E c o n o m i c s B u l l e t i n

On the bias of yield-based capital budgeting methods

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Abstract

The aim of this paper is twofold. First, we present a new capital budgeting method, called the real rate of return (RRR), which has been developed for solving the inconsistency of the modified internal rate of return (MIRR) with shareholders' wealth maximization when costs of capital differ between projects. After surveying the merits of this method over the MIRR, we focus our attention on another interesting feature of the RRR when cash flows are uncertain. We compare the RRR bias with the MIRR bias and demonstrate that the RRR bias is inferior to the MIRR bias. This theoretical finding confirms once again that the RRR is a better capital budgeting method than the MIRR. Knowing that managers exhibit in practice a large preference for comparing the merits of projects with rates of return, this simple and flexible yield-based capital budgeting method has all the qualities to be accepted in practice.

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1 Introduction

Recent capital budgeting surveys indicate that yield-based capital budgeting techniques are widely used in practice by managers (see among others: Arnold and Hatzopoulos 2000, Graham and Harvey 2001, Brounen et al. 2004, Meier and Tarhan 2007). In the United States, for example, the internal rate of return (IRR) is the managers' preferred method for project evaluation before the net present value (NPV)[1]. This managers' preference for yield-based criteria has two explanations: habit and risk perception. The world of business and finance has always thought with rate-based measures: bank loans, financial investment, annual sales growth, etc. As a result, to summarize and compare the merits of a project, it is from a cognitive point of view more natural for managers to communicate with percentages (IRR) rather than amount of money (NPV) or with an index (PI). In addition, managers like to be able to compare the project's expected rate of return with the opportunity cost of capital because this gives them an assessment of the degree of risks they take by accepting the project.

From an academic point of view, this finding is surprising because it is a clearly established fact that the IRR suffers from many drawbacks (multiple IRR and problems of ranking) which could lead firms to bad decisions when capital is unlimited or rationed[2]. Therefore, corporate finance textbooks recommend that managers should use, in place of the IRR, the NPV when the capital is unlimited and the profitability index (PI) when the capital is rationed[3] (see for instance, Fabozzi and Peterson 2003, Ross et al. 2005, Brealey et al. 2006).

Among the many improvements of the IRR, the modified internal rate of return (MIRR) (Lin 1976, Mc Daniel et al. 1988) gives a unique solution and has many qualities explaining its introduction in finance textbooks and its adoption by a large number of companies. Unfortunately, when costs of capital differ between projects, the MIRR does not rank projects consistently with the PI and the NPV[4]. This drawback severely limits the application fields of the MIRR because, in practice, costs of capital are very likely to be different between projects. Indeed, the cost of capital is calculated from the relative weights of each component of the company's capital struc-

[1]The IRR is the "first choice" capital budgeting technique in the survey of Meier and Tarhan (2007), and is the first "always or almost always used" technique according to the questionnaire of Graham and Harvey (2001). In Europe, the IRR is also popular but, according to the country, this method and the NPV are only at the second or third place after the payback period rule (simple payback period, not the discounted payback period).

[2]In this paper, we consider only now or never decisions (a rejected project is not realizable in the future) and ignore the existence of embedded real options.

[3]Notice that the choice of the optimal group of projects is not always easy to determine with the PI. This arises when capital is rationed over more than one period, or when other resources than capital (employees, production capacity, etc.) are also rationed, or even when projects are not independent for some other reasons. In such cases, the optimal group of projects is obtained with tedious trial and error or with dynamic programming.

[4]Notice that this inconsistency with the NPV also arises when there is no scale difference between projects, situation for which the PI is consistent with the NPV.

ture (shares of debt and equity), from the firm's market risk (or from division's market risk) and from the risk of the project. Thus, in a firm, costs of capital of evaluated projects are likely to be different because of the adjustment of the weighted average cost of capital (WACC) to the project's risk. In an investment bank, the probability that costs of capital differ between projects is all the more important since companies wishing to be funded have in principle different WACC.

