E C O N O M I C S   B U L L E T I N

Nonlinear Mean Reversion and Arbitrage in the Gold Futures Market

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Abstract

Previous literatures take transaction costs as being negligible when analyzing the futures basis behavior in linear dynamic framework. However, we argue that the relationship between the futures and spot prices with the conventional linear cointegration approach may not be appropriate after taking transaction costs into account. In this paper, an incorporation of transaction costs presented by Dumas (1992) and Michael (1997) into the exponential smooth transition autoregressive (ESTAR) model developed by Granger and Terasvita (1993) is motivated to examine the dynamic relationship between daily gold futures and spot prices and the nonlinear behavior of the gold futures basis. Transaction costs may lead to the existence of neutral band for futures market speculation within which profitable trading opportunities are impossible. Further, our results indicate that the ESTAR model provides higher forecasting power than the linear AR(1) model.

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1. Introduction

Recently, a large body of both theoretical and empirical research has been focusing on the dynamic equilibrium relationships between spot and futures prices of some financial derivatives. These studies were often motivated based on the effects of arbitrage in futures markets along the lines of models of the type developed by Garbade and Silber (1983), where traders induce movements in spot and futures prices such that the basis returns to a certain equilibrium level (e.g., see Kawaller, Koch, & Koch, 1987; Chan, 1992). However, an incorporation of transaction costs into the dynamic model done as in the papers by Dumas (1992) and Michael et al. (1997) may capture the nonlinear adjustment process for the basis series within and outside bands.

Threshold-type models of the type originally proposed by Tong (1990) are adopted to empirically characterize the behavior of the futures basis. These threshold models allow transactions costs to form bands within which no adjustments take place, so that deviations from the basis may display unit root behavior, while outside the band the process switches abruptly to become stationarily autoregressive. Dumas (1992) suggests that under certain restrictive conditions such as identical transactions costs, identical margin requirements and position limits, and homogeneity of agents, the jump to mean-reverting behavior will tend to smooth the transition between regimes. In examining nonlinear adjustments in real exchange rates, Michael et al. (1997) consider transaction costs in an exponential smooth transition autoregressive model and find strong evidence of mean-reverting behavior for PPP deviations.

Furthermore, some other studies on financial derivatives based on the cost-of-carry model have pointed out that there does exist the actual futures prices deviating from and even lower than their equilibrium prices, which is equivalently to assuming that the futures basis adjusts linearly toward its equilibrium value, but the existence of transaction costs makes it possible for the basis to adjust toward its equilibrium value nonlinearly rather than linearly. In investigating the dynamic behavior of the futures basis in stock index futures markets, Monoyios and Sarno (2002) conduct an empirical study which concentrates on the persistence of deviations of the futures basis from the equilibrium level implied by the cost-of-carry model, and find that nonzero transaction costs on trading the underlying asset of the futures contract may lead to the basis displaying a form of nonlinear mean reversion such that the basis becomes increasingly mean reverting with the size of the deviation from its equilibrium value. The reasoning behind nonlinear mean reversion of the basis is straightforward. Transactions costs create a band of no arbitrage for the basis, but the basis can stray beyond the thresholds. Once beyond the upper or lower threshold, the basis becomes increasingly mean reverting with the distance from the threshold. Within the transactions costs band, when no trade takes place, the process driving the basis is divergent since arbitrage is not profitable. Hence, smooth rather than discrete adjustment may be more appropriate in the presence of proportional transaction costs, and time aggregation and nonsynchronous adjustment by heterogeneous agents are likely to result in smooth aggregate regime switching.

Since the cost-of-carry model with transaction costs predicts that spot and futures prices co-move so that their long run equilibrium is defined by the futures basis, which implies that the basis is mean-reverting and that nonlinear models with transaction costs are more appropriate to characterize the equilibrium relationship between the futures price and the spot price than linear models. According to Monoyios and Sarno (2002), the basis should become increasingly mean reverting with the size of the deviation from the equilibrium level. Intuitively, several factors such as the existence of transaction costs and heterogeneity generate no arbitrage bands, thus implying that a law of motion for the basis is consistent with nonlinear adjustment toward equilibrium. Following the nonlinear models developed by Granger and Terasvirta (1993), Terasvirta(1994) along with transaction costs presented by Dumas (1992) and Michael et al. (1997), this paper examines the dynamic relationship between the futures price and spot price of gold and the nonlinear behavior of the gold futures basis.

The rest of the paper is structured as follows. Section 2 presents the methodology. Section 3 analyzes the empirical results containing evaluation of forecasting performance. Section 4 concludes remarks.

