E C O N O M I C S   B U L L E T I N

On the possibility of licensing in a market with logit demand functions

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Abstract

We analyze the incentives for technology transfer between two firms in a market characterized by a logit demand framework. The available licensing policies of the incumbent innovator are the up front fee, royalty and two-part tariff policies. We show that when the market is covered there is no equilibrium where technology transfer occurs.

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1. Introduction

It is well known that the owner of a patented cost-reducing innovation may increase the return of his innovation by allowing his competitors in the market to acquire the new technology. In most cases, the transfer of technology is based on a licensing policy that includes royalties as then the innovator maintains a competitive advantage over his rivals. The purpose of this note is to identify a simple context where such a possibility for technology transfer does not exist –irrespective of the licensing policy used.

The patent licensing literature was initiated by Arrow (1962) who analyzed licensing of a cost-reducing innovation in a perfectly competitive industry and in a monopolistic industry. Subsequently, Katz and Shapiro (1985, 1986) and Kamien and Tauman (1984, 1986) analyzed optimal licensing for the oligopoly case. Later studies considered models with differentiated goods (Muto 1993, Faulí-Oller and Sandonís 2002), asymmetric information (Gallini and Wright 1990, Macho-Stadler and Pérez-Castrillo 1991, Beggs 1992), strategic delegation, (Mukherjee 2001, Saracho 2002), Stackelberg leader-follower (Filippini 2005).

Licensing of an innovation by an incumbent innovator was first discussed by Shapiro (1985). Wang (1998) analyzed the optimal policy of an incumbent innovator in a Cournot duopoly with homogeneous goods and showed the superiority of the royalty policy over the fee policy. Kamien and Tauman (2002) extended Wang's model for the case of an arbitrary number of firms. Fauli-Oller and Sandonis (2002) examined two-part tariff policies in duopoly models with differentiated goods and showed that the incumbent innovator licenses to his rival even a drastic innovation. Filippini (2005) examined a model where the incumbent innovator acts as the Stackelberg leader and showed that the optimal policy includes only royalty. Sen and Tauman (2007) analyzed two-part tariff policies for an incumbent innovator in a Cournot oligopoly with n firms and showed that the optimal policy depends on both the number of firms and the magnitude of the innovation.

In this note we examine the licensing of a quality-improving innovation by an incumbent innovator too. We focus on a duopoly market where consumers preferences depend on the quality level of the products and on an idiosyncratic random term distributed according to a double exponential distribution. This framework gives rise to the logit demand framework. The innovator either uses the new technology on his own or can license it to his rival as well. The licensing policies available for the latter purpose are the up front fee, royalty and two-part tariff policies.

We restrict attention to the covered market case, i.e., the case where all consumers purchase one of the two products in the market. We show that there is no equilibrium where transfer of technology occurs between the two firms. In particular, we show that whenever the licensing policy involves royalties, the resulting price equilibrium is such that the problem of computing the optimal royalty has no solution. On the other hand, whenever restricted to a mere up front fee policy, the innovator finds it optimal not to license the new technology to his competitor.

2. The Model

We consider a market with N consumers and two firms. Each consumer in the market purchases one unit of just one of the two products that firms offer. Consumer m's evaluation for the product of firm i is given by

$$V_{mi} = y + \theta s_i - p_i + \epsilon_{mi}$$

where y is the income of the consumer, s_i is the quality of the product of firm i, $i = 1, 2$, θ is the marginal valuation of quality (common for all consumers), p_i is the price charged by firm i and ϵ_{mi} is a random term distributed according to the double exponential distribution

$$F(x) = Pr(\epsilon_{mi} \leq x) = \exp\{-\exp -[(x/\mu) + \gamma]\}$$

where γ and μ are positive constants. The ϵ_{mi} random variables are independent across both products and consumers, namely for every consumer m, ϵ_{m1}

and ϵ_{m2} are mutually independent and for every product i consumers obtain independent signals $\epsilon_{1i}, \epsilon_{2i}, ..., \epsilon_{Ni}$. The mean and variance of the random term are given respectively by $E(\epsilon_{mi}) = 0$ and $Var(\epsilon_{mi}) = \mu^2\pi^2/6$. Hence, parameter μ is a measure of the dispersion of consumers' preferences and expresses the degree of horizontal differentiation in the market.

