E c o n o m i c s   B u l l e t i n

Price stability, inflation targeting and public debt policy

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Abstract

This paper studies the implications of inflation targeting (IT) regimes for public debt accumulation. By utilizing a simple dynamic macroeconomic policymaking model, we show that IT regimes may lead to higher public debt. Our results suggest that in countries where there are inherent distortions in the economy all IT regimes can do is shift the burden of adjustment onto other aspects of macroeconomic policymaking. We therefore argue that, adopting an IT regime without carrying out the required reforms towards eliminating the distortions in the economy is not necessarily an effective device for overall macroeconomic stability.

Citation: Cabral, Rene and Gulcin Ozkan, (2008) "Price stability, inflation targeting and public debt policy." *Economics Bulletin*, Vol. 5, No. 32 pp. 1-12
Submitted: September 22, 2008. **Accepted:** November 2, 2008.
URL: http://economicsbulletin.vanderbilt.edu/2008/volume5/EB-08E50027A.pdf

1. Introduction

During the 1990s, inflation targeting (IT) emerged as the preferred form of monetary policy framework in a large number of countries. This was partly due to the failure of targeting monetary aggregates in previous decades and partly to the breakdown of pegged exchange rate regimes throughout the 1990s. The success of initial targeters such as Chile, New Zealand, the UK, Israel and Canada in reducing inflation has made IT an attractive monetary policy option for a wide variety of countries.

Researchers attempting to examine whether IT can be applied more widely, especially in emerging market countries, have highlighted the soundness of fiscal stance and the absence of fiscal dominance as pre-conditions for a successful IT regime (see, for example, Amato and Gerlach, 2002, Giavazzi, 2003 and Fraga *et al.,* 2003).[1] Indeed, the evolution of fiscal stance is not independent of the monetary policy regime. Clearly, the adoption of an IT regime imposes obvious constraints on governments' fiscal balances by restricting seigniorage revenues and preventing central bank's credit. Fiscal implications of an IT regime may be particularly important for emerging market countries with low tax bases and high debt levels.[2] Some researchers have recently pointed to the cases of Brazil and Turkey to highlight the risks of pursuing IT regimes in a high-debt environment and showed how the IT regime may raise the debt even further (for the case of Brazil, see, for example, Blanchard, 2004 and Favero and Giavazzi, 2004; for Turkey's case see Ersel and Ozatay, 2008 and Sahinbeyoglu, 2008). Indeed, a number of other emerging market countries such as Colombia, Czech Republic, Hungary, Mexico, Poland and Thailand experienced sharp rises in debt levels following the adoption of the IT regime.

The increase in emerging markets' public debt-to-GDP ratios has been attracting considerable attention since the early 2000s (see, for instance, the IMF World Economic Outlook 2003 and the Bank of England Stability Review 2003). It has been argued that this increase in debt accumulation resulted from a greater reliance of public finances on domestic debt, a tendency that started in the mid-1990s. Such reliance on domestic debt, in turn, has been linked to the improved cost of domestic borrowing resulting from a benevolent macroeconomic environment, among other factors (Hanson, 2007). Recent evidence suggests that emerging economies have shifted their debt composition towards domestic borrowing, producing a domestic versus foreign debt mix similar to that of advanced countries (Jeanne and Guscina, 2006).[3]

This paper provides one potential explanation for such rises in public debt levels by formally examining the role of delegating monetary policy to an independent central bank and implementing an IT regime on public debt accumulation. Although the implications of existing fiscal environment for a successful implementation of IT regimes have been widely discussed, fiscal consequences of IT regimes have received very little attention in formal

[1] Formal evidence on the implications of the IT regimes is mixed. For example, Ball and Sheridan (2005) argue that there is not sufficient evidence on the role of IT regimes in reducing inflation and inflation volatility. On the other hand, Goncalves and Salles (2008) show that IT regimes have not only reduced inflation in emerging market countries, they also brought down growth volatility.

[2] See, for example, IMF(2003).

