E C O N O M I C S   B U L L E T I N

Is it beneficial for households without children to subsidy the cost of rearing children to increase pension benefits?

Masatoshi Jinno
Nagoya university

Abstract

This paper analyzes the effects of child allowance on households without a child with respect to the pay-as-you-go public pension system. We demonstrate that the child allowance can improve the utilities of households without a child through an increased pension benefits when the rate of households raising the number of children is sufficiently low.

Citation: Jinno, Masatoshi, (2008) "Is it beneficial for households without children to subsidy the cost of rearing children to increase pension benefits?." *Economics Bulletin*, Vol. 8, No. 9 pp. 1-7
Submitted: August 29, 2008. **Accepted:** October 1, 2008.
URL: http://economicsbulletin.vanderbilt.edu/2008/volume8/EB-08H50015A.pdf

1. Introduction

In recent times, the aging of the population combined with the diminishing number of children has become a serious problem faced by advanced countries. What causes a decline in the number of children? Cigno (1993) and Groezen, Leers, and Meijdam (G-L-M) (2003) show that there are two opposite externalities that children have under an economy where the pay-as-you-go pension system exists. An additional child increases the future output, and thus, the pension is beneficial; however, it also causes a decline in the capital-labor ratio. Households that do not consider these externalities tend to have fewer than the optimal number of children. G-L-M and Groezen and Meijdam (2008) present the necessity of subsidizing the cost of child rearing in order to attain the command optimum.

Households, while not considering the social importance of having a child in an economy where the pay-as-you-go pension system exists, tend to have fewer children or no children because the pay-as-you-go public pension system provides insurance for old age. This free-riding behavior on the public pension system results in the decline in the number of children. G-L-M and Groezen and Meijdam both stress the need for child allowance; however, they assume that households are representative and homogeneous. If all households had children, their utilities would rise relatively easily since an increase in the rate of child allowance decreases the cost of child rearing and the burden of households directly decreases and the child allowance indirectly increases the pension benefits.

However, there are some households who will not have any children even if child allowance decreases the cost of child rearing. We will therefore analyze whether the utilities of such households rise or not since the child allowance increasing the tax rate only becomes a direct burden for them, even if it does indirectly increase pension benefit. We will present the condition that indirect benefits from the child allowance system exceed direct burdens for households without a child.

The remainder of this paper is organized as follows. The model is presented in section 2, and section 3 summarizes the discussion and presents issues for further study.

2. The Model

This section develops an overlapping generational model wherein two types of household coexist. The first type has positive savings and zero children and the second type has positive savings and some children. The utility of the former type depends on lifetime consumption only while that of the latter type depends on life time consumption and number of children. Households are characterized by their preferences to having children. The type of household is determined by its own nature. The duration of a household can be categorized into three periods: youth, middle age, and old age. We term the current generation as the t^{th} generation; households are born in period $t-1$, they work, save, and have some or no children in period t, and retire to live on saving returns and pensions in period $t+1$. During middle age, households inelastically supply one unit of labor. The government adopts the pay-as-you-go public pension system and the child allowance system.

We assume that there are only two types of typical household in an economy: (1) those that have positive savings and zero children and (2) those that have positive savings and some children. The former type of household is M_t^1,while the latter one is M_t^2. They earn the same wage rate.

The ratio of the second type of household to the t[th] generation is $\hat{p}$, which is constant and given[1]. Thus, we have

$$M_t^1 = (1-\hat{p})N_t \text{, and} \quad (1\text{-a})$$

$$M_t^2 = \hat{p}N_t . \quad (1\text{-b})$$

Here, N_t is the population of t[th] generation.

When the number of children selected for the first (or second) type of household in period t is $n_t^1 (n_t^2)$, the upper index indicates the type of household, and the generation in period t grows as follows:

$$N_{t+1} = \bar{n}_t N_t = n_t^1 (1-\hat{p})N_t + n_t^2 \hat{p}N_t = n_t^2 \hat{p}N_t . \quad (2)$$

Thus, equation (2) implies that $\bar{n}_t = n_t^2 \hat{p}$.

For analytical convenience, we assume the utility function to be additively separable and logarithmic, and the first type of household has the utility function:

$$U(c_{t1}^1, c_{t2}^1) = a_1 \ln(c_{t1}^1) + a_2 \ln(c_{t2}^1), \quad (3a)$$

while the second type of households has the utility function:

$$U(c_{t1}^2, c_{t2}^2, n_t^2) = a_1 \ln(c_{t1}^2) + a_2 \ln(c_{t2}^2) + a_3 \ln(n_t^2) \quad (3b)$$

where $a_j \, (j = 1 \sim 3)$ are the utility weights of the middle age consumption $c_{t1}^i \, (i = 1,2)$, old age consumption c_{t2}^i, and number of children. We assume that $a_j > 0$ and $a_1 + a_2 + a_3 = 1$.