Despite its importance in practice, this problem was to our knowledge only highlighted by Fabozzi and Peterson (2003) and resolved by Rousse (2008) with the development of a new yield-based capital budgeting technique. This method, called the real rate of return (RRR), starts from the idea that managers need to compare the MIRR of competing projects with respect to their cost of capital. For that, Rousse (2008) applies the Fisher equation (1907, 1930) to this capital budgeting issue. Considering that the cost of capital is the expected inflation rate of the project, i.e. the expected growth in percentage of the expected average value of investments belonging to the same risk class as the project, and that the MIRR is the nominal rate of return on the project, the Fisher equation allows us to calculate the real rate of return (RRR) of the project[5]:

$$(1 + MIRR) = (1 + RRR)(1 + r) \text{ or } RRR = \frac{(1 + MIRR)}{(1 + r)} - 1$$

The decision rule of the RRR is to accept (reject) all projects with a positive (negative) RRR. Between two mutually exclusive projects, the manager must choose the one with the largest RRR.

From an economic point of view and due to its theoretical construct, the RRR corresponds to the rate of return adjusted for the minimum acceptable expected rate of return on a project given its risk. In other words, the RRR is the rate of return at which the firm's value increases in real terms. The RRR differs from the IRR and the MIRR because the latter do not directly express how shareholders' wealth will really increase compared to an investment of the same risk class. The original aspect of the RRR is that it provides managers the same signal as the PI and the NPV: a positive RRR means directly (without comparison) that the return is more than enough to compensate for the opportunity cost of capital. Above all, the RRR is a better investment criterion than other yield-based methods (IRR and its variants) because it gives the same projects ranking as the one obtained with the PI[6]. This property is obtained by showing that the RRR can be calculated

[5]In a trivial sense, we can say that the RRR gives for each project a measure of the rate of return above the cost of capital. On this latter point, notice that the RRR can be easily approximate by subtracting the cost of capital to the MIRR: $RRR \approx MIRR - r$. These approximation works best when both the MIRR and the cost of capital are small. For higher values of the MIRR and the cost of capital, the approximation error (e) becomes large: $e = RRR \times r$ with RRR and r expressed as decimals.

[6]Notice also that the ranking of equal size projects from the RRR or the NPV is the same; the scale problem not being able to be resolved with a logical rate of return (except with the incremental approach of Fisher).

either from the MIRR or from the PI. Indeed, the RRR and the PI verify the following equation:

$$IP = (1 + RRR)^T \text{ or } RRR = PI^{1/T} - 1$$

In other words, the RRR is the annual rate of return at which one euro has to be compounded for the T periods of the project lifetime in order to get the PI. Thus, the RRR maximizes shareholders' wealth when capital is rationed and can be used in place of the PI as a complementary tool to the NPV.

The aim of this paper is to show another interesting feature of the RRR over the MIRR giving managers another good reason to use the RRR as the first choice yield-based investment criterion. Following Anderson and Barber (1994) who have previously found that the MIRR overstates a project's expected random variable MIRR when cash flows are uncertain, we demonstrate that the RRR bias is inferior to the MIRR bias.

2 A comparison of the MIRR and the RRR bias

Consider a project with a life of T periods ($t = 0, \ldots, T$), a cost of capital r and an initial investment I_0 followed by a stream of random cash flows $\tilde{F}_1, \ldots, \tilde{F}_T$. The MIRR and the RRR are the rate of return defined in terms of the expected cash flows:

$$MIRR = \left[\frac{1}{I_0}\sum_{t=1}^{T} E\left(\tilde{F}_t\right)(1+r)^{T-t}\right]^{1/T} - 1 \tag{1}$$

$$RRR = \frac{1}{(1+r)}\left[\frac{1}{I_0}\sum_{t=1}^{T} E\left(\tilde{F}_t\right)(1+r)^{T-t}\right]^{1/T} - 1 \tag{2}$$

The random variable MIRR and the random variable RRR on a project may be written as follows:

$$\widetilde{MIRR} = \left[\frac{1}{I_0}\sum_{t=1}^{T} \tilde{F}_t (1+r)^{T-t}\right]^{1/T} - 1 \tag{3}$$

$$\widetilde{RRR} = \frac{1}{(1+r)}\left[\frac{1}{I_0}\sum_{t=1}^{T} \tilde{F}_t (1+r)^{T-t}\right]^{1/T} - 1 \tag{4}$$

Or after simplifying:

$$\widetilde{MIRR} = (1+r)\left[\frac{1}{I_0}\sum_{t=1}^{T} \tilde{F}_t (1+r)^{-t}\right]^{1/T} - 1 \tag{5}$$

$$\widetilde{RRR} = \left[\frac{1}{I_0} \sum_{t=1}^{T} \tilde{F}_t \left(1+r\right)^{-t} \right]^{1/T} - 1 \tag{6}$$

When cash flows are certain, it is obvious that equations (1) and (3) are equivalent and that equations (2) and (4) are also equivalent.