[3] This is in contrast to the so called 'original sin' argument, which refers to the inability of a country to borrow abroad in its own currency (Eichengreen and Hausmann, 2004).

studies. This is the main motivation for the analysis in this paper. By utilizing a simple dynamic macroeconomic policymaking model, we show that IT regimes may indeed lead to higher public debt. We also derive the condition under which this is more likely.

2. The model

In order to investigate the fiscal implications of IT regimes, we utilize a simple two-period model of discretionary monetary and fiscal policy.[4] The government's disutility function can be represented in the following form:

$$L_t^G = \frac{1}{2}\sum_{t=1}^{T=2}\beta_G^{t-1}\left[\delta_1(\pi_t - \widetilde{\pi}_t)^2 + (x_t - \widetilde{x}_t)^2 + \delta_2(g_t - \widetilde{g}_t)^2\right] \tag{1}$$

where L_t^G denotes the welfare losses incurred by the government, δ_1 and δ_2 represent, respectively, the government's relative dislikes for the deviations of inflation (π_t) and public spending as a share of output (g_t) from their target levels ($\widetilde{\pi}_t$ and $\widetilde{g}_t$ respectively) relative to the deviations of (log of) output (x_t) from its target level ($\widetilde{x}_t$) and β_G is the government's discount factor. A non-zero output target ($\widetilde{x}_t$) represents the bliss point for output in the absence of non-tax distortions, for example, due to labour or commodity market imperfections. The bliss point for public spending ($\widetilde{g}_t$) can be interpreted as the optimal share of non-distortionary output to be spent on public spending. Both weights δ_1 and δ_2 and the bliss points for output and public spending; $\widetilde{x}_t$ and $\widetilde{g}_t$ reflect the political and the institutional structure of the economy.

Similarly, the preferences of the central bank (CB) are summarized by

$$L_t^{CB} = \frac{1}{2}\sum_{t=1}^{T=2}\beta_{CB}^{t-1}\left[\mu_1(\pi_t - \widetilde{\pi}_t)^2 + (x_t - \widetilde{x}_t)^2\right] \tag{2}$$

where L_t^{CB} denotes the welfare losses incurred by the CB at period t, β_{CB} is the CB's discount factor and μ_1 is used for the CB's inflation aversion parameter. In contrast to the government acting through the fiscal authority (FA), the independent CB cares only about deviations of inflation and output from their targets. Under the IT regime, the government assigns an specific target for inflation, π_t^T, to the CB. In that case, this target replaces $\widetilde{\pi}_t$ in equation (2).

Output is given by the following production function: $Y_t = N_t^{\gamma}$, where Y_t and N_t represent output and labour respectively, in period t and $0 < \gamma < 1$. Distortionary taxes, which are the only form of taxes available to the government, are levied on output at the rate τ_t. A representative competitive firm's problem is to maximize profits $P_t(1-\tau_t)N_t^{\gamma} - W_tN_t$, where P_t and W_t represent the price level and the wage rate respectively, in period t. A representative competitive firm chooses labor to maximize profits by taking P_t, W_t and τ_t as given. The resulting output supply function is $y_t = v(p_t - w_t - \tau_t) + z$, where lower case letters represent logs, e.g. $y_t = ln(Y_t)$, $v = \gamma/(1-\gamma)$, $z = v\ln(\gamma)$ and $\ln(1-\tau) \simeq -\tau$.

[4] Similar variants of this model are used by Beetsma and Bovenberg (1997, 1999) and Ozkan (2000).

Normalizing output by subtracting z from y_t, for simplicity and utilizing $w_t = p_t^e$ yields the following normalized output supply function

$$x_t = v(\pi_t - \pi_t^e - \tau_t) \tag{3}$$

where π_t^e is expected inflation and all other variables are as defined above.

