E C O N O M I C S   B U L L E T I N

Threshold effects in Okun's Law: a panel data analysis

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Abstract

Our approach involves the use of switching regime models, to take account of the structural asymmetry and time instability of Okun's coefficient. More precisely, we apply the non-dynamic panel transition regression model introduced by Hansen (1999) to a panel of 20 OECD countries over the last three decades. With all specifications applied, the tests lead to the rejection of the null hypothesis of a linear relationship between cyclical output and cyclical unemployment. The asymmetry implies the existence of four regimes. For lower or higher values of cyclical unemployment, it follows that there is a relatively strong negative correlation between unemployment rate and output. However, when unemployment stands at intermediate levels, this relationship tends to weaken.

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1 Introduction

Since Okun's seminal paper (1962), there have been numerous studies which provide evidence of the correlation between variations in unemployment and real output over the business cycle. Nevertheless, this interest has lately been enhanced by the fact that economic growth has been productive of fewer "jobs" in recent years than had been the case in the past in some OECD countries. As an illustration of this, an increase in French gross domestic product (GDP) of 2.5 % led to the creation of 39,000 jobs in 2004, whereas in 2001, 246,000 jobs were created despite less dynamic economic growth of 2.1 %.

However, most of the existing literature dealing with Okun's law tends to focus on the lack of robustness of the Okun's coefficient, without questioning the linear nature of the relationship. The exceptions are recent papers by Lee (2000), Virèn (2001) and Harris & Silverstone (2001). Responding to this challenge, our approach involves the use of switching regime models, to take account of the structural asymmetry and time instability of Okun's coefficient.

The innovation in the present paper is the application of this approach to a panel of 20 OECD countries over the last three decades. To do this, we have used the non-dynamic panel transition regression model with fixed individual effects introduced by Hansen (1999). Focusing on a panel of countries rather than on a single country allows us to learn about the behavior of individuals by observing the behavior of the others and allows us to derive results for a wide geographical area. Moreover, it highlights non-linear behavior that could not be taken into account by time series, due to the insufficient number of points in each regime.

Applying all specifications, the tests lead to rejection of the null hypothesis of a linear relationship between cyclical output and cyclical unemployment. To be more precise, the asymmetry entails the existence of four regimes. For lower or higher values of cyclical unemployment, a relatively strong negative correlation is observed between unemployment rate and output. However, when unemployment stands at intermediate levels, this relationship tends to weaken. Our results also characterize individual heterogeneity, making it possible for countries not to belong simultaneously to the same regime.

The outline of this paper is the following: the next section describes Okun's law. The third introduces the model specification and the methods of estimation. The fourth presents the data and the empirical results. The final section concludes with the threshold effects on Okun's law.

2 The model specification

Okun's law refers to the empirical regularity that seems to hold between cyclical unemployment and cyclical output. A version of this relationship may be represented by the following set of equations given by Weber (1995):

$$\begin{aligned} y_{it}^c &\equiv y_{it} - y_{it}^n \\ u_{it}^c &\equiv u_{it} - u_{it}^n \\ u_{it}^c &= \alpha y_{it}^c + \epsilon_{it} \quad \alpha < 0 \end{aligned} \tag{1}$$

Where y_{it}^{c}[1] captures the cyclical level of output (output gap), y_{it} is the logarithm of the actual output and y_{it}^n represents the potential or trend level of the output. Similarly, u_{it}^c captures the cyclical unemployment rate (unemployment gap), u_{it} is the observed unemployment rate, u_{it}^n represents the natural unemployment rate and ϵ_{it} is a stochastic error term. The parameter α is commonly known as the Okun coefficient. Initially, Okun (1962) found a significant negative correlation (-0.3) between unemployment and growth for US quarterly data over the period of 1947-1960.

