E C O N O M I C S B U L L E T I N

On the design of an optimal transfer schedule with time inconsistent preferences

Juan Carlos Chavez-Martin del Campo
Universidad de Guanajuato and Centro de Estudios de las Finanzas Publicas

Abstract

This paper incorporates the phenomenon of time inconsistency into the problem of designing an optimal transfer schedule under income shocks and asymmetric information. The optimal solution reflects the dilemma that a principal has to face when playing the roles of self-control enforcer and consumption smoother simultaneously.

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1. Introduction

Economic theories of intertemporal choice generally assume that individuals discount the future exponentially. In other words, the choices made between today and tomorrow should be no different from the choices made between the days 200 and 201 from now, all else equal. Mainstream public economics is not the exception. Particularly, in the context of designing optimal transfer schemes, exponential discounting is usually assumed(Shavell and Weiss, 1979; Thomas and Worrall, 1990; Brito et al., 1991; Hopenhayn and Nicolini, 1997).

However, experimental evidence suggests that many individuals have preferences that reverse as the date of decision making nears (Lowenstein and Thaler, 1989; Ainslie, 1992). Moreover, there exists field evidence of present-biased preferences and time inconsistent behavior (Angeletos et al., 2001; DellaVigna and Malmendier, 2003; Fang and Silverman, 2007). Some empirical studies present evidence of how consumption is often very sensitive to an income transfer in the very short-run (Stephens, 2002, 2003), and even indicate a declining caloric intake over the 30-day period following the receipt of food stamps in the US (Shapiro, 2005) with the policy implication of increasing the frequency of payments to improve welfare. Similar results have been found in developing countries, where commitment devices are used to face the overconsumption problem (Rutheford, 1999; Ashraf et al., 2003, 2006).

If individuals tend to overconsume and are subject to economic shocks that are not publicly observed, how should a transfer schedule be allocated over time? Is it optimal to commit to a transfer schedule before income shocks are realized? Is it preferable to give individuals full control over the budget to be distributed? Fully committing to a transfer schedule is unlikely to be a good idea when income shocks are yet to be realized. In fact, if individuals do not show present-bias, it is in the individual's best interest to access a contingent transfer scheme. On the other hand, if individuals face an over-consumption problem, it might not be optimal to give them total discretionality over the intertemporal budget allocation.

We formally approach these questions within a dynamic principal-agent framework under one-sided asymmetric information, hyperbolic discounting, and income shocks. The agent takes consumption decisions over time and suffers from present-biased preferences. The principal's goal is to allocate an exogenous budget in order to maximize the agent's "long-run"welfare. Since the income shocks are assumed to be not publicly observed, the agent has an incentive to underreport income realizations because of his bias for present consumption. Therefore, not only the principal should consider his role as a consumption smoother, but also as a self-control enforcer who helps the agent face his overconsumption problem. The solution we found represents the existent tradeoff between these two roles.

2. The model

Consider the following economy. Time is discrete and indexed by $t = 1, 2, \ldots, T$. There is one agent and one principal. There is one consumption good x. The agent's preferences belong to the class of constant absolute risk aversion (CARA), and are represented by the following instantaneous utility function

$$u(x_t) = -\exp(-\alpha x_t) \tag{1}$$

where $\alpha > 0$ denotes the coefficient of absolute risk aversion.

In period t, preferences over consumption streams $x = (x_1, \ldots, x_T) \in \mathbb{R}^T_+$ are representable by the utility function

$$U_t(x) = E_t[u(x_t) + \beta \sum_{\tau=t+1}^{T} \delta^{\tau-t} u(x_\tau)] \tag{2}$$

where $(\beta, \delta) \in (0, 1] \times (0, 1]$. The principal has access to a riskless asset with gross return normalized to one. The agent has no access to capital markets. In other words, the agent cannot save nor borrow.

The type of preferences represented by this model incorporates the so-called quasi-geometric discounting. The parameter δ is called the *standard discount factor* and it represents the long-run, time consistent discounting; the parameter β represents a preference for immediate gratification and is known as the *present-biased factor*. For $\beta = 1$ these preferences reduce to exponential discounting. For $\beta < 1$, the (β, δ) formulation implies discount rates that decline as the discounted event is moved further away in time.[1]

We introduce uncertainty into the model by assuming that income, y_t, is independently and identically distributed over time with probability distribution

$$y_t = \begin{cases} y_l & \text{with probability } \gamma \\ y_h & \text{with probability } 1 - \gamma \end{cases}$$

where $y_h > y_l$. We say that the beneficiary receives a negative income shock at time t if $y_t = y_l$. Analogously, we say the beneficiary receives a positive income shock at time t if $y_t = y_h$. Let E_t be the expectation operator conditional on all information available at t, and let $E(-u(y_t)) = \mu < \infty$.