The government imposes a proportional and a lump-sum tax ϕw_t on households for transfers to the elderly as a pay-as-you-go public pension premium and T_t for child allowance. ϕ is the constant pension premium rate and w_t is the wage rate. The public pension benefit depends on the average number of children in period t + 1. Due to the pay-as-you-go pension system and the child allowance system, the budget constraints of the first type become

$$(1-\phi)w_t - T_t = c_{t1}^1 + s_t^1 \text{, and} \quad (4a)$$

$$\left(1+r_{t+1}\right)s_t^1+\eta_{t+1}=c_{t2}^1\,, \tag{5}$$

where T_t is the lump-sum tax, s_t^i the savings, r_{t+1} the interest rate, and η_{t+1} the public pension benefit which is the same for all households. The middle-age budget constraint of the second type becomes

$$\left(1-\phi\right)w_t-T_t=c_{t1}^2+s_t^2+n_t^2b_t\left(1-\tau_t\right) \tag{4b}$$

where b_t is the cost of rearing a child, and τ_t is a subsidy rate per child. The last-age budget constraint is the same as equation (5). Equations (4a) and (4b) express the disposable income when middle age is divided among consumption, savings, and child rearing (in the case of second type households). Equation (5) expresses the return on middle-age savings and the pension benefits are spent on old-age consumption.

The government runs the public pension system denoted by the following budget constraint per household,

$$\eta_{t+1}=\bar{n}_t\phi w_{t+1}\,, \tag{6}$$

If households consider the investment aspect of having children, that is, the public pension premiums they pay, equation (6) should be included in the maximization problem. However, because we assume that children only bring happiness to households and that the effect of changing the number of children is limited to the extent where it can be ignored with respect to the public pension benefit, this paper does not include equation (6) in the problem (see, for example, Groezen *et al.* 2003).

The government lowers the cost of child rearing by a subsidy of $b_t\tau_t^c$ per child. To finance the child allowance or the subsidy for savings, the government imposes the lump-sum tax $\left(T_t\right)$. The budget constraint per household for this system is

$$T_t=\bar{n}_tb_t\tau_t\,. \tag{7}$$

In each period t, the Cobb-Douglas technology is employed for production using two inputs: physical capital K_t and labor N_t; $Y_t=K_t^\alpha N_t^\beta$ such that we have $\left(1+r_t\right)=\alpha Y_t/K_t$ and $w_t=\beta Y_t/N_t$. Perfect competition in the factor markets ensures that, in equilibrium, the return from savings and the wage per labor unit are equal to the respective marginal products. In each period, the capital stock is the result of the households' savings in the preceding period.

$$K_{t+1}=\sum s_t^i=\bar{s}_tN_t\,. \tag{8}$$

The capital stock lasts only for one period and attains zero scrap value in the subsequent period. The initial capital stock (K_0), which belongs to N_{-1} households, is given to those who are old in period 0. Each of them own $s_{-1} = K_0 / N_{-1}$. Using the perfect competition in the factor markets and equations (2), (7) and (8), the following relation is formed:

$$1 + r_{t+1} = \frac{\alpha}{\beta} \frac{\bar{n}_t}{\bar{s}_t} w_{t+1} . \tag{9}$$

A household born in period t – 1 maximizes utility (3) subject to the budget constraints (4) and (5). The household's optimal choices are characterized by the first-order condition:

$$\frac{-a_1}{c_{t1}^i} + \frac{a_2 (1 + r_{t+1})}{c_{t2}^i} = 0 \tag{10a}$$

$$\frac{-a_1 b_t (1 - \tau_t)}{c_{t1}^2} + \frac{a_3}{n_t^2} = 0 . \tag{10b}$$

Equation (10a) indicates that households equate the marginal rate of substitution between current and future consumptions to the rate of return from savings. This is common for all households. The optimal condition (10b) can be interpreted as follows. The marginal disutility of children, resulting from a decrease in consumption during middle age, is equal to the direct marginal benefit of having children. This applies to second type households only.

By substituting equations (3) and (7) into equations (10), we have the optimal savings and the optimal number of children as follows:

$$s_t^1 = \frac{a_2 (1 + r_{t+1}) ((1 - \phi) w_t - T_t) - a_1 \eta_{t+1}}{(a_1 + a_2)(1 + r_{t+1})} , \tag{11a}$$

$$s_t^2 = \frac{a_2 (1 + r_{t+1}) ((1 - \phi) w_t - T_t) - (a_1 + a_3) \eta_{t+1}}{(1 + r_{t+1})} , \tag{11b}$$

$$n_t^2 = a_3 \frac{(1 + r_{t+1}) ((1 - \phi) w_t - T_t) + \eta_{t+1}}{(1 + r_{t+1})(1 - \tau) b} . \tag{12}$$