Recently, the results of various studies have confirmed the negative relationship for USA and other developed countries (Döpke 2001). However, Lee (2000) argues that the coefficient has substantial sensitivity to the choice of method for extraction of the cyclical component. This step is complicated by the presence of a unit root in the GDP and in the unemployment rate. For this reason, we apply four different methods in succession in order to assess the robustness of our result. Firstly, we use a difference model where the output and the unemployment variable are expressed in first differences. However, as suggested by Attfiled & Silverstone (1998), this method suffers from a major drawback. This is so because if the series are not only individually integrated as I (1) but are also cointegrated together, then the model is misspecified. A second solution involves considering Okun's law from the standpoint of the notion of the "gap" between actual and equilibrium output and between actual and equilibrium unemployment. To that end, we apply three filters[2] in succession: Hodrick & Prescott (1980) filter, the Baxter & King (1999) filter and the Beveridge & Nelson (1991) (filters HP, BK and BN in what follows).

Moreover, regarding data decomposition procedures, a major issue for Okun's law is that this relationship generally leaves out of account asymmetry phenomena in the employment dynamic. However, Harris & Silverstone (2001) point to four elements to justify

[1] The subscript t denotes the time period and i the country.

[2] Three methods are used to assess the robustness of our result. However, the HP filter and BK filter could overestimate the importance of transitory shocks in the presence of unit root (Murray 2003). On the other hand, the BN decomposition allows the decomposition of an $I(1)$ series into a stochastic trend and a cyclical component. But, this procedure needs to identify an $ARIMA(p, 1, q)$, and this step is not straightforward in time series. To achieve this goal, we use the methodology of the Extended Sample Autocorrelation Function and the Smallest CANonical correlation (Tsay & Tiao (1984), Tsay & Tiao (1985)).

them: capacity constraints, signal extraction, costly adjustment and downward nominal wage rigidity. For this reason, we apply a class of panel threshold models developed by Hansen (1999) to characterize the relationship between cyclical unemployment and cyclical output. The corresponding model with fixed effects α_i is then defined as follows:

$$U_{it}^c = \mu_i + \beta_0' y_{it}^c \mathbb{I}(q_{it} \leq c) + \beta_1' y_{it}^c \mathbb{I}(q_{it} > c) + \epsilon_{it} \tag{2}$$

where q_{it} is the threshold variable[3] and c the threshold parameter. The transition function is an indicator function $\mathbb{I}(.)$ which equals 1 when the threshold condition in brackets is satisfied and 0 otherwise. In this model, the observations are divided into two regimes depending on whether the threshold variable q_{it} is smaller or greater than the threshold parameter c. The regimes are distinguished by different regression slopes, β_1 and β_2. A first advantage of this model is that it allows parameters to vary across individuals (heterogeneity issue), but also with time (stability issue) depending on the number of regimes. For our specific purpose, the PTR model has a second advantage in investigating the asymmetries in the employment dynamics.

There is no reason to limit our analysis to just two regimes. Such an assumption reduces the possibility of heterogeneity and may be unrealistic even for OECD countries which have different labor market flexibilities. Moreover, the estimation approach proposed by Hansen (1999) allows a more general specification with r thresholds (*i.e.* $r+1$ regimes), which take the form of:

$$U_{it}^c = \mu_i + \beta_0 y_{it}^c \mathbb{I}(q_{it} \leq c_1) + \beta_1 y_{it}^c \mathbb{I}(c_1 < q_{it} \leq c_2) + ...\beta_r y_{it}^c \mathbb{I}(c_{r-1} < q_{it}) + \epsilon_{it} \tag{3}$$

where the threshold parameters c_j are sorted in ascending order, $c_1 < ... < c_r$.

3 Estimation and tests

Estimation of the PTR model involves several stages. First, estimation of the parameters model requires eliminating the individual effects α_i by removing individual-specific means and then applying the least squares sequential procedure(see Hansen (1999) or Hurlin (2006) for more details). For example, if we consider a single threshold model, for a given value of the threshold parameter c, the slope coefficients β_1 and β_2 can be estimated by OLS. Thus, we can compute the sum of squared errors, denoted $S_1(c)$:

$$S_1(c) = \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{\epsilon}_{it}^2 \tag{4}$$

The threshold parameter c is then estimated by minimizing $S_1(c)$.

$$\hat{c} = ArgMin_c \; S_1(\hat{c}) \tag{5}$$

[3]No constraints are imposed on the choice of the threshold variable, except that it cannot be a contemporaneous endogenous variable and time independent. The choice of this threshold variable is discussed further in the next section.