The government budget constraint creates the link between the fiscal and monetary policies chosen by the government and the CB. This constraint is formally given by:

$$g_t + (1 + \rho + (\pi_t^e - \pi_t))d_{t-1} = \tau_t + k\pi_t + d_t \tag{4}$$

where d_{t-1} is the debt issued in period $t-1$ (as a ratio of output) that should be paid back at period t, ρ is the *ex ante* real interest rate, d_t is the debt issued in period t and k measures real money holdings as a share of output.[5] Clearly, surprise inflation erodes the real value of government's obligations. Equation (4) also suggests that a favorable change in inflation expectations relaxes the government's financing requirement.[6]

3. Debt dynamics

In choosing the level of borrowing in $t = 1$, the policymaker weighs the benefits and costs of borrowing *vis-à-vis* those of the other forms of financing. Resorting to borrowing in the first period alleviates the in-period distortionary effects of the financing requirement by reducing the pressure on taxes and seigniorage in $t = 1$, but only to increase them when it is time to re-pay the debt in $t = 2$. Clearly, the greater the use of borrowing in $t = 1$, the greater the required use of inflation and taxes in $t = 2$. Given that $t = 2$ is the final period, no new debt is issued in the second period. Thus, all borrowing is regarded as short-term and matures after one period, which means that all the outstanding liabilities are paid in $t = 2$.[7]

We solve for the equilibrium outcome using backwards induction. Formally, solving the policymaker's loss minimization problem in $t = 2$, calculating the welfare losses in $t = 2$ and substituting the solution into the intertemporal loss function in $t = 1$ yields the following first-period Lagrangian

$$\mathcal{L}_1 = \frac{1}{2}\left[\delta_1 {\pi_1}^2 + (x_1 - \widetilde{x}_1)^2 + \delta_2 (g_1 - \widetilde{g}_1)^2\right]$$
$$+\lambda_2[g_1 - \tau_1 - k\pi_1 + (1 + \rho + (\pi_1^e - \pi_1))d_0 - d_1] + \beta_G L_2^G \tag{5}$$

[5] While seigniorage revenues, $k\pi_t$ tend to be negligible in industrial economies, emerging market countries with less developed financial systems routinely resort to seigniorage as a source of revenue (see, for example, IMF World Economic Outlook, 2001).

[6] This favourable role of reduced inflationary expectations on relaxing the government budget constraint could be a significant benefit associated with successful IT regimes. This is especially the case for emerging market countries that pay high premia on their debts.

[7] There is a possibility that IT regimes, if successful and credible, may increase the average duration of public debt. Given the two-period nature of our framework, incorporating this possibility is beyond the scope this analysis.

where L_2^G is the value of the government's welfare losses in $t = 2$ and λ_2 is the Lagrange multiplier. Solving the minimization problem stated in (10) yields the equilibrium values of the choice variables; π_1, τ_1, g_1 and d_1. Details of this derivation are presented in the Appendix.

Table 1 presents the outcome for public borrowing in $t = 1$ under three different policymaking arrangements and provides the basis of our comparative analysis. The first of these cases is centralized discretion where the government controls both the fiscal and monetary policymaking, which is used as a benchmark. We also consider delegation to a conservative central bank (CCB) with and without an explicit inflation target. Following Svensson (1997), under an IT regime the CB is assigned an explicit inflation target designed to attain the government's inflation bliss point.[8]

Table 1- *Equilibrium borrowing under centralized discretion, CCB and IT*

Policymaking Arrangement	*Debt in $t = 1$*
Centralized discretion	$d_1^D = \frac{\left[\left(\widetilde{K}_1 - k\widetilde{\pi}_1\right) + (1+\rho)d_0 - \left(\widetilde{K}_2 - k\widetilde{\pi}_2\right)\right] + (1-\beta_D^*)\left(\widetilde{K}_2 - k\widetilde{\pi}_2\right)}{[1+\beta_D^*(1+\rho)]}$
CCB	$d_1^{CCB} = \frac{\left[\left(\widetilde{K}_1 - k\widetilde{\pi}_1\right) + (1+\rho)d_0 - \left(\widetilde{K}_2 - k\widetilde{\pi}_2\right)\right] + (1-\beta_C^*)\left(\widetilde{K}_2 - k\widetilde{\pi}_2\right)}{[1+\beta_C^*(1+\rho)]}$
IT	$d_1^{IT} = \frac{\left[\left(\widetilde{K}_1 - k\widetilde{\pi}_1\right) + (1+\rho)d_0 - \left(\widetilde{K}_2 - k\widetilde{\pi}_2\right)\right] + (1-\beta_G^*)\left(\widetilde{K}_2 - k\widetilde{\pi}_2\right)}{[1+\beta_G^*(1+\rho)]}$