Consumer m observes the realization of ϵ_{mi}, $i = 1, 2$ and selects the product which maximizes his net utility. The probability that m will select the product of firm i is $Pr(V_{mi} \geq V_{mj})$. In the absence of an outside (no-purchase) option, the expected demand of firm i is given by the following logit formula (see Anderson et.al, 1992)

$$d_i(p) = \frac{\exp\left[(\theta s_i - p_i)/\mu\right]}{\sum_{j=1}^{2} \exp\left[(\theta s_j - p_j)/\mu\right]} N, \qquad i, j = 1, 2, \;\; i \neq j \tag{1}$$

where $p = (p_1, p_2)$.

The firms in the market compete in prices. The marginal cost of production of firm i is independent of its quantity but depends on the quality level of its product, namely $c_i = c(s_i)$, $i = 1, 2$, where c_i denotes the marginal cost of firm i. Fixed costs are zero. Prior to any innovative activity firms produce commodities of the same quality, $s_1 = s_2 = s$.

Let now firm 1 innovate (in a costless way) a product of higher quality $s^* > s$. To guarantee that the new quality level is profit-enhancing we need to assume that the quality-marginal cost differential (of any firm that uses the new quality) increases in quality, i.e.,

$$\theta s' - c(s') > \theta s - c(s), \quad \forall\, s' > s \tag{A1}$$

Firm 1 can either use the new technology exclusively or it could license it to firm 2 as well. The relevant decision of firm 1 is embedded in the following three-stage game. In the first stage, firm 1 decides whether to license or not to firm 2 and in the former case it decides on the form of the licensing policy. We consider the licensing policies of royalty, up front fee and two-part tariffs.

If firm 1 does offer a licensing contract (based on one of the three policies) then in the second stage firm 2 responses to the offer. Finally, in the last stage, and given the outcomes of the previous stages, the two firms engage in simultaneous price competition. Let G denote this interaction. Let also G_{fr}, G_r and G_f denote the sub-games of G induced by the two-part tariff, royalty and up front fee policies respectively. Finally let G_{-} denote the sub-game of G corresponding to the case where licensing does not occur. In the next section we analyze the price competition stage of each of these games.

2.1 The price stage

Let firm i enter the price stage with quality level s_i. Denote by $c_i = c(s_i)$ its marginal cost, $i = 1, 2$. Let

$$x_i = \exp[(\theta s_i - p_i)/\mu], \quad i = 1, 2 \tag{2}$$

By (1) and (2) the demand functions of the two firms are

$$d_1(p) = \frac{x_1}{\sum_{k=1}^{2} x_k} N, \qquad d_2(p) = \frac{x_2}{\sum_{k=1}^{2} x_k} N \tag{3}$$

The payoff functions are

$$\pi_1(p) = (p_1 - c_1)\frac{x_1}{\sum_{k=1}^{2} x_k} N, \qquad \pi_2(p) = (p_2 - c_2)\frac{x_2}{\sum_{k=1}^{2} x_k} N \tag{4}$$

It can be verified that equilibrium prices are given by the unique solution of the system[1]

$$p_1 = c_1 + \mu + \mu\frac{x_1}{x_2}, \;\; p_2 = c_2 + \mu + \mu\frac{x_2}{x_1} \tag{5}$$

where x_1 and x_2 are given by (2). Let $p_1^*(s_1, s_2)$, $p_2^*(s_1, s_2)$ be the unique solution[2] of the system in (5). By (2) – (5) equilibrium profits are

$$\pi_1^*(s_1, s_2) = \mu N\frac{x_1^*}{x_2^*}, \qquad \pi_2^*(s_1, s_2) = \mu N\frac{x_2^*}{x_1^*} \tag{6}$$

[1]See Anderson et.al (1992).

[2]A closed-form solution for the equilibrium prices of the logit model cannot be computed *unless* firms are symmetric.

where x_1^* and x_2^* are evaluated at $p_1^*(s_1, s_2), p_2^*(s_1, s_2)$ (using (2)).

Notice that in G_-, $s_1 = s^*$ and $s_2 = s$. On the other hand, in G_f we have $s_1 = s_2 = s^*$. In this case it is easy to show that

$$p_1^*(s^*, s^*) = p_2^*(s^*, s^*) = c^* + 2\mu, \qquad \pi_1^*(s^*, s^*) = \pi_2^*(s^*, s^*) = \mu N \tag{7}$$

where $c^* = c(s^*)$.