As Hansen (1999) emphasizes, the minimization problem can be reduced to a search over the values of c equal to the distinct values of q_{it} in the sample. However, it is pointless to select a threshold c, which leads to fewer observations in one regime or another. For this reason, we impose a supplementary constraint: there should be at least $T/2$ observations in a given regime.[4].

The next step is to determine whether the threshold effect is statistically significant relative to a linear specification. The null hypothesis in this case describes the simple linear specification and can be expressed as: $H_0: \ \beta_1 = \beta_2$. This hypothesis could be tested by a likelihood ratio test:

$$F_1 = \frac{S_0 - S_1(\hat{c})}{\hat{\sigma}^2} \tag{6}$$

where S_0 is the sum of the squared residuals of the linear model, S_1 the sum of the squared residuals of the one-threshold model and $\hat{\sigma}^2 = \frac{S_1(\hat{c})}{n(T-1)}$. Unfortunately, the distribution of this test is non-standard since the PTR model contains unidentified nuisance parameters c under H_0 (Davies 1987). A solution consists of simulating by Bootstrap the asymptotic distribution of the statistic F_1. When the threshold effect is proved, the same procedure can be applied to general models (equations 3) in order to determine the number of thresholds required to capture the whole non-linearity. The new null hypothesis consists of testing a specification with r regimes versus a specification with $r+1$ regimes. The procedure starts by testing one threshold versus two, and then two versus three, and so on. The procedure stops when the null hypothesis is not rejected.

4 Data and results

The present study focus on a selection of 20 OECD countries: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Iceland, Italy, Japan, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and USA. Our data were extracted from the OECD database (Economic Outlook N 75.). As recommended by Hansen (1999), we consider only balanced panels. We have therefore used quarterly data over the period of 1970–2004.

For the preliminary results, we were interested in studying the stationary of the GDP and unemployment rate series. It will be recalled that the data decomposition procedures are more complex in the presence of unit roots and the BN method is more appropriate. With this in mind, our econometric methodology is based on second-generation panel data integration tests proposed by Choi (2002) and Pesaran (2007). These tests allow the assumption of the cross-sectional dependence among panel units. The results reported in Table 1 indicate essentially that the null hypothesis of unit-root in the series cannot be

[4]The choice of this constraint is a guarantee that the influence of a given sector in the search of c is not neglected.

rejected by the two tests at the five percent level. There is only an exception for the statistics without trend by Choi (2002) in the unemployment case. This rejection of the null hypothesis implies that at least one country is stationary, the others probably not being so. As a result, we can reach a no-stationary conclusion on the variables.

In the estimation phase of the PTR, determining the threshold variable is an issue. We consider each of two potential "candidates" in turn: cyclical unemployment and output gap lagged by one period[5]. These variables were selected for two reasons. Firstly, it appears logical that past cyclical unemployment influences Okun's law. Secondly, Lee (2000) stresses the non-stability and asymmetry related to this variable. However, it is possible that the transition is induced by the cyclical output: a higher output gap level implies a different impact on cyclical unemployment than a lower level. We discriminate among these "candidates" according to two criteria: we select the threshold variable which minimizes the sum of squared residuals (Hansen, 1999) and which leads to the strongest rejection of the linearity hypothesis[6].

For each threshold variable and data decomposition procedure, the first step is to test the linear specification and, possibly, to determine the number of thresholds. The results of the linearity test and the determination of the number of thresholds are reported in table 2. The linearity tests (F_1) clearly lead to the rejection of the null hypothesis of linearity of the Okun's law, whatever the threshold variable chosen and the filter considered. Moreover, the presence of strong threshold effects detected in the cases of the two selected threshold variables. The likelihood ratio tests F_2 and F_3 are also significant at a level of 10% for the two variables. This means that there are at least four regimes. According to Hansen's procedure, it would be necessary to estimate and test four thresholds, five thresholds and so on, until the corresponding F-test is statistically non-significant. However, we have limited our analysis to a model with at most three threshold parameters (*i.e.* four regimes). This choice can be justified on two grounds (Hurlin, 2006): firstly, computational costs and, secondly, an extra regime will not affect (or only slightly affect) the estimates of the other threshold parameters and the estimates of the slope parameters in the existing regimes. Looking at table 2, we can also determine the "optimal" variable. For all the data decomposition procedures, this is cyclical unemployment lagged by one period. Given this, we continue our analysis using this threshold variable and four regime models.