By using equations (1), (6), (9), (11) and (12), the optimal average savings and number of children become

$$\bar{s}_t = \frac{\alpha a_2 z_1}{z_2} \{(1 - \phi) w_t - T_t\}, \text{ and} \tag{13a}$$

$$\bar{n}_t = \frac{a_2 z_1 \beta \phi + z_2}{(1 - \tau) b z_2} \hat{p} a_3 \{(1 - \phi) w_t - T_t\}, \tag{13b}$$

where $z_1 \equiv (a_1 + a_2)\hat{p} + (1-\hat{p})$ and $z_2 \equiv \beta\phi(a_1 + a_2 a_3 \hat{p}) + \alpha(a_1 + a_2)$. According to equation (13), the capital-labor ratio, the wage rate and the interest rate in period t+1 depend on the subsidy rate of child allowance in period t:

$$k_{t+1} = \frac{\alpha(1-\tau_t)ba_2 z_1}{\hat{p}a_3(a_2 z_1 \beta\phi + z_2)}, \tag{14}$$

$$w_{t+1} = \beta(k_{t+1})^{\alpha} \tag{15}$$

$$r_{t+1} = \alpha(k_{t+1})^{\alpha-1} - 1 \tag{16}$$

We assume that the government imposes a lump-sum tax T_t on households in order to subsidize the cost of child rearing. By substituting equation (13b) into equation (7), the lump-sum tax rate becomes

$$T_t = \tau_t a_3 \hat{p}(1-\phi)w_t \frac{a_2 z_1 \beta\phi + z_2}{a_3 \hat{p}(a_2 z_1 \beta\phi + z_2)\tau_t + (1-\tau_t)z_2}. \tag{17}$$

We investigate the effect of child allowance through the pension system on the first type of household. Substituting equations (11a), (14)~(17) into the utility function leads to the indirect utility function:

$$U(c_{t1}^1, c_{t2}^1) = a_1 \ln\left(a_1 \frac{a_2 z_1 \beta\phi + z_2}{z_2(a_1 + a_2)} W_t\right)$$
$$+ a_2 \ln\left(\left(\frac{a_2 z_1 \beta\phi + z_2}{(a_1 + a_2)\beta}\right)(\alpha a_2)^{\alpha} \frac{\beta}{z_2}\left(\frac{bz_1}{a_3 \hat{p}(a_2 z_1 \beta\phi + z_2)}\right)^{\alpha-1} W_t\right) \tag{18}$$

where $W_t \equiv \dfrac{(1-\tau_t)(1-\phi)w_t z_2}{a_3 \hat{p}(a_2 z_1 \beta\phi + z_2)\tau_t + z_2(1-\tau_t)}$. Differentiating it with respect to τ_t and evaluating it with $\tau_t = 0$, we obtain

$$\left.\frac{dU(c_{t1}^2, c_{t2}^2)}{d\tau_t}\right|_{\tau_t=0} = \frac{-1}{z_2}\left[a_3 \beta\phi\left\{2a_1 a_2 + (a_1)^2 + \alpha(a_1)^2\right\} + \alpha(a_1 + a_2)^2 a_3\right]\hat{p}$$
$$+ \frac{1}{z_2}(1-\alpha)a_2\{a_1 \beta\phi + \alpha(a_1 + a_2)\}. \tag{19}$$

The first term in equation (18) denotes the direct effect of an increase in the lump-sum tax. The second term denotes the indirect effect of an increase in child allowance in the pension system. Child allowance results in an increase in the number of children. Further, it increases the number of workers in the next period but decreases the wage rate. An increase in the number of workers implies a rise in those supporting the system, which in turn implies a rise in pension benefits. A decline in the wage rate implies a decrease in the contributions because the premium rate is constant. Thus, an increase in child allowance has ambiguous effects on the second type of household in the pension system.

The sign of equation (19) depends on $\hat{p}$

$$\left.\frac{dU\left(c_{t1}^{2},c_{t2}^{2}\right)}{d\tau_{t}}\right|_{\tau_t=0} \gtrless 0,$$

$$\text{if} \quad \hat{p} \lessgtr \left[\frac{(1-a)a_2}{a_3}\right]\left[\frac{a_1(\alpha+\beta\phi)+a_2\alpha}{a_1(a_1+2a_2)(\alpha+\beta\phi)+a_2{}^2\alpha(1+\beta\phi)}\right]. \qquad (20)$$

Equation (20) implies that unless the rate of households with some children is small, the utility of households without a child does not rise even if there is child allowance. The higher the rate of households with children, the higher the burden of child allowance. Thus, child allowance does not increase the utility of households without a child through an increase in pension benefits when the rate of households with children is sufficiently high.

Proposition

Child allowance can improve the utility of households without a child through the pay-as-you-go pension system in an economy where the rate of households with children is sufficiently small.

3. Conclusions and Remaining Issues

This paper assumes the case that there are households without a child and analyzes the effects of the child allowance system on such households through an increase in pension benefits. With regard to the returns from an uncontrolled pension system, this paper shows that child allowance increases the utility of households without a child when the rate of households with some children is sufficiently small because of increased pension returns.

When considering an endogenous fertility rate, the relationship between the cost of raising a child and that of education is also very important. Thus, it is important to take into account the relationship between the number of children, savings, the pension system, and investment in education. It remains to be seen whether child allowance or a subsidy for education investment more effectively improves the utility of households without a child through increased pension benefits.

Endnotes

[1] An increase in child allowance offers households without a child an incentive to have one because it lowers the cost of raising a child. This aspect of child allowance is of some importance when the government considers the optimal child allowance rate; however, this has not been considered in this

paper. This paper focuses on whether child allowance improves the utility of the last household that has no child even if the cost of child rearing is lowered.