When cash flows are uncertain, we demonstrate that the MIRR exceeds the expected random variable MIRR and that the RRR exceeds the expected random variable RRR. Consider f and g two functions of $\tilde{F}_1, \ldots, \tilde{F}_t$ with:

$$\widetilde{MIRR} = \ f\left(\tilde{F}_1, \ldots, \tilde{F}_t\right) \text{ and } MIRR = f\left[E\left(\tilde{F}_1\right), \ldots, E\left(\tilde{F}_t\right)\right]$$

$$\widetilde{RRR} = \ g\left(\tilde{F}_1, \ldots, \tilde{F}_T\right) \text{ and } RRR = g\left[E\left(\tilde{F}_1\right), \ldots, E\left(\tilde{F}_T\right)\right]$$

Because f and g are concave functions (root functions), then by Jensen's inequality the following inequalities are verified:

$$E\left[f\left(\tilde{F}_1, \ldots, \tilde{F}_T\right)\right] < f\left[E\left(\tilde{F}_1\right), \ldots, E\left(\tilde{F}_T\right)\right]$$

$$E\left[g\left(\tilde{F}_1, \ldots, \tilde{F}_T\right)\right] < g\left[E\left(\tilde{F}_1\right), \ldots, E\left(\tilde{F}_T\right)\right]$$

Thus, we conclude that:

$$E\left(\widetilde{MIRR}\right) < \ MIRR \text{ and } E\left(\widetilde{RRR}\right) < \ RRR$$

In order to determine if the RRR is a better capital budgeting method than the MIRR when cash flows are uncertain, we have to approximate and compare the MIRR and the RRR bias. For that, we express equations (5) and (6) in terms of the present value of the stream of random cash flows at the cost of capital, $\tilde{P} = \sum_{t=1}^{T} \tilde{F}_t \left(1+r\right)^{-t}$, and we use Taylor series expansions to approximate about the expected value of $\tilde{P}$, denoted $\bar{P}$:

$$\widetilde{MIRR} = u\left(\tilde{P}\right) = \left(1+r\right)\left(\frac{\tilde{P}}{I_0}\right)^{1/T} - 1$$

$$\widetilde{RRR} = v\left(\tilde{P}\right) = \left(\frac{\tilde{P}}{I_0}\right)^{1/T} - 1$$

Thus we have:

$$\widetilde{MIRR} = u\left(\bar{P}\right) + u'\left(\bar{P}\right)\left(\tilde{P} - \bar{P}\right) + \frac{1}{2}u''\left(\bar{P}\right)\left(\tilde{P} - \bar{P}\right)^2 + \ldots + \frac{1}{n!}u^n\left(\bar{P}\right)\left(\tilde{P} - \bar{P}\right)^n + R_n\left(\tilde{P}\right) \tag{7}$$

$$\widetilde{RRR} = v\left(\bar{P}\right) + v'\left(\bar{P}\right)\left(\tilde{P} - \bar{P}\right) + \frac{1}{2}v''\left(\bar{P}\right)\left(\tilde{P} - \bar{P}\right)^2 + \ldots + \frac{1}{n!}v^n\left(\bar{P}\right)\left(\tilde{P} - \bar{P}\right)^n + S_n\left(\tilde{P}\right) \tag{8}$$

where $u^n\left(\bar{P}\right)$ and $v^n\left(\bar{P}\right)$ are the n^{th} derivatives of $u\left(.\right)$ and $v\left(.\right)$ evaluated at $\bar{P}$; and $R_n\left(\tilde{P}\right)$ and $S_n\left(\tilde{P}\right)$ are the remainder terms.

As the expected value of $\tilde{P}$ is $\bar{P} = \sum_{t=1}^{T} E\left(\tilde{F}_t\right)(1+r)^{-t}$, then $u\left(\bar{P}\right) = MIRR$ and $v\left(\bar{P}\right) = RRR$. Consider that σ_n^n is the n^{th} moment of $\tilde{P}$ about $\bar{P}$ ($\sigma_2^2 = E\left(\tilde{P} - \bar{P}\right)^2$ and $\sigma_3^3 = E\left(\tilde{P} - \bar{P}\right)^3$ are respectively the variance and skewness of $\tilde{P}$), the expectations of equations (7) and (8) can be expressed as follows:

$$E\left(\widetilde{MIRR}\right) = MIRR + \frac{1}{2}u''\left(\bar{P}\right)\sigma_2^2 + \frac{1}{6}u'''\left(\bar{P}\right)\sigma_3^3 + \ldots + \frac{1}{n!}u^n\left(\bar{P}\right)\sigma_n^n + R_n\left(\tilde{P}\right)$$

$$E\left(\widetilde{RRR}\right) = RRR + \frac{1}{2}v''\left(\bar{P}\right)\sigma_2^2 + \frac{1}{6}v'''\left(\bar{P}\right)\sigma_3^3 + \ldots + \frac{1}{n!}v^n\left(\bar{P}\right)\sigma_n^n + S_n\left(\tilde{P}\right)$$

By differentiating $u\left(.\right)$ and $v\left(.\right)$, we find that $u^n\left(\bar{P}\right) = (1+r)\,v^n\left(\bar{P}\right)$. So, the MIRR and RRR bias may be written as follows:

$$MIRR - E\left(\widetilde{MIRR}\right) = -\frac{1}{2}(1+r)\,v''\left(\bar{P}\right)\sigma_2^2 - \frac{1}{6}(1+r)\,v'''\left(\bar{P}\right)\sigma_3^3 - \ldots - \frac{1}{n!}(1+r)\,v^n\left(\bar{P}\right)\sigma_n^n - R_n\left(\tilde{P}\right) \tag{9}$$

$$RRR - E\left(\widetilde{RRR}\right) = -\frac{1}{2}v''\left(\bar{P}\right)\sigma_2^2 - \frac{1}{6}v'''\left(\bar{P}\right)\sigma_3^3 - \ldots - \frac{1}{n!}v^n\left(\bar{P}\right)\sigma_n^n - S_n\left(\tilde{P}\right) \tag{10}$$

Factoring equation (9) by $(1+r)$ and neglecting the remainder terms, we find:

$$MIRR - E\left(\widetilde{MIRR}\right) = (1+r)\left[RRR - E\left(\widetilde{RRR}\right)\right]$$

Thus we can conclude that the MIRR bias is $(1+r)$ times higher than the RRR bias.

To give a simple approximation of the expected random variable MIRR and expected random variable RRR, we neglect for the sake of simplicity the third and higher terms in equations (9) and (10). As Anderson and Barber (1994) have previously stated for the MIRR case, these approximations are accurate if the variance of the distributions are finite and if higher moments are not extreme. Differentiating twice $v\left(.\right)$, we have:

$$v''\left(\bar{V}\right) = -\frac{(T-1)}{T^2}\left(\frac{1}{I_0}\right)^{1/T}\left(\bar{V}\right)^{1/T-2} \tag{11}$$

Substituting (11) into equations (9) and (10) and rearranging terms, we obtain these following approximations of the MIRR and RRR bias:

$$MIRR - E\left(\widetilde{MIRR}\right) = (1+r)\,\frac{(T-1)}{2T^2}CV^2PI^{1/T}$$

$$RRR - E\left(\widetilde{RRR}\right) = \frac{(T-1)}{2T^2}CV^2PI^{1/T}$$

where $CV = \frac{\sigma_2}{\bar{V}}$ is the coefficient of variation (scaled standard deviation) of the present value of future cash flows and $PI = \frac{\bar{P}}{I_0}$ is the profitability index.

3 Conclusion

Developed to solve the MIRR failure to maximize the value of the firm when costs of capital differ between projects, the RRR is an improvement of the MIRR and also a variant of the PI. In addition to maximize the value of the firm, the RRR combines many advantages and so has a good chance to be accepted by managers in practice. Indeed, it is an easy to calculate and interpret criterion that satisfies managers' needs, habits and way of thinking. It is also a flexible criterion in its use because managers can adapt it to many practices and situations: different treatments of positive and negative cash flows, different borrowing and reinvestment rates, replacement projects, risk studies as sensitivity analysis and Monte Carlo simulations, etc. Finally, we show in this paper that when cash flows are uncertain, the RRR is once again a better method than the MIRR because the RRR bias is inferior to the MIRR bias. This property may be important for post-audit because when comparing the project's results with expectations, auditors do not necessarily take into account this bias and rely solely on rates of return defined in terms of the expected cash flows. As post-audit is one of the most important elements in a good capital budgeting system, using a less biased investment criterion is preferable to design better capital budgeting programs.

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