The following remarks relate solely to Beveridge Nelson decomposition, but we will see later that the other filters largely confirm these conclusions. The estimates of the parameters of the PTR models with four regimes and the corresponding t-statistics based on

[5]We have also estimated the lag between one and four. But we have not reported these results because they lead the same number of regimes and these threshold variables are statistically lower "optimal". However, the results are available upon request.

[6]The "optimal" threshold variable in a panel smooth transition model corresponds to the variable which leads to the strongest rejection of the linearity hypothesis. We extend here this result to the PTR class of model.

standard errors corrected for heteroskedasticity are reported in Table 3, together with the threshold values. These parameters are important because they show when the transition between the two regimes occurs. For instance, if cyclical unemployment is greater than -0.0180 and less than -0.0123, the country concerned switches to the second regime. In light of these results, we note that the relationship between cyclical unemployment and output gap (-0.611 and -0.354) is the strongest in the two lower regimes and subsequently in the higher regime (-0.211). Conversely, the link is extremely weak in the higher intermediate regime, even if the coefficient is significantly different from zero. Okun's law may apply in extreme regimes, like Purchasing Power Parity.

According to the estimated threshold values, we can deduce the distribution of the countries among the different regimes (table 4) and plot this transition, taking into consideration time and countries[7] (figure 1). We observe that the majority of observations are in the third regime, which corresponds to a weak Okun's law. The results for Japan are aligned with those generally admitted in time series analysis. This country is placed solely in the third regime and consequently confirms Lee's explanations of high rigidity in the labor market. Conversely, observations for the United Kingdom and USA are often in extreme regimes. This point highlights a degree of flexibility and confirms the high coefficients generally admitted in time series. France and Germany are in intermediate positions, but belong largely in the third regime. As a final comment on this table, we may note that the panel data investigation has revealed threshold effects that it would not have been possible to obtain in times series covering too few observations.

To check the robustness of our results, it is however useful to analyze not only the results using Beveridge Nelson data decomposition but also those of the other methods used. Looking at table 3, we once again see the four significant regimes including three extreme regimes with high coefficients compared to the remaining one, which encompasses a lower central regime. The only difference between these decompositions is to be found in the location of this regime in the HP filter (lower intermediate regime). However, the majority of the observations are located in this regime. Like Lee (2000), our results are qualitatively similar, but differ in quantitative terms. Moreover, there is a gap between the values of the coefficients in different data decomposition methods. However, the economics are intuitively the same. For more precision, we have plotted the transition for time and countries in figure 2. We may observe once again that this threshold allows heterogeneity and time instability to be taken into account. Japan is still in the lower regime of Okun's law, and the US and United Kingdom are often outside this regime.

5 Conclusion

The purpose of this study was twofold: to prove that the law of Okun is not linear, and then represent it with threshold panel data models. Firstly, the methodology of Hansen

[7] We have reported only eight countries. Others are available on upon request.

(1999) allows us to show the presence of threshold effects in Okun's law, irrespective of the data decomposition method or the threshold variable chosen. These results demonstrate the existence of an asymmetry in the relationship between cyclical unemployment and output gap. Secondly, this study offers an original approach by clustering countries with same dynamic and thus solving the heterogeneity problem and by introducing an Okun law which may vary in time.