Note: $\widetilde{K}_t = \widetilde{g}_t + \widetilde{x}_t/v$, $\beta_D^* = \beta_G^* \frac{D^* D_0}{D_1^2}$, $\beta_G^* = \beta_G(1+\rho)$, $D^* = \frac{(1+k)^2}{\delta_1} + \frac{1}{v^2} + \frac{1}{\delta_2}$, $D_0 = \frac{k(1+k+d_0)}{\delta_1} + \frac{1}{v^2} + \frac{1}{\delta_2}$, $D_1 = \frac{k(1+k+d_1)}{\delta_1} + \frac{1}{v^2} + \frac{1}{\delta_2}$, $\beta_C^* = \beta_G(1+\rho)\frac{C^*}{C}$, $C^* = \frac{\delta_1}{\mu_1^2} + \frac{1}{v^2} + \frac{1}{\delta_2}$, $C = \frac{k}{\mu_1} + \frac{1}{v^2} + \frac{1}{\delta_2}$ and all other parameters are as defined earlier.

The levels of borrowing under the three arrangements can be formally ranked as follows.

Proposition 1 *Provided that the CB is more conservative than the government; $\delta_1 < \mu_1$, the ranking of debt under the three arrangements is as follows;*

(a) if $k > \delta_1/\mu_1 \Rightarrow d_1^{CCB} > d_1^{IT} > d_1^D$,

(b) if $k < \delta_1/\mu_1 \Rightarrow d_1^{IT} > d_1^{CCB} > d_1^D$.

Proof. *Equilibrium debt levels presented above suggest that the ranking of d_1^{CCB}, d_1^{IT} and d_1^D is determined by the ranking of β_C^*, β_G^** and *β_D^*. It is straightforward to establish that $\beta_D^* > \beta_G^*$ given $\frac{D^* D_0}{D_1^2} > 1$. Since* both *$\partial d_1^{IT}/\partial \beta_G^*$ and $\partial d_1^D/\partial \beta_D^*$* are *strictly non-positive, $d_1^{IT} > d_1^D$. Similarly, given $\frac{C*}{C} < \frac{D^* D_0}{D_1^2}$ it follows that $d_1^{CCB} > d_1^D$. Finally, the ranking between d_1^{IT} and*

[8] IT regimes may vary in practice according to the degree of committment to the inflation target. In a recent paper, Carare and Stone (2006) show that the degree of committment to an IT regime is determined by the existing level of credibility, which in turn, is shaped by the underlying economic structure.

d_1^{CCB} *depends upon the ranking between* β_C^* *and* β_G^*. *Clearly,* $d_1^{IT} > d_1^{CCB}$ *when* $\beta_C^* > \beta_G^*$, *which holds when* $\frac{C*}{C} > 1$. *This condition, in turn, is satisfied when* $k < \delta_1/\mu_1$. *And vice versa holds when* $k > \delta_1/\mu_1$. ■

Outcomes presented in Table 1 suggest that there are two components in determining the equilibrium debt levels. The first, $[(\widetilde{K}_1 - k\widetilde{\pi}_1) + (1+\rho)d_0 - (\widetilde{K}_2 - k\widetilde{\pi}_2)]$, is the gap between the current and the future financing requirements nets of targeted seigniorage revenue. Clearly, the greater the current net requirement relative to the future one, the higher is the required borrowing. The second component, $(1-\beta_i^*)(\widetilde{K}_2 - k\widetilde{\pi}_2)$ measures the importance of the second period's net financing requirement taking into account the policymaker's discount factor ($i = D, C$ and G). Common to both components are the net distortions in the economy, as represented by $\widetilde{K}_t - k\widetilde{\pi}_t = \widetilde{x}_t/v + \widetilde{g}_t - k\widetilde{\pi}_t$, that are the main source of borrowing in all three regimes. Table 1 also reveals that debt varies among the three regimes due to different effective discount factors in each case. Under discretion, the effective discount factor is given by β_D^* which includes a *credibility effect*, $\frac{D^*D_0}{D_1^2}$. This credibility effect originates from the government's attempt to reduce public debt in order to mitigate inflation in $t = 2$. By doing so, the government exchanges additional output distortions, resulting from higher taxes in $t = 1$, for credibility gains in $t = 2$.