In contrast to a naive or partially naive person who believes he will behave like a time-consistent individual in the future, we assume that the agent is sophisticated in the sense that he is fully aware of his time inconsistency problem. We formally model the agent as a sequence of T autonomous selves making choices in a dynamic game (Strotz, 1956; Phelps and Pollak, 1968; Peleg and Yaari, 1973; Goldman, 1980;

[1] See Frederick et al. (2002), for review of the (β, δ) formulation and its relation to hyperbolic discounting.

Laibson, 1997, 1998; O'Donoghue and Rabin, 1993, 2001). The principal allocates an exogenous budget $B \in \mathbb{R}$ through a transfer schedule $\{\tau_t\}_{t=1}^{T}$.[2] Transfers are allocated over time in order to maximize the "long-run"welfare of the agent represented by the function

$$W_t(x) = E_t[u(x_t) + \sum_{\tau=t+1}^{T} \delta^{\tau-t} u(x_\tau)] \tag{3}$$

O'Donoghue and Rabin (1999) argue that this approach involves the existence of a "...(fictitious) period 0 where the person has no decision to make and weights all future periods equally."This approach incorporates the fact that most models of present-biased preferences try to capture situations in which people pursue immediate gratification. This type of analysis, where the principal has an objective function that is different from that of the agent, is not new in public economics. There is a long tradition of non-welfarist welfare economics where the outcomes of individual behavior are evaluated using a preference function different from the one that generate the outcomes (Kanbur et al., 2006).

It is assumed that income realizations are not public information. Therefore, the first-best allocation of resources is impeded by the problem of incentive compatibility, since an agent with income realization $y_t = y_l$ and relatively strong degree of present-biased preferences might have an incentive to claim a negative income shock in order to receive a higher transfer at period t.

3. Optimal transfer schedule

Based on the revelation principle, the principal can restrict attention to direct revelation mechanisms with the property that the agent truthfully reports his true income y_t.

For any period t, let τ_t^h represent the transfer when the agent reports a positive income shock, and τ_t^l represent the transfer when he reports a negative income shock. In period $T-1$, the policymaker solves the problem

$$\max_{\tau_l^t, \tau_t^h} \gamma[u(\tau_{T-1}^l + y_l) + \delta E_{T-1} u(B_T^l + y_T)] + (1-\gamma)[u(\tau_{T-1}^h + y_h) + \delta E_{T-1} u(B_T^h + y_T)]$$

subject to the following incentive-compatibility and resource constraints

$$\begin{aligned} u(\tau_{T-1}^l + y_l) + \beta\delta E_{T-1} u(B_T^l + y_T) &\geq u(\tau_{T-1}^h + y_l) + \beta\delta E_{T-1} u(B_T^h + y_T) \\ u(\tau_{T-1}^h + y_h) + \beta\delta E_{T-1} u(B_T^h + y_T) &\geq u(\tau_{T-1}^l + y_h) + \beta\delta E_{T-1} u(B_T^l + y_T) \\ \tau_{T-1}^l + B_T^l &\leq B_{T-1} \\ \tau_{T-1}^h + B_T^h &\leq B_{T-1} \end{aligned}$$

[2]Following a tradition in the public economics literature, we set aside the revenue-raising implications to finance this budget.

Define by $v_{T-1}(B_{T-1})$ the value function of this problem. By standard arguments, $v_{T-1}(B_{T-1})$ is strictly concave and differentiable.

Next, take any period t and suppose $v_{t+1}(\cdot)$ is strictly concave and differentiable. Although the principal and the agent disagree on the amount of discounting applied between t and $t+1$, they both agree on the utility obtained from $t+1$ on. Therefore, by applying a standard induction argument, we have that for all t the principal solves the problem:

$$\max_{\tau_l,\tau_h} \gamma[u(\tau_t^l + y_l) + \delta v_{t+1}(B_{t+1}^l)] + (1-\gamma)[u(\tau_t^h + y_h) + \delta v_{t+1}(B_{t+1}^h)]$$

subject to the following incentive compatible and budget constraints:

$$\begin{aligned} u(\tau_t^l + y_l) + \beta\delta v_{t+1}(B_{t+1}^l) &\geq u(\tau_t^h + y_l) + \beta\delta v_{t+1}(B_{t+1}^h) \\ u(\tau_t^h + y_h) + \beta\delta v_{t+1}(B_{t+1}^h) &\geq u(\tau_t^l + y_h) + \beta\delta v_{t+1}(B_{t+1}^l) \\ \tau_t^l + B_{t+1}^l &\leq B_t \\ \tau_t^h + B_{t+1}^h &\leq B_t \end{aligned}$$

The following proposition characterizes the optimal transfer scheme:

Proposition 1 *Let $\beta^* = \exp(-\alpha(y_h - y_l))$. The optimal transfer scheme is characterized as follows*

1. $\tau_h = \tau_l$ *if* $\beta \leq \beta^*$
2. $\tau_h < \tau_l$ *if* $\beta > \beta^*$
3. $\tau_l - \tau_h$ *is not decreasing in* β

Proof: See Appendix.