2. **Methodology**

The no-arbitrage condition between the futures and spot prices of the gold implied by the cost-of-carry model with zero transaction cost is

$$F_{t,T} = S_t \exp[(r-q)(T-t)] \tag{1}$$

where $F_{t,T}$ is the futures price of the gold underlying a futures contract at time t that expires at time T, S_t is the price at time t on the spot market for the same gold, r is the ratio of cost of carry to spot price, and q is the convenience yield. Alternatively, the conventional version of the cost of carry model can be written in logarithm form as:

$$F_t = \alpha + \beta S_t + b_t \tag{2}$$

where F_t is the log of futures price, S_t is the log of spot price, and b_t is a stationary error term which denotes the log of the futures basis under the restrictions $\alpha = 0$ and $\beta = 1$. The conventional view represents the cost of carry model to hold as long as b_t is stationary. Assume a linear process for b_t implies that the adjustment process is continuous with a constant speed of adjustment. However, Michael et al. (1997) indicates that the process of transaction costs implies that a nonlinear adjustment process of b_t has implications for the conventional linear cointegration test of the cost of carry model.

Granger and Terasvirta (1993) suggest that the non-linear adjustment process can be characterized based on a smooth transition autoregressive (hereafter STAR) model which is specified by:

$$b_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i b_{t-i} + F(b_{t-d})(\beta_0 + \sum_{i=1}^{p} \beta_i b_{t-i}) + \varepsilon_t \quad (3)$$

where ε_t is an independently and normally distributed random variable with a zero mean and constant variance σ^2, and $F(b_{t-d})$ is a transition function which, by convention, is bounded by zero and one. If the adjustment for a time series is smooth instead of jumping abruptly, then the STAR model seems to be the more attractive option than the TAR model in describing the non-linear adjustment of b_t.

In general, there are two different transition functions in $F(\cdot)$, the exponential function and the logistic function, so the STAR model can be distinguished in the exponential smooth transition autoregressive (ESTAR) model and the logistic smooth transition autoregressive (LSTAR) model :

$$b_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i b_{t-i} + (1 - \exp(b_{t-d} - c)^2)(\beta_0 + \sum_{i=1}^{p} \beta_i b_{t-i}) + \varepsilon_t \quad (4)$$

$$b_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i b_{t-i} + (1 + \exp(b_{t-d} - c)))^{-1}(\beta_0 + \sum_{i=1}^{p} \beta_i b_{t-i}) + \varepsilon_t \quad (5)$$

According to Michael et al. (1997), reparameterizing the STAR model (Eqs. (4) - (5)) gives：

$$\Delta b_t = \alpha_0 + \rho b_{t-1} + \sum_{i=1}^{p-1} \alpha_i \Delta b_{t-i} + F(b_{t-d})\{(\beta_0 + \rho^* b_{t-1} + \sum_{i=1}^{p-1} \beta_i \Delta b_{t-i})\} + \varepsilon_t \quad (6)$$

where $\Delta b_{t-j} = b_{t-j} - b_{t-j-1}$. In this form, the crucial parameters are ρ and ρ^*.

Monoyios (2002) further indicates that the incorporation of transaction costs suggests that the larger the deviation from the equilibrium value of the basis, the stronger will be the tendency to move back to equilibrium, thus implying that while $\rho \geq 0$ is admissible, one must have $\rho^{*} < 0$ and $(\rho + \rho^{*} < 0)$. That is, for small deviations the adjustment process may be characterized by unit root or even explosive behavior, but for large deviations the process is mean reverting.

Based on Granger and Teräsvirta (1993) and Teräsvirta (1994) to examine the model's appropriateness, it is required to estimate of the auxiliary regression:

$$y_t = \alpha + \sum_{j=1}^{p} (\beta_{0j} y_{t-j} + \beta_{1j} y_{t-j} y_{t-d} + \beta_{2j} y_{t-j} y_{t-d}^{2} + \beta_{3j} y_{t-j} y_{t-d}^{3}) + \varepsilon_t \qquad (7)$$

where y_t is the residual of the AR(p) model. The linearity test is $H_0 : \beta_{1j} = \beta_{2j} = \beta_{3j}$. The rejection of null hypothesis implies the appropriateness of a STAR specification in modeling y_t. To choose between the LSTAR and ESTAR models through a sequence of test of nested hypotheses, the sequence of hypotheses to be tested is as follows:

$$H_{01} : \beta_{3j} = 0 \qquad \text{(for all j=1,…,p)} \qquad (8a)$$

$$H_{02} : \beta_{2j} = 0 \,|\, \beta_{3j} = 0 \qquad \text{(for all j=1,…,p)} \qquad (8b)$$

$$H_{03} : \beta_{1j} = 0 \,|\, \beta_{2j} = \beta_{3j} = 0 \qquad \text{(for all j=1,…,p)} \qquad (8c)$$