Under CCB, the effective discount factor features a *strategic effect*, $\frac{C^*}{C}$ arising from the disagreement between the CB and the FA with respect to the optimal level of inflation in $t = 2$. It is straightforward to show that, the strategic effect is smaller than the credibility effect, that is $\frac{C^*}{C} < \frac{D^*D_0}{D_1^2}$, thus delegating monetary policy to a CCB invariably produces a higher level of debt than under centralized discretion. This is because delegating monetary policy to a CCB alleviates the credibility problems faced by policymakers associated with lack of commitment, thus reducing the need to cut down borrowing as an attempt towards better inflation performance.

Note that the effective discount factor under the IT regime is β_G^* in which neither the credibility effect nor the strategic effect features. Put differently, the IT regime which works as a commitment device eliminates both the strategic and the credibility effects. Given that $\frac{D^*D_0}{D_1^2} > 1$, the IT regime unambiguously raises the equilibrium level of debt relative to that under discretion.

Hence, a regime of centralized discretion always provides less debt accumulation than delegating monetary policy to a CCB or adopting an IT regime. This result provides one potential explanation for the increase in debt accumulation experienced by emerging market countries during the last decade. It is commonly observed that both independent CBs and explicit and implicit IT regimes have characterized monetary policy design in many emerging market countries since the early 1990s (see, for example, Cukierman, 2007). Our analysis suggests that this process might have contributed to the observed higher debt levels in these countries.

In terms of the debt ranking between the CCB and the IT regimes, Proposition 1 suggests that when k is relatively large as compared with δ_1/μ_1, $d_1^{CCB} > d_1^{IT}$. That is, when k is large the FA raises borrowing in $t = 1$ under the CCB regime given that the required inflation to pay for additional debt is small when k is high. In contrast, when k is low the opposite holds, $d_1^{IT} > d_1^{CCB}$. Given that the IT regime eliminates the strategic effect the outcome would be an increase in the equilibrium debt level as compared with that under a CCB regime. In other words, since the IT regime assures lower inflation, attempting to reduce public debt in order to mitigate inflation in $t = 2$ is no longer an optimal strategy for the FA. In practice, it is unlikely that k would be higher than δ_1/μ_1. Modern economies with efficient payment systems maintain low money holdings-to-GDP ratios (i.e. k is close to zero). As a result, public debt under an IT regime would be expected to be higher than that with a CCB.

4. Concluding remarks

Our formal analysis shows that increased debt accumulation may indeed follow the delegation of monetary policy to an independent CCB and the adoption of an IT regime. Our analysis also reveals that the main source of borrowing is the distortions in the economy that may arise from labour markets, tax systems and political preferences. It, therefore, follows that in countries where there are inherent distortions in the economy all IT regimes can do is shift the burden of revenue raising from inflation to other sources of finance such as borrowing. This, in turn, implies that adopting an IT regime without carrying out the required reforms towards eliminating the distortions in the economy is not necessarily an effective device for overall macroeconomic stability.