Consider next the price stage under the assumption that firm 1 has licensed the new technology to firm 2 using a royalty rate r (which could be accompanied or not by an up front fee). The payoff function of firm 1 in the price stage of *either* G_{fr} *or* G_r is

$$\pi_1(p) = (p_1 - c^*)\frac{x_1}{\sum_{k=1}^2 x_k}N + r\frac{x_2}{\sum_{k=1}^2 x_k}N,$$

while that of firm 2 is

$$\pi_2(p) = (p_2 - c^* - r)\frac{x_2}{\sum_{k=1}^2 x_k}N$$

Equilibrium prices are given implicitly by the (unique) solution of the system

$$p_1 = c^* + \mu + \mu\frac{x_1}{x_2} + r, \quad p_2 = c^* + \mu + \mu\frac{x_2}{x_1} + r \tag{8}$$

Let $p_1^*(r)$ and $p_2^*(r)$ denote the solution of (8). Equilibrium payoffs are[3]

$$\pi_1^*(r) = \mu\frac{x_1^*(r)}{x_2^*(r)}N + rN, \qquad \pi_2^*(r) = \mu\frac{x_2^*(r)}{x_1^*(r)}N \tag{9}$$

where $x_1^*(r)$ and $x_2^*(r)$ are computed at the unique solution of (8) (using again (2)).

2.2 The licensing stage

Let us now analyze the licensing stage of the interaction. Consider first the two-part tariff licensing policy where firm 2, if acquires the new technology has

[3] We use different notations for the equilibrium prices and payoffs in G_{fr}-G_r and in G_f hoping that this won't cause a confusion.

to pay a royalty r per unit of its production plus an up front fee α. Clearly, the fee cannot exceed the difference between the profit of firm 2 when it uses the new technology s^* and pays r and its profit when it unilaterally uses the old technology s. Hence $\alpha = \pi_2^*(r) - \pi_2^*(s^*, s)$ where $\pi_2^*(r)$ is given by (9) and $\pi_2^*(s^*, s)$ by (6). The problem facing firm 1 in the licensing stage of G_{fr} is to find the royalty rate that solves

$$max_r\{\pi_1^*(r) + \pi_2^*(r) - \pi_2^*(s^*, s)\} \quad s.t. \;\; \pi_2^*(r) \geq \pi_2^*(s^*, s) \tag{10}$$

Lemma 1. *The optimization problem in (10) has no solution.*

Proof. First we note that for all $r \geq 0$, $p_1^*(r) = p_2^*(r)$. This is proved in the Appendix (Lemma A1). Given this equality of prices and since $s_1 = s_2 = s^*$, (2) implies that $x_1^*(r) = x_2^*(r)$. Hence by (6) and (9) the total payoff of firm 1 in G_{fr} is

$$\Pi_1(r) \equiv 2\mu N - \mu N \frac{x_2^*}{x_1^*} + rN \tag{11}$$

Note that x_1^* and x_2^* do not depend on r (as x_1^*, x_2^* emerge in G_-).

Claim 1. *The constraint in (10) is always satisfied, i.e., irrespective of r, $\pi_2^*(r) \geq \pi_2^*(s^*, s)$.*

Proof. Appears in the Appendix.

Note in (11) that $\Pi_1(r)$ is strictly increasing in r. This fact combined with Claim 1 imply that the problem in (10) has no solution in r. ■

It is straightforward to show that the optimal royalty cannot be computed in G_r as well. On the other hand, under an up front fee policy the conclusion is clear: under this policy firm 1 prefers not to sell its technology.

Lemma 2. *Irrespective of s^*, firm 1 prefers G_- over G_f, i.e., firm 1 does not sell its technology under the up front fee policy.*

Proof. The fee firm 1 can charge in G_f is given by $\pi_2(s^*, s^*) - \pi_2(s^*, s)$. Hence by (6) and (7), firm 1 obtains in G_f a total payoff equal to

$$\pi_1(s^*, s^*) + \pi_2^*(s^*, s^*) - \pi_2^*(s^*, s) = 2\mu N - \mu N \frac{x_2^*}{x_1^*}$$

On the other hand, in G_- firm 1 obtains $\mu N \frac{x_1^*}{x_2^*}$. Hence, licensing occurs if and only if $2\mu N - \mu N \frac{x_2^*}{x_1^*} \geq \mu N \frac{x_1^*}{x_2^*}$ or if and only if $2x_1^* x_2^* \geq (x_1^*)^2 + (x_2^*)^2$ which cannot hold. ■

Lemmas 1 and 2 imply the following.

Proposition 1. *The game G has no equilibrium where transfer of technology between the two firms takes place.*

Appendix

Lemma A1. *For all* $r \geq 0$, $p_1^*(r) = p_2^*(r)$.