Rejecting Eq.(8a) H_{01} implies selecting the STAR model. If we accept Eq. (8a) H_{01} and reject Eq. (8b) H_{02}, we choose the ESTAR model. Accepting Eq. (8a) H_{01} and Eq. (8b) H_{02} and rejecting Eq.(8b) H_{03} leads to the choice of the LSTAR model.

Lastly, to investigate the futures basis behavior of the cost-of-carry model with transaction costs, the restricted ESTAR model as suggested by Michael (1997) is employed and is thus estimated after successively testing and imposing the following hypotheses (Eqs. (10a) – (10c)) on the ESTAR model:

$$\Delta b_t = \alpha_0 + \rho b_{t-1} + \sum_{i=1}^{p-1} \alpha_i \Delta b_{t-i} + \{1 - \exp(-\gamma (b_{t-d} - c)^2)\}\{(\beta_0 + \rho^{*} b_{t-1} + \sum_{i=1}^{p-1} \beta_i \Delta b_{t-i})\} + \varepsilon_t \qquad (9)$$

$$H_0^{a} : \alpha_0 = \beta_0 = c = 0, \qquad (10a)$$

$$H_0^b: 1+\rho = -\rho^*,\ \alpha_i = -\beta_i \ \text{(for all i=1,…,p-1) given } H_0^a, \qquad (10b)$$

$$H_0^c: \rho = 0 \ \text{ given } H_0^a \text{ and } H_0^b \qquad (10c)$$

Since the series b_t are the mean-corrected deviations from equilibrium basis, we may reasonably expect that the ESTAR model satisfies the restrictions of $\alpha_0 = \beta_0 = c = 0$. Therefore, H_0^a is expected to hold. The implication of H_0^b and H_0^c is interesting. H_0^b implies that in the outer regime, when F(・) = 1, b_t is a white noise. However, H_0^c implies that when F(・) = 0, the process of b_t in the middle regime has a unit root. The likelihood ratio statistics LR^a, LR^b and LR^c are applied to test the null hypotheses of H_0^a, H_0^b and H_0^c, respectively.

3. Empirical Results

3.1 Linear Unit Root and Cointegration Tests

Our daily data on futures and spot closing prices of gold are collected from the Datastream. The observation period spans over 2001/1/1 through 2005/12/31 during which the data patterns on the spot and futures prices of the gold appear to move smoothly, and the shape of the estimated transition function of the futures basis of gold as shown in Fig. 1 matches the type of ESTAR model. In this paper, the log futures basis is calculated as the difference between the log spot price and the log futures price.

We now implement test for unit root behavior of each of the futures price, spot price and the futures basis time series for the gold by calculating standard augmented Dickey-Fuller (ADF) test statistics, reported in Table I. In keeping with the very large number of studies of unit root behavior for these time series and conventional finance theory, we are in each case unable to reject the unit root null hypothesis applied to each of the futures price and the spot price for both indices at the 5% level of significance. However, after differencing the two price series appear to induce stationarity in each case, clearly indicating that both F_t and S_t are realizations from stochastic processes integrated of order one. Furthermore, the results strongly suggest a rejection of the unit root of null hypothesis applied to b_t in levels as well as in differences, thus implying stationarity of the basis and possibly the existence of a cointegrating relationship between the futures price and the spot price for the gold.

Table I: Unit Root Tests (sample period : 2001/1/1~2005/12/31)

	$F_t^{(c)}$	$\Delta F_t^{(c)}$	$S_t^{(c)}$	$\Delta S_t^{(c)}$	$b_t^{(c)}$	$\Delta b_t^{(c)}$
Gold	-0.306	-37.825*	-0.231	-36.373*	-21.261*	-17.195*

Notes. F_t , S_t and B_t denote the log-level of the futures price, the log-level of the spot price, and the demeaned log-level of the basis, respectively. Statistics are augmented Dickey-Fuller test statistics for the null hypothesis of a unit root process; the (c) superscript indicates that a constant was included in the augmented Dickey-Fuller regression; Δ is the first-difference operator. The critical value at the 5% level of significance is -2.864.