Proposition 1 establishes that if the agent's self-control problem, parameterized by β, is relatively more serious than his risk problem, parameterized by α and the income shock range $y_h - y_l$, then the principal optimally opts for an equilibrium where the transfer is independent of the value taken by the income shock. Intuitively, if the degree of present-bias is too high, the principal's optimal response is to offer a non-contingent transfer schedule. This is equivalent to committing to a transfer schedule before the realizations of income shocks, implying that the value of information is zero when the agent shows a strong bias for present consumption.

4. Conclusions

Our analysis has several implications and possible extensions. Firstly, since we are assuming a sophisticated agent, we could use other types of commitment devices. For instance, the agent could be provided with an illiquid instrument a la Laibson (1997) or he could choose a contingent transfer schedule from a menu. If he is aware of his self-control problem, the final consumption allocation would be the best from a current perspective; secondly, the paper could be extended to incorporate preferences outside the neighborhood of constant absolute risk aversion; thirdly, it could be assumed that income shocks are not i.i.d., following instead another type of random process; finally, one could introduce naivete into the model and design an optimal mechanism that takes into account the possibility of facing a mixture of sophisticated and naive individuals within the population.

Ultimately, the most important message from this paper is that an optimal transfer scheme should consider the existent tradeoff between facing the beneficiary's bias for present consumption and his exposure to income shocks. In other words, the principal should offer a package flexibly enough to balance both consumption smoothing and commitment motives.

5. Appendix

Proof of Proposition 1 Let $\varphi_l = -u(y_l)$ and $\varphi_h = -u(y_h)$. We make the following change of variables: instead of having τ_l and τ_h as our decision variables, let the decision variables be $u_h = u(\tau_h)$, $u'_h = v(B^h)$, $u_l = u(\tau_l)$ and $u'_l = v(B^l)$. The program becomes

$$\max \gamma[\varphi_l u_l + \delta u'_l] + (1-\gamma)[\varphi_h u_h + \delta u'_h]$$
$$\text{s.t.}$$

$$\begin{aligned}
\varphi_l u_l + \beta\delta u'_l - \varphi_l u_h - \beta\delta u'_h &\geq 0 \\
\varphi_h u_h + \beta\delta u'_h - \varphi_h u_l - \beta\delta u'_l &\geq 0 \\
B - V_1(u_l) - V_2(u'_l) &\geq 0 \\
B - V_1(u_h) - V_2(u'_h) &\geq 0
\end{aligned}$$

where V_1 and V_2 are the inverse functions of $u(\cdot)$ and $v(\cdot)$, respectively. Because $u(\cdot)$ and $v(\cdot)$ are concave functions, V_1 and V_2 are convex and, in consequence, the incentive compatible and borrowing constraints are concave. Since the objective function is linear, $u^* = (u_h, u'_h, u_l, u'_l)$ is a solution to the program if and only if there is $\lambda^* = (\lambda^*_1, \lambda^*_2, \lambda^*_3, \lambda^*_4) \in R^4_+$, such that the Kuhn-Tucker conditions are satisfied. In particular, the Kuhn-Tucker first order conditions are

$$\gamma\varphi_l + \lambda_1\varphi_l - \lambda_2\varphi_h = \frac{\partial V_1(u_l)}{\partial u_l}\lambda_3 \tag{4}$$

$$\gamma + \lambda_1\beta - \lambda_2\beta = \delta^{-1}\frac{\partial V_2(u_l')}{\partial u_l'}\lambda_3 \tag{5}$$

$$(1-\gamma)\varphi_h - \lambda_1\varphi_l + \lambda_2\varphi_h = \frac{\partial V_1(u_h)}{\partial u_h}\lambda_4 \tag{6}$$

$$(1-\gamma) - \lambda_1\beta + \lambda_2\beta = \delta^{-1}\frac{\partial V_2(u_h')}{\partial u_h'}\lambda_4 \tag{7}$$

In a pooling equilibrium, the first order conditions imply

$$\lambda_2 = \frac{\gamma(\mu - \varphi_l)}{\mu\beta - \varphi_h} + \frac{\mu\beta - \varphi_l}{\mu\beta - \varphi_h}\lambda_1 \tag{8}$$