tables and figures

Table 1: Unit root tests in panel data

		GDP		Unemployment	
	Statistic	without trend	trend	without trend	Trend
Choi (2002)	P_m	$-2,15$	$0,93$	7.96	$1,31$
		(0,98)	(0,17)	(0,00)	$-0{,}09$
	Z	$1,6$	$0,36$	$-6,06$	$-0,96$
		(0,94)	(0,64)	(0,00)	$-0{,}17$
	$L^\star$	$1,48$	$(0;56)$	$-6,37$	$-0,78$
		(0,93)	(0,57)	(0,00)	$-0{,}21$
Pesaran (2003)	$CIPS_1$	$-1,95$	$-2,03$	$-2,01$	$-2,27$
		(0,27)	(0,93)	(0,20)	(0,64)
	$CIPS_2$	$-1,89$	$-1,89$	$-1,9$	$-2,27$
		(0,34)	(0,98)	(0,33)	(0,65)
	$CIPS_3$	$-1,96$	$-1,95$	$-1,86$	$2,27$
		(0,25)	(0,97)	(0,38)	(0,63)

Notes : The p-value are bracketed. All specifications are estimated with fixed effects.

Table 2: Linearity test and determination of regime number

	Threshold Variables							
	B.N Filter		First Dif.		B.K Filter		H.P filter	
	u_{t-1}	y_{t-1}	u_{t-1}	y_{t-1}	u_{t-1}	y_{t-1}	u_{t-1}	y_{t-1}
Two Rgimes								
RSS	0.181	0.227	0.0253	0.0261	0,052	0,061	0.0714	0.0811
F_1	1047	292	108	16.4	397	51,7	382	14.3
p-value	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)
Three Regimes								
RSS	0.163	0.224	0.025	0.026	0,045	0,060	0.063	0.802
F_2	301	46.6	24.84	15.71	386	19.2	372	30.1
p-value	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)
Four Regimes								
RSS	0.161	0.220	0.025	0.026	0,044	0,059	0.061	0.080
F_3	30.1	7.33	17.0	16.9	59,6	35,0	80.5	19.2
p-value	(0,00)	(0,08)	(0.04)	(0.08)	(0,00)	(0,00)	(0,00)	(0,00)

Note: F_1, F_2 F_3 are the likelihood ratio statistics. $p - values$ are obtained with 300 simulations (Hansen, 1999) and with 200 simulations for the models with four regimes because computational costs. RSS: Residuals Sum of Squared.

Table 3: Threshold model with four regimes

Filter	BN	H.P	First Dif.	B.K
Lower Threshold	-0.0180	-0,0075	-0,0047	-0,0093
Middle Threshold	-0.0123	0,0031	-0,0028	-0,0065
Higher Threshold	0.0067	0,0070	0,0076	0,0068
RSS	0.1616	0,0609	0,0249	0.0441
Lower regime Coef. β_0	-0.611^{***} (-34.0)	$-0,421^{***}$ (-30.5)	-0.264^{***} (-14.2)	-0.533^{***} (-28.4)
Lower Middle Coef. β_1	-0.355^{***} (-8.69)	$-0,100^{***}$ (-13.5)	-0.160^{***} (-9.75)	-0.334^{***} (-15.6)
Higher Middle Coef. β_2	-0.010^{***} (-3.22)	$-0,273^{***}$ (-15.6)	-0.066^{***} (-11.1)	-0.127^{***} (-17.0)
Higher Coef. β_3	-0.211^{***} (-18.4)	$-0,554^{***}$ (-33.2)	-0.251^{***} (-6.90)	-0.572^{***} (-33.7)