References

Amato, J. D. and Gerlach, S. (2002) "Inflation Targeting in Emerging Market and Transition Economies: Lessons After a Decade" *European Economic Review* **46**, 781—790

Ball, L. and Sheridan, N. (2005) "Does Inflation Targeting Matter?" NBER working paper 9577

Bank of England (2003). "Financial Stability Conjectures and Outlook" *Financial Stability Review* **14,** 5—72

Beetsma, R. M. and Bovenberg, A. L. (1999) "Does Monetary Unification Lead to Excessive Debt Accumulation?" *Journal of Public Economics* **74**, 299—325

Beetsma, R. M. and Bovenberg, A. L., (1997) "Central Bank Independence and Public Debt Policy" *Journal of Economic Dynamics and Control* **21**, 873—894

Blanchard, O. (2004) "Fiscal Dominance and Inflation Targeting: Lessons From Brazil" NBER working paper 10384

Carare, A. and Stone, M. R. (2006) "Inflation Targeting Regimes" *European Economic Review* **50**, 1297—1315

Cukierman, A. (2007) "Central Bank Independence and Monetary Policy-making Institutions" CEPR Discussion Paper 6441

Eichengreen, B. and Hausmann, R. (2004) *Other People's Money: Debt Denomination and Financial Inestability in Emerging Market Economies.* The University of Chicago Press, Chicago, IL.

Ersel, H. and Ozatay, F. (2008). "Fiscal Dominance and Inflation Targeting: Lessons from Turkey", *Emerging Markets Finance and Trade* (forthcoming).

Favero, C. A. and Giavazzi, F. (2004) "Inflation Targeting and Debt: Lessons from Brazil" NBER working paper 10390

Fraga, A. Goldfajn, I. and Minella, A. (2003) "Inflation Targeting in Emerging Market Countries" in *NBER Macroeconomics Annual* by Gertler, M. and Rogoff, K., Eds., MIT Press, Cambridge, MA, 365—399

Giavazzi, F. (2003) "Inflation Targeting and the Fiscal Policy Regime: The Experience of Brazil" *Bank of England Quarterly Bulletin* **43**, 334—342

Goncalves, C. E. S. and Salles, J. M. (2008) "Inflation Targeting in Emerging Market Countries: What Do the Data Say?" *Journal of Development Economics* **85**, 312—318

Hanson, J. A. (2007) "The Growth in Government Domestic Debt: Changing Burdens and Risks" World Bank Policy Research working paper 4348

International Monetary Fund (2001) "The Decline of Inflation in Emerging Markets: Can it be Maintained?" in *IMF World Economic Outlook* May, Washington D. C., 116—144

International Monetary Fund (2003) "Public Debt in Emerging Markets: Is it to High?" in *IMF World Economic Outlook* September, Washington D. C., 113—152

Guscina, A. and Jeanne, O. (2006) "Government Debt in Emerging Markets: A New Data Set" IMF working paper 0698

Ozkan, F. G. (2000) "Who Wants an Independent Central Bank: Monetary Policy Making and Politics" *Scandinavian Journal of Economics* **102**, 621—643

Sahinbeyoglu, G. (2008). "From Exchange Rate Stability to Inflation Targeting: Turkey's Quest for Price Stability", in *Monetary Policy and Inflation Targeting in Emerging Markets* by de Mello, Luiz, Ed., OECD, 143—172.

Svensson, L. E. O. (1997) "Optimal Inflation Targets, Conservative Central Banks, and Linear Inflation Contracts" *American Economic Review* **87**, 98—114

Appendix

The model in its dynamic set up is solved using backwards induction. To simplify the analytical solutions presented later on, following Beetsma and Bovenberg (1997) we define the Government Financing Requirement (GFR) by re-expressing (4) as:

$$GFR_t = \widetilde{K}_t + (1 + \rho + (\pi_t^e - \pi_t))d_{t-1} - d_t = [\tau_t + \widetilde{x}_t/v] + k\pi_t + [\widetilde{g}_t - g_t] \tag{A1}$$

where $\widetilde{K}_t = \widetilde{g}_t + \widetilde{x}_t/v$. The GFR is given by the sum of government spending target, the labour subsidy that aims at compensating the implicit tax on output, $\widetilde{x}_t/v$, and the outstanding debt obligations net of new borrowing, $(1+\rho+(\pi_t^e-\pi_t))d_{t-1}-d_t$. On the right are the sources of finance for these expenditures; net tax revenues, $[\tau_t + \widetilde{x}_t/v]$, seigniorage, $k\pi_t$, and the shortfall of public spending relative to its target.