Proof. First note that if $r = 0$ then firms 1 and 2 produce commodities of the same quality s^* and with the same marginal cost; hence by (7), when $r = 0$ the two firms charge the same price. Let now r increase from 0. The effects of a marginal change of r in the prices of the two firms are given by the solution of the system

$$p_{1r} = 1 + \frac{-x_1^*(r)p_{1r} + x_1^*(r)p_{2r}}{x_2^*(r)}, \qquad p_{2r} = 1 + \frac{-x_2^*(r)p_{2r} + x_2^*(r)p_{1r}}{x_1^*(r)} \tag{12}$$

in p_{1r}, p_{2r} where $p_{ir} = \dfrac{\partial p_i^*(r)}{\partial r}$, $i = 1, 2$. Note that in the unique solution of (12) we have $p_{1r} = p_{2r} = 1$. By this last fact and by the fact that when $r = 0$ prices are equal, we conclude that for all $r \geq 0$, $p_1^*(r) = p_2^*(r)$.

Proof of Claim 1. Note that $\pi_2^*(r) = \mu N$ (by (9) and since $x_1^*(r) = x_2^*(r)$). On the other hand $\pi_2^*(s^*, s) = \mu N \dfrac{x_1^*}{x_2^*}$. Hence to show the validity of the claim it suffices to show that $x_2^* < x_1^*$. This holds by Proposition 1 of Anderson and de Palma (2001) according to which in the logit model the profit of firm i is higher than that of firm j if and only if $\theta s_i - c_i > \theta s_j - c_j$ or if and only if $\theta s_i - p_i^* > \theta s_j - p_j^*$. Notice that in G_- we have $\theta s_1 - c_1 = \theta s^* - c^*$ and $\theta s_2 - c_2 = \theta s - c$, where $c = c(s)$. By assumption (A1), $\theta s^* - c^* > \theta s - c$ and hence using Proposition 1 of Anderson and de Palma (2001), we conclude that $\theta s^* - p_1^* > \theta s - p_2^*$. This last inequality implies that $x_1^* > x_2^*$ (by (2)). This proves Claim 1. ■

References

Anderson S., Palma A. and Thisse J.F. (1992), Discrete Choice Theory of Product Differentiation, MIT Press.

Anderson S., Palma A. (1992), The logit as a model of product differentiation, Oxford Economic Papers 44, 51-67.

Anderson S., Palma A. (2001), Product diversity in asymmetric oligopoly: is the quality of consumer goods too low?, Journal of Industrial Economics, vol. 49, 113-135.

Arrow K.J. (1962), Economic welfare and the allocation of resources for invention. In: R.R. Nelson (Ed.), The Rate and Direction of Inventive Activity: Economic and Social Factors, Princeton Univ. Press, pp. 609-625.

Beggs, A.W. (1992), The licensing of patents under asymmetric information, International Journal of Industrial Organization, 10, 171-191.

Gallini N.T., Wright B.D. (1990), Technology transfer under asymmetric information, RAND Journal of Economics, 21, 147-160.

Fauli-Oller R., Sandonis J. (2002), Welfare reducing licensing, Games and Economic Behavior $41, 192 - 205$.

Filippini L. (2005), Licensing contract in a Stackelberg model. The Manchester School, 73, 582-598.

Kamien M. and Tauman Y. (1984), The private value of a patent: a game theoretic analysis, Zeitschrift fur Nationalokonomie/Journal of Economics, 4, 93-118.

Kamien M. and Tauman Y. (1986), Fees versus royalties and the private value of a patent, The Quarterly Journal of Economics, vol. 101, no.3, $471 - 492$.

Kamien M., and Tauman Y. (2002), Patent licensing: The inside story, The Manchester School, 70, 7-15.

Katz M.L. and Shapiro C. (1985), On the licensing of innovations, Rand Journal of Economics, XVI, $504 - 520$.

Katz M.L. and Shapiro C. (1986), How to license intangible property, Quarterly Journal of Economics, vol. 101, $567 - 589$.

Macho-Stadler, I., Pérez-Castrillo, J.D. (1991), Contrats de licence et asymétrie d'information. Annales d'Economie et de Statistique, 24, 189-208.

Mukherjee A. (2001), Technology transfer with commitment, Economic Theory

17, 345-369.

Muto S. (1993), On licensing policies in Bertrand competition, Games and Economic Behavior, 5, 257-267.

Saracho A.I. (2002), Patent licensing under strategic delegation, Journal of Economics and Management Strategy 11, 225-251.

Sen D., Tauman Y. (2007), General licensing schemes for a cost-reducing innovation. Games and Economic Behavior, 59, 163-186.

Shapiro C. (1985), Patent licensing and R&D rivalry, The American Economic Review, vol. 75, No. 2, Papers and Proceedings 25-30.

Wang X.H (1998), Fee vs. royalty licensing in a Cournot duopoly model, Economics Letters, vol. 60, 55-62.