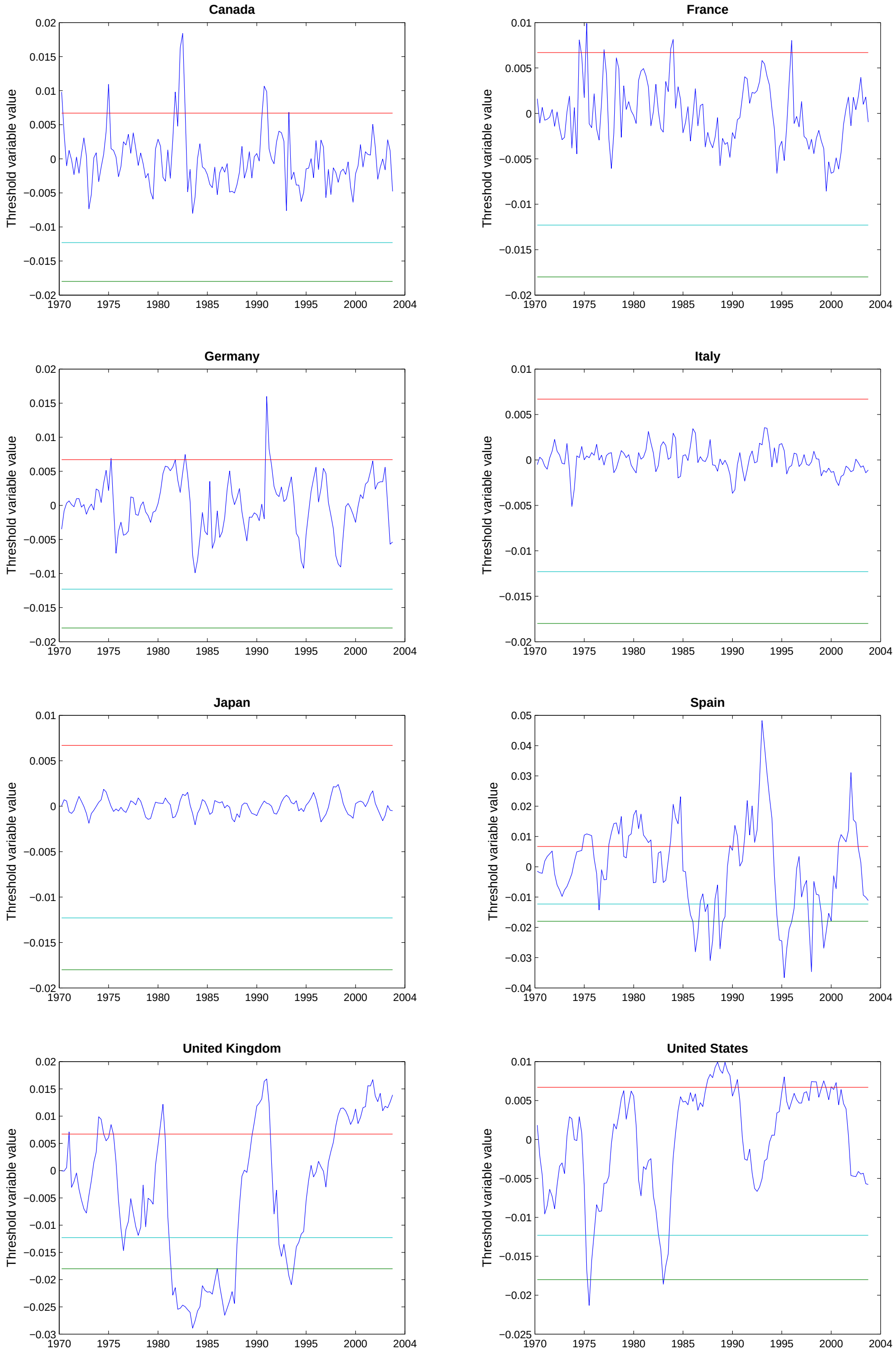
Note: Residuals Sum of Squared. $t-statistics$ corrected for heteroskedasticity are bracketed.***: significance level at 1%, **: significance level at 5%,*: significance level at 10%.

Table 4: Data distribution between regimes and countries

	Lower	Lower Middle	Upper Middle	Superior
Australia	0	2	129	4
Belgium	2	9	92	32
Canada	0	0	126	9
Denmark	0	2	124	9
Finland	29	5	44	57
France	0	0	129	6
Germany	0	0	131	4
Ireland	1	4	98	32
Island	0	0	103	32
Italy	0	0	135	0
Japan	0	0	135	0
Luxembourg	0	0	135	0
Netherland	6	12	79	38
Norway	0	0	135	0
Portugal	3	5	98	29
Spain	17	10	60	48
Sweden	0	0	129	6
Switzerland	0	0	134	1
United Kingdom	27	11	59	38
USA	2	5	110	18