Solution in $t = 2$

Under the IT regime, the government (fiscal authority) and the independent central bank play a Nash game in both periods acting simultaneously to choose their respective instruments.

Starting in period $t = 2$, the monetary authority chooses inflation (π_2) to minimize its loss function

$$L_2^{CB} = \frac{1}{2}\left[\mu_1(\pi_2 - \pi_2^T)^2 + (x_2 - \widetilde{x}_2)^2\right] \tag{A2}$$

Similarly, the fiscal authority attempts to minimize its welfare losses subject to the budget constraint, as defined by the following Lagrangian

$$\mathcal{L}_2 = \frac{1}{2}\left[\delta_1(\pi_2 - \widetilde{\pi}_2)^2 + (x_2 - \widetilde{x}_2)^2 + \delta_2(g_2 - \widetilde{g})^2\right] + \lambda_2[g_2 - \tau_2 - k\pi_2 + (1+\rho+(\pi_2^e - \pi_2))d_1] \tag{A3}$$

where λ_2 is the Lagrangian multiplier associated with the government budget constraint in period $t = 2$.

Substituting the output supply function from (3) into (A2) and (A3), then differentiating the resulting expressions w.r.to π_2, τ_2 and g_2 yields the following FOCs

$$\frac{\partial L_2^{CB}}{\partial \pi_2} = \mu_1(\pi_2 - \pi_2^T) + v(v(\pi_2 - \pi_2^e - \tau_2) - \widetilde{x}) = 0$$

$$\frac{\partial \mathcal{L}_2}{\partial \tau_2} = -v(v(\pi_2 - \pi_2^e - \tau_2) - \widetilde{x}_2) - \lambda_2 = 0$$

$$\frac{\partial \mathcal{L}_2}{\partial g_2} = \delta_2(g_2 - \widetilde{g}_2) + \lambda_2 = 0$$

Eliminating λ_2 from the above system and imposing rational expectations ($\pi_2 = \pi_2^e$), yields

$$\pi_2 = \frac{v^2}{\mu_1}\left[\tau_2 + \frac{\widetilde{x}_2}{v}\right] + \pi_2^T \tag{A4}$$

$$\widetilde{g}_2 - g_2 = \frac{v^2}{\delta_2}\left[\tau_2 + \frac{\widetilde{x}_2}{v}\right] \tag{A5}$$

Substituting (A4) and (A5) into (A1) for $t = 2$, and solving for π_2, we obtain

$$\pi_2 = \frac{1/\mu_1}{C}[\widetilde{K}_2 + (1+\rho)d_1] + \frac{\widetilde{C}}{C}\pi_2^T$$

where $\widetilde{C} = \frac{1}{v^2} + \frac{1}{\delta_2}$.

The inflation target ensuring that equilibrium inflation matches the government target, $\pi_2 = \widetilde{\pi}_{2,}$ is thus given by

$$\pi_2^T = \frac{C}{\widetilde{C}}\widetilde{\pi}_2 - \frac{1/\mu_1}{\widetilde{C}}[\widetilde{K}_2 + (1+\rho)d_1] \tag{A6}$$

Substituting this target into the *FOC*s produces the following equilibrium outcomes under the *IT* regime for period $t = 2$:

$$\pi_2 = \widetilde{\pi}_2 \tag{A7}$$

$$\widetilde{x}_2 - x_2 = \frac{1/v}{\widetilde{C}}[\widetilde{K}_2 + (1+\rho)d_1 - k\widetilde{\pi}_2] \tag{A8}$$

$$\widetilde{g}_2 - g_2 = \frac{1/\delta_2}{\widetilde{C}}[\widetilde{K}_2 + (1+\rho)d_1 - k\widetilde{\pi}_2] \tag{A9}$$

Substituting (A7)—(A9) into (1) and rearranging yields the welfare losses of the fiscal authority for period $t = 2$:

$$L_2^{FA} = \frac{1}{2\widetilde{C}}\left[\widetilde{K}_2 + (1+\rho)d_1 - k\widetilde{\pi}_2\right]^2$$

