52.83. This is greater than the critical value associated with 0.001 for a $\chi^2$ with 2 degrees of freedom (13.82). Therefore, the null of the test is rejected, in accordance to our hypothesis (a*). The opposite applies when comparing players 1 in treatments B1 and B2: we cannot reject the null of no difference between the two treatments since the test statistics is equal to 4.45, which is smaller than the critical value associated with the 0.1 significance level with 2 degrees of freedom (4.60). Again this support hypothesis (a*).

The same applies when we test hypothesis b*: the test values for the difference between players 2 between treatments A1 and A2, on the one hand, and B1 and B2, on the other hand, are 1.08 and 1.93 respectively, which are smaller than the critical values of the $\chi^2$ with 1 degree of freedom associated with significance levels of 0.20 (1.64) and 0.10 (2.71). Therefore in the first case the null of no difference is rejected, while in the second case it is accepted: in both cases our hypothesis (b*) is supported by data.

## 4. Conclusions

This paper has tested in the laboratory the theory of drift that concerns equilibrium selection in which second-order disturbances may have first-order effects in the emergence of one equilibrium over the other. We have matched human players with computer players programmed to bring some players to one equilibrium and the remaining players to the other equilibrium. Then computerised players have been removed and humans matched against each other. Our findings are consistent with the theory. When we slightly modify game payoffs just to cause a smaller set of player-2 play distributions supporting Exit as player-1 equilibrium choice, drift becomes less compatible with the outcome (Exit, Right). This equilibrium tends to be displaced even in presence of the divergent conditioning by computer players.

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