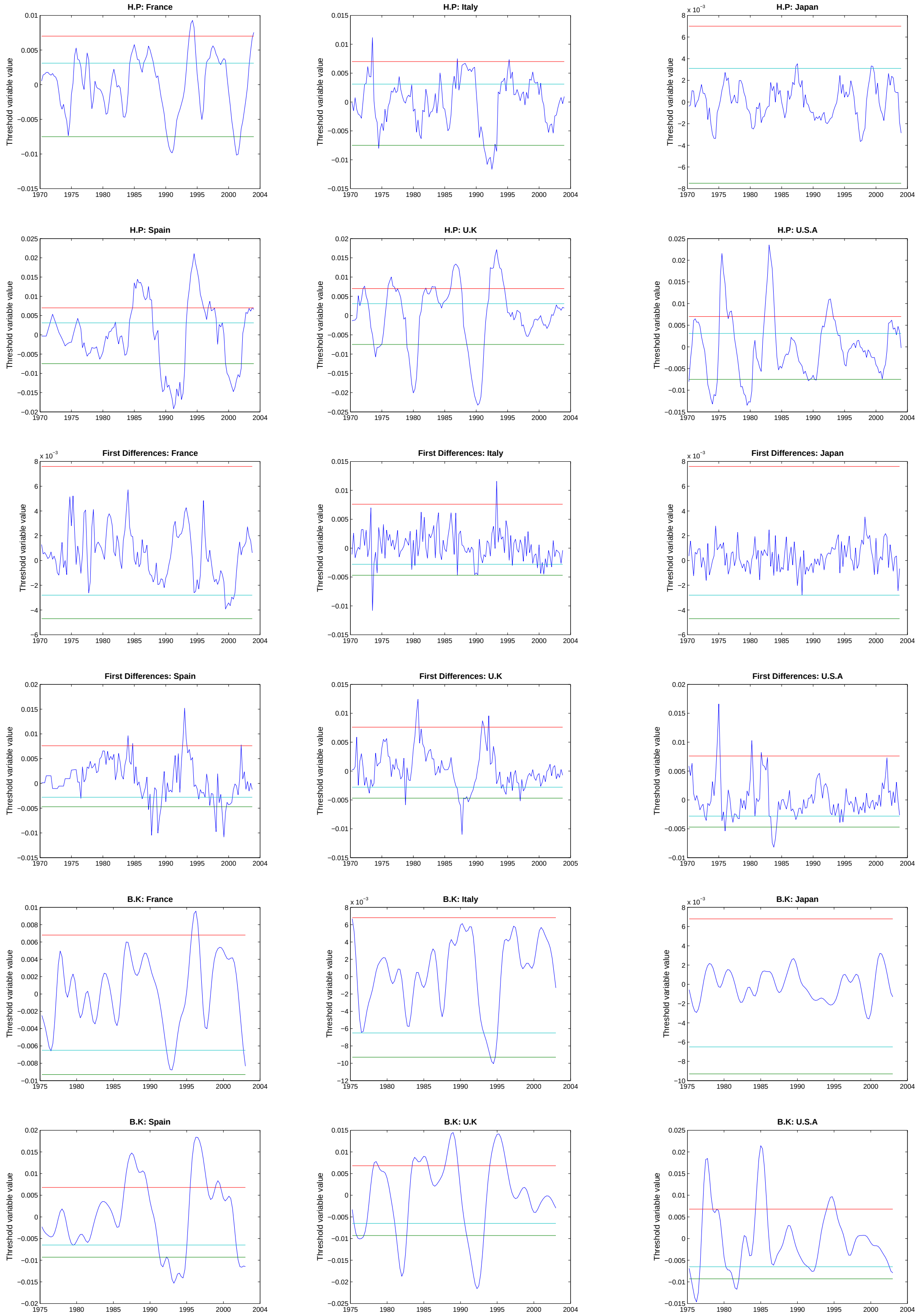
Note: Data decomposition procedure is Beveridge Nelson decomposition. The threshold variable is the cyclical unemployment lagged by one period.

Figure 1: Distribution of eight countries among the different regimes



Note: Data decomposition procedure is Beveridge Nelson decomposition. The straight lines are the threshold values.

Figure 2: Distribution of the countries applying other decomposition procedures



Note: B.K : Baxter King filter (1999).The straight lines are the threshold values.

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