Solution in $t = 1$

First period Lagrangian of the fiscal authority can be written as

$$\mathcal{L}_1 = \frac{1}{2}\left[\delta_1(\pi_1 - \widetilde{\pi}_1)^2 + (x_1 - \widetilde{x}_1)^2 + \delta_2(g_1 - \widetilde{g}_1)^2\right] +$$

$$\lambda_1[g_1 - \tau_1 - k\pi_1 + (1 + \rho + +(\pi_1^e - \pi_1))d_0 - d_1] + \frac{1}{2\widetilde{C}}\beta_G\left[\widetilde{K}_2 + (1+\rho)d_1 - k\widetilde{\pi}_2\right]^2 \tag{A10}$$

The monetary authority attempts to minimize its loss function given by

$$L_1^{CB} = \frac{1}{2}\left[\mu_1(\pi_1 - \pi_1^T)^2 + (x_1 - \widetilde{x}_1)^2\right] \tag{A11}$$

Differentiating (A10) and (A11) w.r.to the policymaker's choice variables in $t = 1$ (i.e. π_1, τ_1, g_1 and d_1), then combining the resulting *FOC*s and the rational expectations condition

($\pi_1 = \pi_1^e$), yields the following expressions:

$$\pi_1 = \frac{v^2}{\mu_1}\left[\tau_1 + \frac{\widetilde{x}_1}{v}\right] + \pi_1^T \tag{A12}$$

$$\widetilde{g}_1 - g_1 = \frac{v^2}{\delta_2}\left[\tau_1 + \frac{\widetilde{x}_1}{v}\right] \tag{A13}$$

$$\left[\tau_1 + \frac{\widetilde{x}_1}{v}\right] = \beta_G^* \frac{1/v^2}{\widetilde{C}}\left[\widetilde{K}_2 + (1+\rho)d_1 - k\widetilde{\pi}_2\right] \tag{A14}$$

where $\beta_G^* = \beta_G(1+\rho)$. By substituting (A12)—(A14) into (A1) for $t = 1$ and solving for π_1, we obtain

$$\pi_1 = \frac{1/\mu_1}{C}[\widetilde{K}_1 + (1+\rho)d_0 - d_1] + \frac{\widetilde{C}}{C}\pi_1^T$$

The inflation target ensuring that equilibrium inflation matches the government target, $\pi_1 = \widetilde{\pi}_1$, is then given by

$$\pi_1^T = \frac{C}{\widetilde{C}}\widetilde{\pi}_1 - \frac{1/\mu_1}{\widetilde{C}}[\widetilde{K}_1 + (1+\rho)d_0 - d_1] \tag{A15}$$

Substituting this target into the set of *FOC*s under rational expectations ((A12)—(A14)) leads to the following equilibrium outcomes for $t = 1$:

$$\pi_1 = \widetilde{\pi}_1 \tag{A16}$$

$$\widetilde{x}_1 - x_1 = \frac{1/v}{\widetilde{C}}[\widetilde{K}_1 + (1+\rho)d_0 - d_1 - k\widetilde{\pi}_1] \tag{A17}$$

$$\widetilde{g}_1 - g_1 = \frac{1/\delta_2}{\widetilde{C}}[\widetilde{K}_1 + (1+\rho)d_0 - d_1 - k\widetilde{\pi}_1] \tag{A18}$$

$$\left[\widetilde{K}_1 + (1+\rho)d_0 - d_1 - k\widetilde{\pi}_1\right] = \beta_G^*\left[\widetilde{K}_2 + (1+\rho)d_1 - k\widetilde{\pi}_2\right] \tag{A19}$$

By solving (A19) for d_1 and re-arranging, we obtain the following expression

$$d_1 = \frac{\left[\left(\widetilde{K}_1 - k\widetilde{\pi}_1\right) + (1+\rho)d_0 - \left(\widetilde{K}_2 - k\widetilde{\pi}_2\right)\right] + (1-\beta_G^*)\left(\widetilde{K}_2 - k\widetilde{\pi}_2\right)}{\beta_G^*(1+\rho)}$$

This is the equilibrium debt level under *IT* presented in Table 1.