From where it can be concluded that the necessary and sufficient condition for having a positive λ_2 for any $\lambda_1 \geq 0$ is $\beta < \varphi_h/\mu$. Moreover, positive λ_3 and λ_4 are obtained if and only if the following condition holds

$$-(1-\gamma) + \frac{\varphi_l}{\varphi_h}\lambda_1 \leq \lambda_2 \leq \min\{\frac{\varphi_l}{\varphi_h} + \frac{\varphi_l}{\varphi_h}\lambda_1, \frac{\gamma}{\beta} + \lambda_1\} \tag{9}$$

It is not difficult to show that conditions (8) and (9) are satisfied for positive λ_1 and λ_2 if and only if $\beta \leq \varphi_h^{-1}\varphi_L$. Therefore, a separating equilibrium exists if $\beta > \varphi_h^{-1}\varphi_L$. From the incentive compatible constraints it follows that $\tau_l > \tau_h$. Finally, in a separating equilibrium the incentive compatible constraint holds with strict inequality when $y_t = y_l$, hence $\lambda_2 \geq \lambda_1$. That $\tau_l - \tau_h$ is weakly increasing in β follows from (5), (7), and the convexity of V_2. $\square$

References

Ainslie, G. (1992). *Picoeconomics.* Cambridge University Press.

Angeletos, G.-M., D. Laibson, A. Repetto, J. Tobacman, and S. Weinberg (2001). The hyperbolic consumption model: Calibration, simulation, and empirical evaluation. *Journal of Economic Perspectives 15*(3), 47–68.

Ashraf, N., N. Gons, D. Karlan, and W. Yin (2003). A review of commitment savings products in developing countries. Asian Development Bank Economics and Research Department Working Paper, 45.

Ashraf, N., D. Karlan, and W. Yin (2006). Tying Odysseus to the Mast: Evidence from a Commitment Savings Product in the Philippines*. *Quarterly Journal of Economics 121*(2), 635–672.

Brito, D. L., J. H. Hamilton, S. M. Slutsky, and J. E. Stiglitz (1991). Dynamic Optimal Income Taxation with Government Commitment. *Journal of Public Economics 44*, 15–35.

DellaVigna, S. and U. Malmendier (2003). Overestimating self-control: Evidence from the health-club industry. mimeo, University of California Berkeley.

Fang, H. and D. Silverman (2007). Measuring time-inconsistency: Evidence from work-welfare decisions in the nsly. mimeo, Yale University.

Frederick, S., G. Loewenstein, and T. O'Donoghue (2002). Time discounting and time preference: a critical review. *Journal of Economic Literature 40*, 351–401.

Goldman, S. (1980). Consistent plans. *Review of Economic Studies 47*, 533–401.

Hopenhayn, H. and J. Nicolini (1997). Optimal Unemployment Insurance. *Journal of Political Economy 105*(2), 412.

Kanbur, R., J. Pirttila, and M. Tuomala (2006). Non-welfarist optimal taxation and behavioural public economics. *Journal of Economic Surveys 20*(5), 849–868.

Laibson, D. (1997). Hyperbolic discounting and golden eggs. *Quarterly Journal of Economics 112*(2), 443–447.

Laibson, D. (1998). Life cycle consumption and hyperbolic discount functions. *European Economic Review 42*, 861–871.

Lowenstein, G. and R. Thaler (1989). Anomalies: Intertemporal choice. *Journal of Economic Perspectives 3*, 181–193.

O'Donoghue, T. and M. Rabin (1993). Doing it now or later. *American Economic Review 89*(1), 103–124.

O'Donoghue, T. and M. Rabin (2001). Choice and procrastination. *Quarterly Journal of Economics 116*(1), 121–160.

Peleg, B. and M. Yaari (1973). On the existence of a consistent course of action when tastes are changing. *Review of Economic Studies 40*, 391–401.

Phelps, E. and R. Pollak (1968). On second-best national saving and game-equilibrium growth. *Review of Economic Studies 35*(1), 185–199.

Rutheford, S. (1999). Savings and the poor: The methods, use and impact of savings by the poor of east africa. Report prepared for MicroSave-Africa.

Shapiro, J. M. (2005). Is there a daily discount rate? evidence from the food stamp nutrition cycle. *Journal of Public Economics 89*, 303–325.

Shavell, S. and L. Weiss (1979). The optimal payment of unemployment insurance benefits over time. *Journal of Political Economy 87*, 1347–1362.

Stephens, M. (2002). Paycheck receipt and the timing of consumption. NBER, working paper 9356.

Stephens, M. (2003). 3rd of the month: do social security recipients smooth consumption between checks? *American Economic Review 93*, 406–422.

Strotz, R. H. (1956). Myopia and inconsistency in dynamic utility maximization. *Review of Economic Studies 23*(3), 165–180.

Thomas, J. and T. Worrall (1990). Income Fluctuation and Asymmetric Information: An Example of a Repeated Principal-agent Problem. *Journal of Economic Theory 51*(2), 367–390.