To accomplish the analysis of the long-run properties of the data, we proceed with test for cointegration between F_t and S_t , employing the well-known Johansen(1988, 1991) maximum likelihood procedure in a vector autoregression comprising F_t and S_t. Both Johansen likelihood ratio (LR) test statistics based on the maximum eigenvalue and on the trace of the stochastic matrix, respectively, clearly suggest that a cointegration relationship exists for both price series under investigation. The results in Table II show that there exists a unique cointegrating vector such that the long-run equilibrium relationship between the futures price and the spot price for the gold exists, which implies the two price series for the gold co-move in the long run.

Table II: Johansen Maximum Likelihood Cointegration Procedure (sample period : 2001/1/1~2005/12/31)

		λ_{MAX}		*Trace*	
		LR	5% Critical Value	LR	5% Critical Value
Gold	v=0	200.454	14.265	200.566	15.495
(1 Cointegrating vector)	v=1	0.112*	3.841	0.112*	3.841

Notes：1. The numbers in parentheses are the 5% finite-sample critical values, as constructed from the asymptotic critical values from Osterwald-Lenum (1992) employing the method in Cheung and Lai (1993). 2. Term v indicates the number of cointegrating vectors. 3. Terms λ_{MAX} and $Trace$ are the maximum eigenvalue statistic and the trace statistic, respectively.

3.2 Linearity Tests and Specification of the Nonlinear Model

Table III summarizes the results of linearity tests, which shows that the null of linearity has been rejected, at standard significance levels, in favor of the ESTAR specification when employing the Granger and Teräsvirta (1993) and Teräsvirta (1994) procedure for examining the model's appropriateness.

Table III: Linearity Test Results and Specification of the Nonlinear Model

Futures basis	Lag (p)	Delay (d)	$H_{01}:\beta_{3j}=0$	$H_{02}:\beta_{2j}=0 \mid \beta_{3j}=0$	$H_{03}:\beta_{1j}=0 \mid \beta_{2j}=\beta_{3j}=0$	F stat / p lue	Type of model
Gold	2	1	0.079	0.009*	0.001	6.94 /0.000*	ESTAR

Note. The values for the nested test H_{01}, H_{02} and H_{03} are P-values. An asterisk indicates the lowest P-value for the three tests. The threshold value for the linearity and the specification of the STAR model is 0.05.

Table VI only reports the most parsimonious form of the estimated equations. Obviously, most of the estimated coefficients are significant at the 5% level, and the likelihood ratio statistics LR^a, LR^b and LR^c are applied to test the null hypothesis of H_0^a, H_0^b and H_0^c respectively. Results from these statistics unanimously indicate that the relevant restrictions are clearly supported by the data of gold, **which implies that when the equilibrium relationship between the futures price and spot price of gold exists, they fall within stochastic bands where the futures price seems to be mispriced and transaction costs may result in the existence of neural band for gold futures market speculation within which profitable trading opportunities are impossible. However, when the futures basis are outside neutral bands, the existence of arbitrage opportunities induces traders to buy long positions of gold futures contracts and sell short positions of spot contracts.** .

We are also interested in the behavior of the estimated residuals. The residual diagnostic tests are satisfactory apart from the failure of the normality test. In general, it is difficult to pass the normality test in nonlinear modeling as evidenced by Michael et al. (1997). The estimated transition function for the basis is plotted in Figure 1. It seems to be a reasonable number of observations above and below the equilibrium. We therefore are reasonably confident in our selection of the ESTAR model

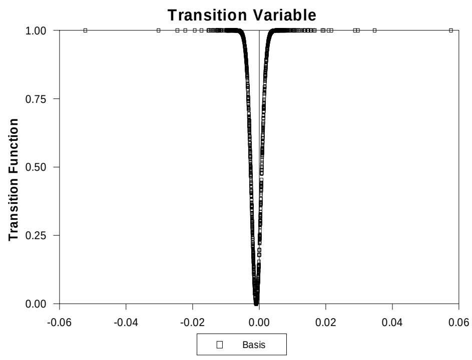
(Teräsvirta, 1994).

Table VI: The Estimates of the Restricted ESTAR Model

	Futures basis
ρ	-0.663 (0.000)*
β_1	-0.221 (0.000)*
γ	6.815 (0.116)
c	-0.001(0.0498)*
Diagnostic tests：	
JB	0.000*
Q(10)	0.341
ARCH(4)	0.032
LR^a	3.741
LR^b	2.136
LR^c	0.545

Note: 1.The number in parentheses is the standard deviation of the estimate. 2. LR^a , LR^b , LR^c are likelihood ratio test statistics corresponding to the tests of Eqs. (12a)-(12c)(with degrees of freedom three, p, and one), respectively. 3. JB, Q(n) and ARCH(n) are the Jarque-Bera normality test, the Ljung-Box autocorrelation test and the autoregressive conditional heteroscedasticity test of Engle(1982).

Fig. 1. Estimated Transition Function



3.3 Forecasting Performance

The results of our forecasts are presented in Table V. This table contains the root mean square error (RMSE) and mean absolute error (MAE) of the forecasts for the linear and nonlinear models. **Our results indicate that both values of RMSE and MAE from the ESTAR model are smaller than those from the linear models, and the ESTAR model clearly provides higher forecasting power than the linear AR(1) model.**

Table V: RMSE and MAE of the Forecasts for the Linear and Nonlinear Model (2006/1/1 ~ 2007/05/20)

	Linear model (AR(1))	Nonlinear model (ESTAR)
Futures basis of gold	Root mean squared error (RMSE)	
	0.010027	0.007708
	Mean absolute error (MAE)	
	0.007365	0.005673

4. Concluding Remarks

Previous studies on financial derivatives based on the cost-of-carry model have pointed out that there exist the actual futures prices deviating from and even lower than their equilibrium prices, and that the futures basis adjusts linearly toward its equilibrium value. However, the existence of transaction costs makes it possible for the basis to adjust toward its equilibrium value nonlinearly rather than linearly. Following the nonlinear ESTAR model developed by Granger and Terasvirta(1993), Terasvirta(1994) along with Dumas (1992) and Michael et al. (1997), this paper finds the nonlinearly cointegrating relation between the futures price and spot price of gold, which indicates that they are within stochastic bands where the futures price seems to be mispriced. However, the existence of transaction costs tends to eliminate profitable arbitrage opportunities, but there exists the tendency for market traders to arbitrage to restore to equilibrium levels when the basis falls outside the band. In evaluating out-of-sample forecasting performance, this paper also finds that the ESTAR model provides higher forecasting power than the linear model.

References

Chan, K. (1992). “A Further Analysis of the Lead–lag Relationship between the Cash Market and Stock Index Futures Market,” Review of Financial Studies, 5, 123–52.

Chen, S. and J. Wu (2000). ”A Re-Examination of Purchasing Power Parity in Japan and Taiwan,” ***Journal of Macroeconomics*****, 22,271-284.**

Dumas, B. (1992). “Dynamic Equilibrium and the Real Exchange Rate in a Spatially Separated World,” Review of Financial Studies, 5, 2, 153-180.

Garbade, K. D., & Silber, W. L. (1983). “Price Movements and Price Discovery in Futures and Cash markets,” Review of Economics and Statistics, 65, 289–297.

Granger, C. W. J. and Terävirta, T.（1993）, Modeling Nonlinear Economic Relationships, Oxford University Press.

Johansen, S. (1988). “Statistical Analysis of Cointegrating Vectors,” Journal of Economic Dynamics and Control, 12, 231–254.

Johansen, S. (1991). “Estimation and Hypothesis Testing of Cointegrating Vectors in Gaussian Vector Autoregressive Models,” Econometrica, 59, 1551–1580.

Kawaller, I. G., Koch, P. D., & Koch, T.-W. (1987). “The Temporal Price Relationship between S&P 500 Futures and the S&P 500 Index,” Journal of Finance, 42, 1309–29.

Michael, P., Nobay, A. R., & Peel, D. A. (1997). “Transactions Costs and Nonlinear Adjustment in Real Exchange Rates: An Empirical Investigation,” Journal of Political Economy, 105, 862–879.

Monoyios , M. & L. Sarno (2002). “Mean Reversion in Stock Index Futures Markets: A Nonlinear Analysis,” Journal of Futures Markets, 22, 4, 285-314.

Terasvirta, T. (1994). “Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models,” Journal of the American Statistical Association, 89. 208-18.

Terasvirta, T. and H. M. Anderson (1992). “Characterizing Nonlinearities in Business

Cycles Using Smooth Transition Autoregressive Models," Journal of Applied Econometrics, 7, 119-139.

Tong, H. (1990). Nonlinear Time Series: A Dynamical System Approach. Oxford: Clarendon Press.

Van Dijk, D., Terasvirta, T. and Franses, P.H. (2002), "Smooth Transition Autoregressive Models－A Survey of Recent Developments," Econometrics Reviews 21, 1-47.